The Study of Magnetic Flux Shunts Effects on the Leakage Reactance of Transformers via FEM

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ABSTRACT:
The influence of arrangement, dimensions, and magnetic permeability of the magnetic flux shunts on the flux distribution and leakage reactance of the power transformers is studied in this paper by using a finite elements method and a simple modeling approach. By using magneto-static analysis and finite element method, first the flux distribution in the 2D model of a core-type three phase power transformer and then using the magnetic stored energy method the leakage reactance of the transformer windings is calculated. By studying the different models including magnetic flux shunts, the effect of the arrangement, geometric dimensions as well as the magnetic permeability of the magnetic flux shunt on the leakage reactance of the transformer are studied and some interesting results are obtained. It is shown that the variation of these parameters in the transformer model has significant effects on the leakage reactance of the transformer.

KEYWORDS: FEM, Leakage Reactance, Magnetic Shunt, Modeling, Power Transformer.

1. INTRODUCTION

The leakage reactance is one of the most important parameters of a transformer that must be exactly calculated for the design stage before making it. This is done in order to model and study the performance characteristics of the transformer so that the performance of the transformer will be ensured under normal and unavoidable abnormal conditions such as the short-circuit fault of the transformer terminal. To calculate leakage reactance using FEM, the proper modeling and post-processing operation are of great importance. In [1] because of the symmetry of the transformer model, only half of it was modeled that lead to less complexity and time needed for calculation compared with considering the whole model of the three phase transformer.

In this paper, a simple modeling was used in order to use energy storage post-processing method to calculate the leakage reactance by using FEM. In order to use energy storage post-processing method, just one half of the transformer core window was modeled and exact results were obtained. First, the given model was discretized and then by using magneto-static analysis, the magnetic potential of the model nodes was calculated, and then the flux distribution over the model was obtained. Then, in the post-processing stage, by using the energy storage method, the leakage reactance of the transformer windings was calculated.

2. LEAKAGE REACTANCE CALCULATION

2.1. Defining a Proper Model

The transformer that was considered in this study is a 30MVA, (63/20) KV, primary winding star-connected with neutral grounded and the secondary winding delta-connected (YnD) three-phase core-type power transformer which its HV winding has 480 turns with the nominal current of 275 A and its LV winding has 264 turns and the nominal current of 500 A.

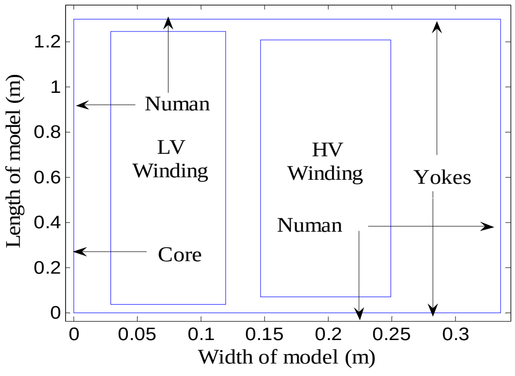

Fig. 1. Transformer model for energy method

To calculate the leakage reactance by using the energy storage method, a model was designed considering the half core window shown in Fig.1.

2.2. Required Equations for Analysis

To solve the problem of the leakage flux distribution and to calculate the leakage reactance, the partial differential equations in the problem are obtained by using Maxwell rules and natural relations between the magnetic field intensity and magnetic flux density as follows [1]:

$$\nabla \times (\upsilon . \nabla \times A) = J \qquad (1)$$

Where: υ is the inverse of the magnetic permeability (μ), J is the current density, and A is the magnetic vector potential.

For the 2D models in x-y plane, the non-zero component of A is the z component of the magnetic potential which is a function of x and y only. Therefore, (1) takes the following scalar form:

$$\frac{\partial}{\partial x}(\upsilon \frac{\partial A}{\partial x}) + \frac{\partial}{\partial y}(\upsilon \frac{\partial A}{\partial y}) = J_S \qquad (2)$$

By solving (2) using FEM on the defined model, the magnetic vector potential and therefore the magnetic flux density can be obtained. Having the magnetic flux density on the defined model, the stored energy and the leakage reactance of the transformer can be easily calculated. Moreover, the stored magnetic energy can be obtained after calculating the magnetic potential of the model nodes.

