Fuzzy Jaccard with Degree of Optimism Ranking Index Based on Function Principle Approach

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ABSTRACT:
Jaccard index similarity measure which applies the extension principle approach to obtain fuzzy maximum and fuzzy minimum has been proposed in ranking fuzzy numbers. However, the extension principle used is only applicable to normal fuzzy numbers and, therefore, fails to rank non-normal ones. Apart from that, the extension principle does not preserve the type of membership function of fuzzy numbers and also involves laborious mathematical operations. In this paper, a simple vertex fuzzy arithmetic operation, namely the function principle is applied. This paper also proposes the degree of optimism concept in aggregating the fuzzy evidence. The method is capable to rank both normal and non-normal fuzzy numbers in a simpler manner from all decision makers' perspectives.

KEYWORDS: Extension principle, function principle, normal fuzzy numbers, non-normal fuzzy numbers.

1. INTRODUCTION

Various techniques of ranking fuzzy numbers which range from the trivial to the complex, including one fuzzy number attribute to many fuzzy number attributes have been proposed since it was invented by Jain [1]. The Jaccard index similarity measure which is a class of fuzzy preference relation ranking methods has also been proposed in ranking fuzzy numbers. The method was first introduced by Setnes and Cross [2] with the agreement between each pair of fuzzy numbers in similarity manner being evaluated. The extension principle (EP) concept is applied in obtaining the fuzzy maximum and fuzzy minimum which then was used in determining the ranking of the fuzzy numbers. However, the conventional arithmetic operation EP is only applicable to normal fuzzy numbers which means that the Jaccard index fails to rank the non-normal fuzzy numbers. Besides, the fuzzy arithmetic operations by EP will change the type of membership function of the fuzzy numbers and also require complex and laborious mathematical operations [3].

In 1985, Chen [4] proposed a simple vertex fuzzy arithmetic operation, namely the function principle (FP). The FP can deal with both normal and non-normal fuzzy number arithmetic operations. Apart from that, as pointed out by Hsieh and Chen [5], the FP not only preserved the type of membership function of the fuzzy number after arithmetical operations, but could also reduce the troublesome and tediousness of the arithmetical operations. The FP operation is only a corresponding real operation on function parameters which has easier and simple calculations compared to EP [4]. Furthermore, the EP is observed as a form of convolution, while the FP is akin to a point wise operation [6]. According to [3], the FP operation linearizes the complicated non-linear membership functions given by the EP operation which eases the calculation without introducing any significant error. Besides, [3] also stated that FP operation is conceptually straightforward, simple to implement, and any concept applying it becomes intelligent as it takes into consideration the degree of confidence of the decision makers' opinions which are represented by the different height of the fuzzy numbers.

A number of authors have applied the FP in solving their fuzzy arithmetical operations. Chen and Wang [7] used the FP in calculating the fuzzy inventory cost in a case of permitting backorder under fuzzy environment. Along the same line, [5] applied the FP in developing a fuzzy product positioning model and designing an algorithm for evaluating the positions of each product. Sheen [3] also used the FP approach in evaluating the fuzzy financial profitability of load management alternatives in Taiwan. In addition, Chen et al. [8]

applied the FP operations in developing a fuzzy production model with emphasize on imperfect products.

In this paper, the FP operation is applied to the Jaccard ranking index to determine the fuzzy maximum and fuzzy minimum. This paper also proposes the degree of optimism concept in calculating the total fuzzy evidence instead of using the mean aggregation since the ranking of fuzzy numbers is commonly implemented in decision-making problems. Furthermore, the usage of the mean aggregation in the original Jaccard ranking index represents the neutral decision maker which is part of the degree of optimism concept. This paper has improved not only the Jaccard ranking method but some of the previous ranking methods for both normal and non-normal fuzzy numbers.

2. PRELIMINARIES

In this section, we briefly review the definition of fuzzy numbers, extension principle (EP), function principle (FP), some properties of EP and FP operations, fuzzy maximum and minimum under the concept of FP.

2.1. Fuzzy Numbers

A fuzzy number is a fuzzy subset in the universe discourse that is both convex and normal. The membership function of a fuzzy number A can be defined as:

$$f_A(x)=\begin{cases} f_A^L(x) & ,\, a\le x\le b \\ 1 & ,\, b\le x\le c \\ f_A^R(x) & ,\, c\le x\le d \\ 0 & ,\, \text{otherwise}\end{cases} \tag{1}$$

where $f_A^L:[a,b]\rightarrow[0,1]$, $f_A^R:[c,d]\rightarrow[0,1]$, and f_A^L and f_A^R are the left and the right membership functions of the fuzzy number A, respectively. Trapezoidal fuzzy numbers are denoted by (a,b,c,d) and triangular fuzzy numbers which are special cases of trapezoidal fuzzy numbers with $b=c$ are denoted by (a,b,d).

2.2. The Extension Principle (EP)

Based on [9], the extension principle introduced by [10] is defined as follows: Let X be a Cartesian product of universes, $X=X_1\times X_2\times\ldots\times X_r$, and $A_1,A_2,\ldots,A_r$ be r fuzzy sets in $X=X_1,X_2,\ldots,X_r$, respectively. Let f be a mapping from $X=X_1\times X_2\times\ldots\times X_r$ to a universe Y such that $y=f(x_1,x_2,\ldots,x_n)$. A fuzzy set B on Y induced from r fuzzy sets A_i through f is:

$$\mu_B(y)=\sup_{y=f(x_i,x_2,\ldots,x_r)}\min\left(\mu_{A_1}(x_1),\mu_{A_2}(x_2),\ldots,\mu_{A_r}(x_r)\right) \tag{2}$$

2.3. The Function Principle (FP)

The FP proposed by [4] is defined as follows: Let g be an arithmetical mapping from n-dimension real number $\Re^n$ into real line $\Re$, and f_g is a corresponding mapping from n-dimension fuzzy numbers into fuzzy number. Suppose that $A_i=(a_i,b_i,c_i,d_i;h_i)$, $i=1,2,\ldots,n$ be n trapezoidal fuzzy numbers. The fuzzy number B on $\Re$ induced from these fuzzy numbers A_i through function f_g is:

$$f_g(A_1,A_2,\ldots,A_n)=B=(a,b,c,d;h) \tag{3}$$

where

$h=\min\{h_1,h_2,\ldots,h_n\}$, $A_{i,s}=\min\{x\mid\mu_{A_i}(x)\ge h\}$,

$A_{i,t}=\max\{x\mid\mu_{A_i}(x)\ge h\}$,

$T=\{g(x_1,x_2,\ldots,x_n)\mid x_i=a_i \text{ or } d_i, i=1,2,\ldots,n\}$

$T_1=\{g(x_1,x_2,\ldots,x_n)\mid x_i=A_{i,s} \text{ or } A_{i,t}, i=1,2,\ldots,n\}$

$a=\min T$, $b=\min T_1$, $c=\max T_1$, $d=\max T$,

$\min T\le\min T_1$ and $\max T_1\le\max T$.

For a special case of trapezoidal fuzzy numbers with the same height h, $A_{i,s}=b_i$ and $A_{i,t}=c_i$. In the following the FP of trapezoidal fuzzy number with the same height h is presented. Suppose that $A_i=(a_i,b_i,c_i,d_i;h)$, $i=1,2,\ldots,n$ be n trapezoidal fuzzy numbers with the same height of h. The fuzzy number B on $\Re$ induced from these fuzzy numbers A_i through function f_g is:

$$f_g(A_1,A_2,\ldots,A_n)=B=(a,b,c,d;h)$$

where

$T=\{g(x_1,x_2,\ldots,x_n)\mid x_i=a_i \text{ or } d_i, i=1,2,\ldots,n\}$

$T_1=\{g(x_1,x_2,\ldots,x_n)\mid x_i=b_i \text{ or } c_i, i=1,2,\ldots,n\}$

$a=\min T$, $b=\min T_1$, $c=\max T_1$, $d=\max T$,

$\min T\le\min T_1$ and $\max T_1\le\max T$.