2.3. Discretization of the Defined Model and Calculating the Flux Distribution

After defining a proper model for the transformer, the model is discretized by triangular finite elements. The internal magnetic potential of each triangle is considered as a first order linear function. For example, if the magnetic potential of the nodes of a triangular element are A1, A2, and A3 respectively, then the approximating function of magnetic potential for each triangular element will be as the following:

$$A = N_1 A_1 + N_2 A_2 + N_3 A_3 \qquad (3)$$

As the above approximating function is different from the exact answer to each triangular element, so by substituting this function in the obtained final partial differential equation, there will be a remaining. By using Galerkin method from weighted residual methods, a series of matrix relations is obtained. After forming the matrix functions for each of the triangular elements, all the elements will be added to each other and a final function is formed. Then the related boundary conditions will be applied and the final matrix function will be modified.

Finally after solving the modified matrix function, the magnetic potential on each node will be estimated. The magnetic potential in each triangular element is a linear function of x and y. In this case, the flux distribution can be calculated over the defined model. The flux density can also be obtained in different parts of the model.

2.4. Reactance Calculation Using the Magnetic Energy Storage Method

In order to obtain the leakage flux pattern and then calculate the leakage reactance of the transformer using this method, it is required to consider a proper model.

2.4.1. The Appropriate Model for This Method

If one of the lateral limbs of a three phase core-type transformer is considered, then the stored energy in the internal space of the transformer window will be a little more than the stored energy in the external space of the core window. This occurs because of the core effect on the leakage flux [4]. However, it is shown that this difference is so little that the core effects can be neglected [4]. Therefore, considering this fact in calculating the leakage reactance using the energy storage method, the modeling of the core window of the transformer will be sufficient. Besides, by applying symmetry effects, the modeling can be limited to half of the core window. In the model defined for the calculation of the leakage reactance of the transformer under study presented in Fig.1, the homogeneous Numan boundary condition was applied to all the boundaries of the model. Using this type of boundary condition in the up, down, and also left boundary of the model is reasonable, because the flux lines enter these boundaries vertically because of the high magnetic permeability of the limbs and yokes of the transformer.

After defining the model, by utilizing the partial differential equation toolbox of the MATLAB software, the model was discretized and then by a magneto-static analysis, the magnetic potential as well as the flux distribution on the given model was obtained as presented in Fig.2.

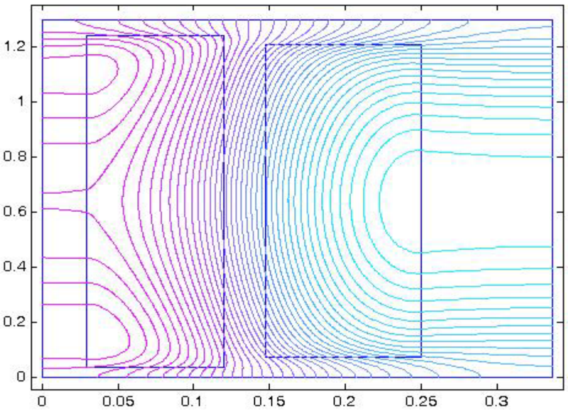

Fig. 2. Flux pattern obtained for the defined model.

The flux in the space between the two windings is mainly axial and the flux deviation occurs at the end of the windings, and the radial component of leakage flux appears. Moreover, since the magnetic centers of the HV and LV windings don’t coincide, in the central areas of the LV winding, the flux deviation and therefore the radial component of the leakage flux density can be seen.

The axial and radial components of the leakage flux density can be obtained simply from the following equations:

$$z \qquad (4)$$

$$B_y = \frac{\partial A}{\partial x} \qquad (5)$$

$$B = \sqrt{B_x^2 + B_y^2} \qquad (6)$$

2.4.2. Required Relations for Reactance Calculation

Once the absolute value of B is obtained for each element, the magnetic energy stored in the window space can be calculated by,

$$W = \frac{1}{2} d.\iint B.H.dxdy \qquad (7)$$

$$W = \frac{1}{2} d.\iint J.A.dxdy \qquad (8)$$

Where: d is the depth of the defined model.

To calculate the stored energy using (7), the integration should be done on the whole model, while if we use (8), the integration is only applied on the conductive parts of the current or on the windings.