2.4. Some Properties of FP and EP Operations

This section presents some properties under FP and EP operations from [4] and [6]. Assume A and B are two trapezoidal fuzzy numbers with the same height.

i. The membership functions of the addition of A and B have the same result under FP and EP operations.
ii. The membership functions of multiplication of A and B have the same four vertices under FP and EP operations.
iii. For more than four trapezoidal fuzzy numbers, the EP cannot solve the multiplication operation but FP can easily calculate it.

2.5. Fuzzy Maximum and Minimum under FP

In this paper, the FP is applied to deal with fuzzy maximum and fuzzy minimum operations in obtaining the ranking results of Jaccard with the degree of optimism index. Thus, this section presents the fuzzy maximum and fuzzy minimum under FP.

For two trapezoidal fuzzy numbers:

$A_1 = (a_1, b_1, c_1, d_1; h_1)$ and $A_2 = (a_2, b_2, c_2, d_2; h_2)$ with $h = \min\{h_1, h_2\}$, $A_{1,s} = \min\{x \mid \mu_{A_1}(x) \geq h\}$, $A_{2,s} = \min\{x \mid \mu_{A_2}(x) \geq h\}$, $A_{1,t} = \max\{x \mid \mu_{A_1}(x) \geq h\}$, $A_{2,t} = \max\{x \mid \mu_{A_2}(x) \geq h\}$, the fuzzy maximum and fuzzy minimum are as follows:

Fuzzy maximum: $MAX(A_1, A_2) = (a, b, c, d; h)$ (4)

with $a = \min T$, $b = \min T_1$, $c = \max T_1$, $d = \max T$ and

$T = \{\max(a_1, a_2), \max(a_1, d_2), \max(d_1, a_2), \max(d_1, d_2)\}$,

$T_1 = \{\max(A_{1,s}, A_{2,s}), \max(A_{1,s}, A_{2,t}), \max(A_{1,t}, A_{2,s}), \max(A_{1,t}, A_{2,t})\}$

Fuzzy minimum: $MIN(A_1, A_2) = (a, b, c, d; h)$ (5)

with $a = \min T$, $b = \min T_1$, $c = \max T_1$, $d = \max T$ and

$T = \{\min(a_1, a_2), \min(a_1, d_2), \min(d_1, a_2), \min(d_1, d_2)\}$,

$T_1 = \{\min(A_{1,s}, A_{2,s}), \min(A_{1,s}, A_{2,t}), \min(A_{1,t}, A_{2,s}), \min(A_{1,t}, A_{2,t})\}$

For a special case of trapezoidal fuzzy numbers with the same height of h, the fuzzy maximum and fuzzy minimum are as follows:

Fuzzy maximum: $MAX(A_1, A_2) = (a, b, c, d; h)$

with $a = \min T$, $b = \min T_1$, $c = \max T_1$, $d = \max T$ and

$T = \{\max(a_1, a_2), \max(a_1, d_2), \max(d_1, a_2), \max(d_1, d_2)\}$,

$T_1 = \{\max(b_1, b_2), \max(b_1, c_2), \max(c_1, b_2), \max(c_1, c_2)\}$.

Fuzzy minimum: $MIN(A_1, A_2) = (a, b, c, d; h)$

with $a = \min T$, $b = \min T_1$, $c = \max T_1$, $d = \max T$ and

$T = \{\min(a_1, a_2), \min(a_1, d_2), \min(d_1, a_2), \min(d_1, d_2)\}$,

$T_1 = \{\min(b_1, b_2), \min(b_1, c_2), \min(c_1, b_2), \min(c_1, c_2)\}$.

The fuzzy minimum of non-normal fuzzy numbers, A_1 and A_2 under FP is shown in Fig. 1. The fuzzy minimum obtained is still in the type of trapezoidal membership function. The fuzzy minimum of A_1 and A_2 cannot be obtained by the EP which is only applicable to normal fuzzy numbers.

The fuzzy maximum of normal fuzzy numbers, A_1 and A_2 under FP and EP are shown in Fig. 2. The result of fuzzy maximum under EP has changed to pentagonal shape but FP is still in trapezoidal shape. However, the membership functions of the fuzzy maximum of A_1 and A_2 under FP and EP have the same four vertices.

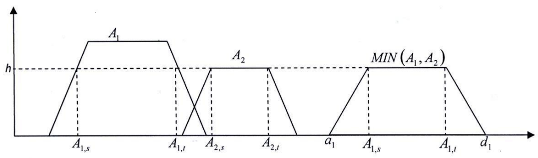

Fig. 1. Fuzzy minimum of non-normal fuzzy sets under FP

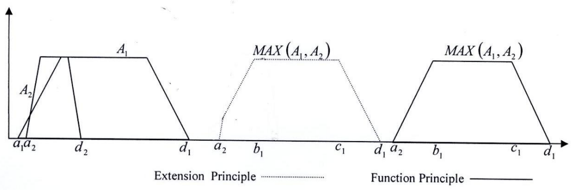

Fig. 2. Comparison of fuzzy maximum under EP and FP

3. A REVIEW ON FUZZY JACCARD RANKING METHOD

Based on the psychological ratio model of similarity from [11], which is defined as:

$$S_{\alpha,\beta}(X, Y) = \frac{f(X \cap Y)}{f(X \cap Y) + \alpha f(X \cap \overline{Y}) + \beta f(Y \cap \overline{X})} \quad (6)$$

various indices of similarity measures have been proposed which depend on the value of α and β. For $\alpha = \beta = 1$, the psychological ratio model of similarity becomes the Jaccard index similarity measure which is defined as:

$$S_{1,1}(X, Y) = \frac{f(X \cap Y)}{f(X \cup Y)} \quad (7)$$

Typically, the function f is taken to be the cardinality function. In extending the Jaccard index similarity measure of psychology to similarity measure for fuzzy sets, the objects X and Y described by the features are replaced with fuzzy sets A and B which are described by the membership functions. The fuzzy Jaccard index similarity measure is defined as:

$$S_J(A, B) = \frac{|A \cap B|}{|A \cup B|} \quad (8)$$

where $|A|$ denotes the cardinality of fuzzy set A, $\cap$ and $\cup$ can be any t-norm and s-norm, respectively.

In this section, the method in ranking fuzzy numbers using Jaccard similarity measure introduced by [2] is briefly reviewed. The procedure is presented as follows:

Step 1: For each pair of triangular fuzzy numbers A_i and A_j where $i, j = 1, 2, \ldots, n$, find the fuzzy minimum and fuzzy maximum between A_i and A_j by using the EP.

Step 2: Calculate the evidences of $E(A_i \geq A_j)$, $E(A_j \leq A_i)$, $E(A_j \geq A_i)$ and $E(A_i \leq A_j)$ which are defined based on the fuzzy Jaccard index as

$$E(A_i \geq A_j) = S_J(MAX(A_i, A_j), A_i), \quad (9)$$

$$E(A_j \leq A_i) = S_J(MIN(A_i, A_j), A_j), \quad (10)$$

$$E(A_j \geq A_i) = S_J(MAX(A_i, A_j), A_j) \quad (11)$$

and

$$E(A_i \leq A_j) = S_J(MIN(A_i, A_j), A_i) \quad (12)$$

where $MAX(A_i, A_j)$ and $MIN(A_i, A_j)$ are the fuzzy maximum and fuzzy minimum between A_i and A_j, respectively. Here, C_{ij} and c_{ji} are used to represent $E(A_i \geq A_j)$ and $E(A_j \leq A_i)$, respectively. Likewise, C_{ji} and c_{ij} are used to denote $E(A_j \geq A_i)$ and $E(A_i \leq A_j)$, respectively.