Once the magnetic energy is calculated, the leakage reactance of transformer for each phase referred to the primary side is calculated by,

$$X_l = \frac{(4 \times \pi \times f \times W)}{i_{p1}^2 + i_{s1}'^2} \qquad (9)$$

Where: X_l is the leakage reactance of the transformer referred to the primary side and f is the supply frequency. i_{p1} is the instantaneous current of one phase of the primary winding and i_{s1}' is the instantaneous current of the same phase of the secondary winding reflected to the primary winding.

3. ANALYSIS OF THE EFFECTS OF THE MAGNETIC SHUNTS ON THE LEAKAGE REACTANCE

In order to do this analysis, first a proper model must be defined for the transformer, and then using the finite elements method, the leakage flux distribution must be obtained. The flux pattern is obtained once in the absence of the magnetic flux shunt and the again in the presence of the magnetic shunt. After that, using the energy storage method, the leakage reactance of each model is calculated.

The flux distribution for the defined model for the transformer being studied in the case of the absence of the magnetic flux shunt is obtained as shown in Fig.2. The leakage reactance of this model using energy storage method is estimated as 12.2874 percent.

3.1. The Effects of the Magnetic Shunt Position on the Leakage Reactance

The magnetic shunt considered for this analysis, has a rectangular cuboid shape with 5 mm height, 220 mm length and its relative magnetic permeability is 2000. In this case, by assuming that the dimension and permeability were constant and that only the position of the shunt was changed, the related effects were studied.

3.1.1. Model-1

In this model, the distance of magnetic shunt from yokes is 5 mm. The leakage reactance of this model using energy storage method is calculated to be 12.6197 percent. The flux distribution for this model is obtained as presented in Fig.3.

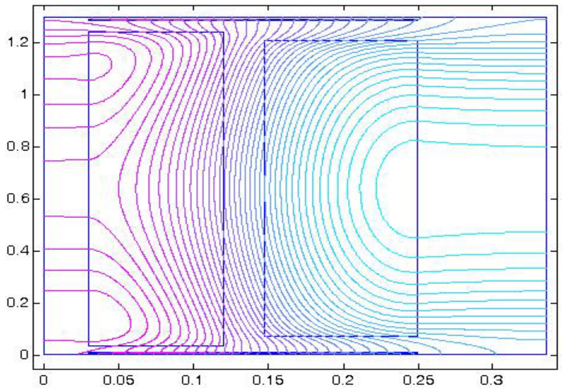

Fig. 3. Flux pattern obtained over model-1.

3.1.2. Other Models

In addition to the above model, some other models have also been considered which in each model the distance of the magnetic shunt from transformer yokes is increased by 2 mm in compare to the previous model. The obtained results are given in table-1.

3.2. The Effects of the Magnetic Shunt Thickness on the Leakage Reactance

To evaluate the effects of the magnetic flux shunt height on the leakage reactance, the position of the magnetic shunt in the model is considered constant at 10 mm from yokes and the thickness of the shunt varies between 1 to 9 mm and for each case, the flux pattern and the leakage reactance are calculated.

3.2.1. Model-12

In this model, the height of the shunt is 1 mm. The leakage reactance of this model is calculated to be 13.8117%. The flux pattern for this model is obtained as shown in Fig.4.

Table. 1. The effect of the shunt position on the reactance

Model No.	Distance of the shunt from yokes (mm)	Leakage reactance (%)
1	5	12.6197
2	7	12.6995
3	9	12.7663
4	11	12.8383
5	13	12.9162
6	15	13.0010
7	17	13.0939
8	19	13.1948
9	21	13.3048
10	23	13.4250
11	25	13.5558

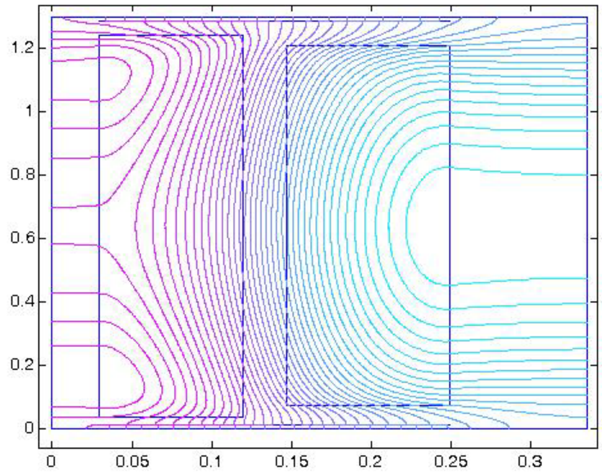

Fig. 4. Flux pattern obtained over model-12.