Step 3: Calculate the total evidences $E_{total}(A_i \geq A_j)$ and $E_{total}(A_j \geq A_i)$ which are defined based on the mean aggregation concept as

$$E_{total}(A_i \geq A_j) = \frac{C_{ij} + c_{ji}}{2} \quad (13)$$

and

$$E_{total}(A_j \geq A_i) = \frac{C_{ji} + c_{ij}}{2}. \quad (14)$$

Here, $E_{\geq}(i,j)$ and $E_{\geq}(j,i)$ are used to represent $E_{total}(A_i \geq A_j)$ and $E_{total}(A_j \geq A_i)$, respectively.

Step 4: For two triangular fuzzy numbers, compare the total evidences in *Step 3* which will result in the ranking of the two triangular fuzzy numbers A_i and A_j as follows:

i. $A_i \succ A_j$ if and only if $E_{\geq}(i,j) > E_{\geq}(j,i)$.

ii. $A_i \prec A_j$ if and only if $E_{\geq}(i,j) < E_{\geq}(j,i)$.

iii. $A_i \approx A_j$ if and only if $E_{\geq}(i,j) = E_{\geq}(j,i)$.

Step 5: For *n* triangular fuzzy numbers, develop $n \times n$ binary ranking relation $R_{>}(i,j)$, which is defined as

$$R_{>}(i,j) = \begin{cases} 1 & , E_{\geq}(i,j) > E_{\geq}(j,i) \\ 0 & , \text{otherwise} \end{cases}. \quad (15)$$

Step 6: Develop a column vector $[O_i]$ where O_i is the total element of each row of $R_{>}(i,j)$ and is defined as

$$O_i = \sum_{j=1}^{n} R_{>}(i,j) \text{ for } j = 1,2,\ldots,n. \quad (16)$$

Step 7: The total ordering of the triangular fuzzy numbers A_i corresponds to the order of the elements O_i in the column vector $[O_i]$.

4. AN EXTENSION OF JACCARD RANKING METHOD

An extension of Jaccard procedure which applies the function principle concept and at the same time improves [2] can be expressed in a series of steps:

Step 1: For each pair of generalized trapezoidal fuzzy numbers A_i and A_j where $i, j = 1,2,\ldots,n$, find the fuzzy minimum and fuzzy maximum between A_i and A_j by using the FP.

Step 2: Calculate the evidences of $E(A_i \geq A_j)$, $E(A_j \leq A_i)$, $E(A_j \geq A_i)$ and $E(A_i \leq A_j)$ which are defined based on the fuzzy Jaccard index as in (9), (10), (11) and (12), respectively. Here, C_{ij} and c_{ji} are used to represent $E(A_i \geq A_j)$ and $E(A_j \leq A_i)$, respectively. Likewise, C_{ji} and c_{ij} are used to denote $E(A_j \geq A_i)$ and $E(A_i \leq A_j)$, respectively.

Step 3: Calculate the total evidences $E_{total}(A_i \geq A_j)$ and $E_{total}(A_j \geq A_i)$ which are defined based on the degree of optimism concept as

$$E_{total}(A_i \geq A_j) = \beta C_{ij} + (1-\beta)c_{ji} \quad (17)$$

and

$$E_{total}(A_j \geq A_i) = \beta C_{ji} + (1-\beta)c_{ij} \quad (18)$$

where $\beta \in [0,1]$ represents the degree of optimism. Conventionally, $\beta = 0$, $\beta = 0.5$, and $\beta = 1$ represent pessimistic, neutral, and optimistic decision maker's perspective, respectively. $E_{\geq}(i,j)$ and $E_{\geq}(j,i)$ are used to represent $E_{total}(A_i \geq A_j)$ and $E_{total}(A_j \geq A_i)$, respectively.