3.2.2. Other Models

In addition, some other models have also been considered and in each model, the height of the shunt is increased by 1 mm. The obtained results are given in table 2.

Table 2. The effects of the shunt height on the reactance

Model No.	Height of the shunt (mm)	Leakage reactance (%)
12	1	13.8117
13	2	13.2018
14	3	12.9764
15	4	12.8646
16	5	12.8015
17	6	12.7645
18	7	12.7422
19	8	12.7305
20	9	12.7255

3.3. The Effects of the Magnetic Shunt Length on the Leakage Reactance

To study the effects of the magnetic flux shunt length on the leakage reactance, the position of the shunt in the model is considered constant (10 mm from yokes) with the height as 5 mm. The length of the shunt is changed between 60 to 220 mm and for each case, leakage reactance is calculated.

3.3.1. Model-21

In this model, the length of the magnetic shunt is 220 mm. leakage reactance of this model is calculated to be 12.8015 percent using energy storage method. The flux distribution for this model is obtained as shown in Fig.5.

3.3.2. Other Models

In addition to the above model, some other models have also been considered and in each model, the length of the magnetic shunt is decreased by 20 mm in compare to the previous model. The obtained results are given in table 3.

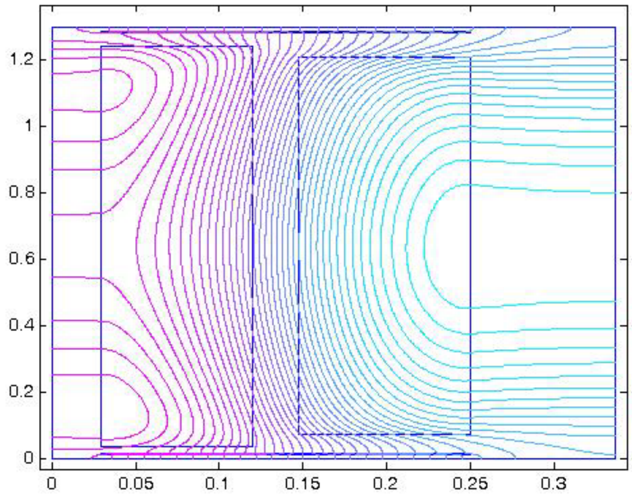

Fig. 5. Flux pattern obtained over model-21.

Table 3. The effects of the shunt length on the reactance

Model No.	Length of the shunt (mm)	Leakage reactance (%)
21	220	12.8015
22	200	12.6345
23	180	12.5176
24	160	12.4399
25	140	12.3911
26	120	12.3627
27	100	12.3458
28	80	12.3348
29	60	12.3225

3.4. The Effects of Magnetic Permeability of the Magnetic Shunt on the Leakage Reactance

To evaluate the effects of the magnetic flux shunt permeability on the leakage reactance, the position of the shunt in the model is considered constant (10 mm from yokes) with the height of 5 mm and length of 220 mm. The relative magnetic permeability of the shunt is variable and changes between 1000 and 2000. Now for each case, the leakage reactance is calculated.

3.4.1. Model-30

In this model, the relative permeability of the magnetic shunt is 1000. Leakage reactance of this model is calculated as 12.7811 percent. The flux distribution for this model is obtained as shown in Fig.6.

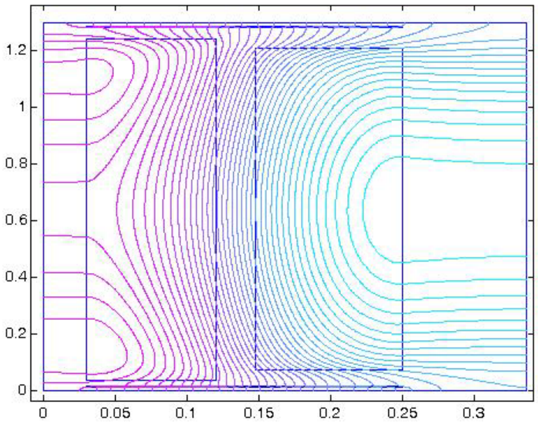

Fig. 6. Flux pattern obtained over model-30.