Step 4: For each pair of fuzzy numbers, compare the total evidences found in *Step 3* which will result in the ranking of the fuzzy numbers A_i and A_j as follows:

i. $A_i \succ A_j$ if and only if $E_{\geq}(i,j) > E_{\geq}(j,i)$.

ii. $A_i \prec A_j$ if and only if $E_{\geq}(i,j) < E_{\geq}(j,i)$.

iii. $A_i \approx A_j$ if and only if $E_{\geq}(i,j) = E_{\geq}(j,i)$.

Step 5: For *n* fuzzy numbers, develop $n \times n$ binary ranking relation $R_{>}(i,j)$, which is defined as in (15).

Step 6: Develop a column vector $[O_i]$ as defined in (16).

Step 7: The total ordering of the fuzzy number A_i corresponds to the order of the element O_i in the column vector $\left[O_i\right]$.

5. COMPARATIVE EXAMPLES

In this section, seven sets of numerical examples are presented to illustrate the validity and advantages of the Jaccard by the FP ranking method. Sets 1, 2, 5, 6 and 7 are adopted from [12], [13], [14], [15] and [16], respectively. Sets 3 and 4 are self-designed numerical examples. Sets 1-4 and Sets 5-7 involve normal and non-normal fuzzy numbers, respectively.

Set 1 : $A = (2, 3, 6; 1)$, $B = (1, 4, 5; 1)$.
Set 2 : $A = (3, 6, 9; 1)$, $B = (5, 6, 7; 1)$.
Set 3 : $A = (0.1, 0.2, 0.4, 0.5; 1)$,
$B = (0.2, 0.3, 0.4; 1)$.
Set 4 : $A = (2, 6.5, 9, 12.5; 1)$, $B = (5, 6, 13; 1)$,
$C = (1, 7, 10, 12; 1)$.
Set 5 : $A = (0.1, 0.3, 0.5; 0.8)$,
$B = (0.1, 0.3, 0.5; 1)$.
Set 6 : $A = (1, 2, 3; 1)$, $B = \left(0.5, 2.5, 3; \dfrac{27}{28}\right)$.
Set 7 : $A = (5, 7, 9, 10; 1)$, $B = (6, 7, 9, 10; 0.6)$,
$C = (7, 8, 9, 10; 0.4)$.

Tables 1 and 2 show the ranking results for normal and non-normal fuzzy numbers respectively.

6. DISCUSSION

Based on Table 1, the Jaccard ranking index with FP produces consistent consequences for all types of decision makers for Set 1, which give results as $A \succ B$. The ranking results are also consistent with the Jaccard by EP, [14], [16], [18] and [21]. The Jaccard with FP improves [12], [17], [19] and [20] as they cannot discriminate the ranking between the two fuzzy numbers. Although the fuzzy maximum and fuzzy minimum for the Jaccard under FP and EP are not equal, they share the same three vertices where the fuzzy maximum vertices are 2, 4 and 6 and the fuzzy minimum vertices are 1, 3 and 5. The fuzzy maximum and fuzzy minimum for Jaccard with FP are still triangular shaped, but for the Jaccard with EP, the shapes of the membership functions have changed.

For Sets 2 and 3, almost all previous methods cannot discriminate the ranking between A and B except for [14] and [18]. The Jaccard with EP also cannot discriminate the ranking between the two fuzzy numbers which is consistent with the statement presented by [22] where for two fuzzy numbers with the same core where one is symmetrical included in the other; the sets are regarded as equal even though they are not identical. The ranking results for Jaccard with FP depending on the index of optimism with optimistic decision maker produces $A \succ B$, pessimistic decision maker $A \prec B$, and neutral decision maker $A \approx B$. Both the Jaccard with FP and EP have the same fuzzy máximum and fuzzy mínimum.

Table 1. Comparative results of the Jaccard FP index with the existing ranking methods for Sets 1-4 (normal fuzzy numbers)

Index		Set 1	Set 2	Set 3	Set 4
[12]	A	3.5	6	0.3	7.5
	B	3.5	6	0.3	7.5
	C				7.5
		$A \approx B$	$A \approx B$	$A \approx B$	$A \approx B \approx C$
[14]	A	0.342	0.316	0.424	1.161
	B	0.077	0.228	0.473	1.042
	C				1.317
		$A \succ B$	$A \succ B$	$A \prec B$	$C \succ A \succ B$
[16]	A	3.697	6.02	0.583	7.466
	B	3.374	6.02	0.583	8.014
	C				7.328
		$A \succ B$	$A \approx B$	$A \approx B$	$B \succ A \succ C$
[17]	A	1.746	3	0.15	3.766
	B	1.746	3	0.15	3.733
	C				3.817
		$A \approx B$	$A \approx B$	$A \approx B$	$C \succ A \succ B$
[18]	A	4.162	6.37	1.206	7.564
	B	3.829	6.718	1.267	8.223
	C				7.360
		$A \succ B$	$A \succ B$	$A \prec B$	$B \succ A \succ C$
[19]	A	7	12	0.6	15
	B	7	12	0.6	15
	C				15
		$A \approx B$	$A \approx B$	$A \approx B$	$A \approx B \approx C$
[20]	A	3.5	6	0.3	7.5
	B	3.5	6	0.3	7.5
	C				7.5
		$A \approx B$	$A \approx B$	$A \approx B$	$A \approx B \approx C$
[21]	A	3.667	6, 0.5	0.3, 0.5	7.449, 0.506
	B	3.333	6, 0.5	0.3, 0.5	8, 0.467
	C				7.310, 0.522
		$A \succ B$	$A \approx B$	$A \approx B$	$B \succ A \succ C$
Jaccard with EP	$E_{\geq}(A,B)$	0.846	0.583	0.583	0
	$E_{\geq}(B,A)$	0.777	0.583	0.583	2
					0
		$A \succ B$	$A \approx B$	$A \approx B$	$B \succ A \approx C$
Jaccard with FP $\beta = 0$	$E_{\geq}(A,B)$	0.667	0.5	0.5	1
	$E_{\geq}(B,A)$	0.2	0.667	0.667	2
					0
		$A \succ B$	$A \prec B$	$A \prec B$	$B \succ A \succ C$
Jaccard with FP $\beta = 0.5$	$E_{\geq}(A,B)$	0.667	0.583	0.583	1
	$E_{\geq}(B,A)$	0.4	0.583	0.583	2
					0
		$A \succ B$	$A \approx B$	$A \approx B$	$B \succ A \succ C$
Jaccard with FP $\beta = 1$	$E_{\geq}(A,B)$	0.667	0.667	0.667	1
	$E_{\geq}(B,A)$	0.6	0.5	0.5	0
					2
		$A \succ B$	$A \succ B$	$A \succ B$	$C \succ A \succ B$

Table 2. Comparative results of the Jaccard FP index with the existing ranking methods for Sets 5-7 (non-normal fuzzy numbers)

Index		Set 5	Set 6	Set 7
[12]	A	*	2	7.25
	B	0.3	*	*
	C			*
		-	-	-
[14]	A	0.356	0.228	0.387
	B	0.446	0.126	0.338
	C			0.8
		$A \prec B$	$A \succ B$	$B \prec A \prec C$
[16]	A	0.583	2.062	7.731
	B	0.583	2.064	8.006
	C			8.502
		$A \approx B$	$A \prec B$	$A \prec B \prec C$
[17]	A	0.12	1.000	3.899
	B	0.15	1.021	2.400
	C			1.700
		$A \prec B$	$A \prec B$	$A \succ B \succ C$
[18]	A	1.251	2.718	8.022
	B	1.236	2.650	8.591
	C			9.304
		$A \succ B$	$A \succ B$	$A \prec B \prec C$
[19]	A	*	4	15.5
	B	0.6	*	*
	C			*
		-	-	-
[20]	A	*	2	7.25
	B	0.3	*	*
	C			*
		-	-	-
[21]	A	0.4	0.500	7.714
	B	0.5	0.510	8.000
	C			8.500
		$A \prec B$	$A \prec B$	$A \prec B \prec C$
Jaccard with EP	$E_{\geq}(A,B)$	*	*	*
	$E_{\geq}(B,A)$	*	*	*
				*
		-	-	-
Jaccard with FP $\beta = 0$	$E_{\geq}(A,B)$	0.88	0.712	2
	$E_{\geq}(B,A)$	0.909	0.805	0
				1
		$A \prec B$	$A \prec B$	$A \succ C \succ B$
Jaccard with FP $\beta = 0.5$	$E_{\geq}(A,B)$	0.894	0.687	0
	$E_{\geq}(B,A)$	0.894	0.802	1
				2
		$A \approx B$	$A \prec B$	$A \prec B \prec C$
Jaccard with FP $\beta = 1$	$E_{\geq}(A,B)$	0.909	0.662	0
	$E_{\geq}(B,A)$	0.808	0.800	1
				2
		$A \succ B$	$A \prec B$	$A \prec B \prec C$