3.4.2. Other Models

In addition, some other models are also considered and the obtained results are given in table-4.

Table 4. The effects of the shunt permeability on the reactance

Model No.	Relative permeability	Leakage reactance (%)
30	1000	12.7811
31	1200	12.7877
32	1400	12.7925
33	1600	12.7963
34	1800	12.7992
35	2000	12.8015

4. RESULTS

The summaries of results are:

- As shown in Fig.7, the leakage reactance is proportional to the distance of the magnetic shunt from the yoke.
- As shown in Fig.8, the leakage reactance is inversely proportional to the height of the magnetic flux shunt.

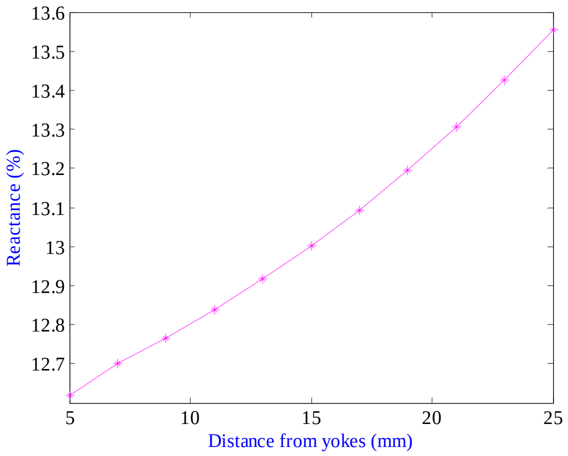

Fig. 7. Leakage reactance versus distance of shunt from yoke.

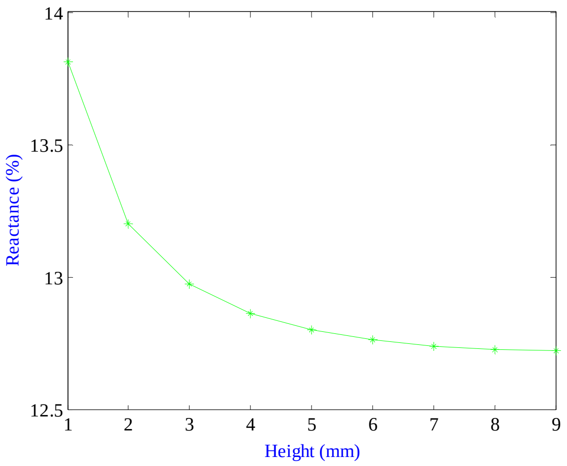

Fig. 8. Leakage reactance versus shunt height.

- As shown in Fig.9, the leakage reactance of transformer is directly proportional to the length of magnetic shunt.

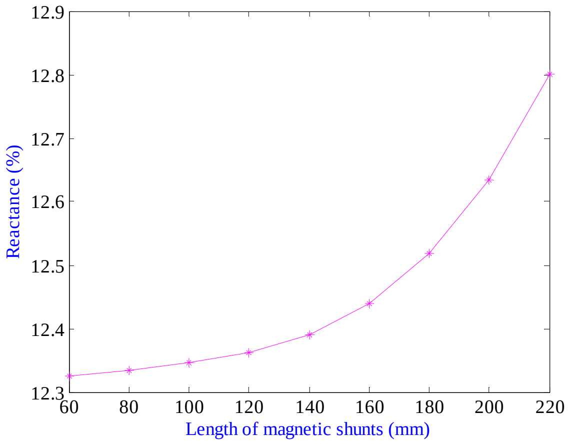

Fig. 9. Leakage reactance versus shunt length.

- As shown in Fig.10, the leakage reactance is directly proportional to the magnetic relative permeability of the magnetic shunt.