'*' : the ranking method cannot calculate the ranking value
'-' : no conclusion for the ranking result

For Set 4, again the Jaccard index with FP improves the ranking indices by [12], [19], [20] and Jaccard with EP. The pessimistic and neutral decision makers produce the ranking as $B \succ A \succ C$, which is consistent with [16], [18] and [21]. The ranking result for optimistic decision maker is consistent with [14] and [17].

As [12], [19], [20] and Jaccard with EP indices cannot rank the non-normal fuzzy numbers, the ranking results for Sets 5, 6 and 7 (in Table 2) by the aforementioned methods cannot be obtained. For Set 5, the ranking result for the pessimistic decision maker of Jaccard with FP is consistent with [14], [17] and [21], while the optimistic is consistent with [18]. The neutral decision maker equally ranks the fuzzy number which is consistent with [16].

For Set 6, the Jaccard with FP ranks the two fuzzy numbers consistently for all types of decision makers with the ranking result consistent with [16], [17] and [21]. For Set 7, the optimistic and neutral decision makers of Jaccard with FP produce $A \prec B \prec C$ which is consistent with [16], [18] and [21]. While the pessimistic decision maker ranks them as $A \succ C \succ B$.

7. CONCLUSION

This paper improves the Jaccard index similarity measure proposed by [2] in ranking both normal and non-normal fuzzy numbers. Instead of using the EP in determining the fuzzy maximum and fuzzy minimum, this paper applies the FP operations which are capable in solving the arithmetic operations of both normal and non-normal fuzzy numbers. It is found that, the vertices of the fuzzy maximum and fuzzy minimum for normal fuzzy numbers between the EP and FP are similar. In fact, in some cases such as for comparable fuzzy numbers where one fuzzy number is included in the other (Sets 2 and 3), the fuzzy maximum and fuzzy minimum between the EP and FP are equal. Besides, the type of membership function for fuzzy maximum and minimum is preserved and the calculation of operation is easier and simpler under the operation of FP rather than EP. According to [6], the EP is observed as a form of convolution, while the FP is akin to a point wise operation.

The Jaccard ranking index with FP produces consistent results with some of the previous ranking indices and, in fact, has improved some of the results. For instance, the Jaccard ranking index with FP can rank normal fuzzy numbers effectively which cannot be distinguished from some of the previous ranking methods such as [16], [12], [17], [19], [20], [21] and the Jaccard with EP indices. Similarly, the Jaccard index with FP can also successfully rank the non-normal fuzzy numbers which failed to be ranked by [12], [19], [20] and the Jaccard with EP methods. The usage of degree of optimism concept in aggregating the fuzzy total evidences provides the ranking results for all types of decision makers (pessimistic, neutral, and optimistic) which is crucial in the decision-making problems. Thus, the Jaccard with FP index is flexible and more intelligent than the Jaccard with EP as it considers the degree of confidence and the degree of optimism of the decision makers' opinions.

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