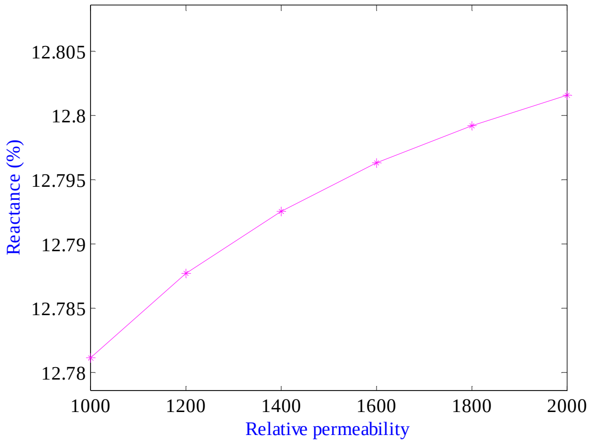

Fig. 10. Leakage reactance versus shunt relative permeability

5. CONCLUSION

It can be deduced from the performed studies that the position, magnetic permeability as well as the geometric parameters of the magnetic flux shunt has notable effects on the leakage reactance of the transformer. Leakage reactance of the transformer is directly proportional to the distance of the shunt from the yoke, the length and the magnetic permeability of the shunt and is inversely proportional to the shunt height. Thus, in the design stage of a transformer, by considering the required value of leakage reactance, the magnetic shunt can be optimized in regard of its position, its geometrical parameters and its magnetic permeability.

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REFERENCES

[1] Jamali S., Ardebili M., Abbaszadeh K.; **“Calculation of Short Circuit Reactance and Electromagnetic Forces upon the Three Phase Transformer Coils Using FEM”**, *ICEMS 2005, Proceedings of the 8th International Conference on Electrical Machines and Systems*, Vol. 3, pp. 1725-1730, (27-29 September 2005)

[2] Wang J., Witulski A.F., Vollin J.L., Phelps T.K., Gardwell G.I.; **“Derivation, Calculation and Measurement of Parameters for a Multi–Winding Transformer Electrical Model”**, *Applied Power Electronics Conference and Exposition, 1999. APEC '99. Fourteenth Annual*, Vol. 1, pp. 220-226, (14-18 March 1999)

[3] Hayek J.E.; **“Short-Circuit Reactance of Multi-Secondaries Concentric Winding Transformers, Electric Machines and Drives Conference. IEMDC 2001”**, IEEE International Electric Machines and. Drives Conference, pp. 462-465, (2001)

[4] Magot D., Margueron X., Keradec J.P.; **“PEEC-Like Analytical Calculation of Static Leakage Inductances of H.F Transformers**”, Industry Applications Conference, 2004. 39th IAS Annual Meeting. Conference Record of the 2004 IEEE, Vol. 1, (3-7 October 2004)

[5] Longfu L., Ziya W., Tiaosheng T.; **“The Method Solving Short-Circuit Impedance between Two Interleave Windings in the Multi-Winding Core-Type Transformer with Finite Element Method, Electrical Machines and Systems”**, ICEMS 2001. Proceedings of the Fifth International Conference, Vol. 1, pp. 254-256, (18-20 Aug. 2001)

[6] Smajic J., Madzarevic V., Berberevic S.; **“Numerical Calculation of Power Transformers Equivalent Circuit Parameters, Electric Power Engineering”**, PowerTech Budapest 99. International Conference, pp. 96, (29 August-2 September 1999)

[7] Alonso G., Antonio J.; **“A New Method for Calculating of Leakage Reactance and Iron Losses in Transformers**”, Electrical Machines and Systems, ICEMS 2001, Proceedings of the Fifth International Conference, Vol. 1, pp. 178 – 181, (18-20 Aug. 2001)

[8] Salon S., Lamattina B., Sivasbramaniam K.; **“Comparison of Assumptions of Short Circuit Forces in Transformers”**, *IEEE Transaction on magnetics*, Vol. 36, No.5, pp.3521-3523, (September 2000)

[9] Zhou P., Fu W.N., Lin D., Stanton S., Cendes Z.J., Xu L.; **“Numerical Modeling of Electrical Machines and Its Application”**, Industry Applications Conference, 2002. 37th IAS Annual Meeting, Vol. 3, pp. 1936-1942, (13-18 October 2002)

[10] Lakhiani V.K., **Transformers** McGraw-Hill Publishing Company**,** (1987)

[11] Martin J. Heathcote, **J&P Transformer,** Reed Educational and Professional publishing Ltd**,** (1998)