Transmission Loss Allocation Based on Buses Injected Currents

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ABSTRACT:
In this paper, a new method is proposed to allocate transmission loss in pool-based electricity markets. This method is based on using the impedance matrix of the network and the admittance equivalent circuit seen from the network buses. After performing load flow equations, the losses of each bus are calculated using the impedance matrix of the network and the reduced admittance matrix and the injected currents from each bus. These losses are properly and fairly shared between network buses for fair loss allocation in proportion to the percent of penetration the currents of each bus. Furthermore, using partial derivatives of the active power losses with respect to the bus currents' coefficients, a sensitivity analysis has been done for proving the fairness of the proposed method. In addition to its simplicity, the suggested method assigns the losses properly and fairly between the buses. Finally, this method has been tested on a benchmark IEEE 14-bus network, and the results are compared with other existing methods.

KEYWORDS: Loss allocation, Pool-based electricity market, Admittance equivalent circuit, Loss matrix.

1. INTRODUCTION

The loss reduction study has always been a matter of interest in non-de-regulated power systems. After creation of the de-regulation concept in the power industry and separation of the parts of the power system, various subjects were introduced in the power industry and power system and the clarification concept in the transmission loss costs of the power system was one of these subjects. Actually, after the separation of the generation, transmission and distribution parts, calculation of the share of each of the customers from the transmission loss costs became an important matter of interest. During the recent years, various methods have been introduced for loss allocation of each part of the network. Each of these methods has its own advantages and disadvantages. However, no general and fair method has been proposed to determine the share of each part of the network from the transmission losses.

In the method proposed in [1] (known as the Pro-rata method), the share of each bus of the network from the losses is expressed in terms of the network utilization. In this method, the total losses of the network are divided to two completely equivalent parts between the generators and the loads of the system. Loss allocation is performed based on the amount of power absorbed or supplied from/to the grid. Another approach is the power tracing method [2]. This method is based on KCL law and assumes that the power entering a bus from a certain branch is divided between the other branches delivering power in proportion to load flow. Determining the share of each of the loads and generators from the currents passing through the lines, the losses are divided between the generators and loads. In Ref [3], the share of each part of the network from the transmission loss is determined using the game theory. Actually, loss allocation is done minimizing a cost function.

Several other methods such as neural networks are also used [4-7], but the Z-bus method seems to be more suitable and fair to all these methods due to using the network topology and the equations of the power system [8]. However, the share of each of the buses of the network from the losses is not divided fairly. It should be noted that a suitable method should have the following properties in order to have a proper and fair loss allocation:

1- The allocated share to each of the buses of the network should be a really reflective of the losses of that bus.

2- Not allocate or minimize negative loss share to the buses of the network.

3- The method be able to be performed with the load flow results.

4- Be simple and can be implemented in power markets.

In this paper, the share of each of the players from the network losses has been proposed using the network's impedance matrix and the Thevenin's equivalent circuit from the view of two of the buses of the network. In the next section, the Thevenin's circuit equations from the view of the buses of the network are introduced for fair allocation. Afterwards, in the third section, the share of each of the buses of the network from the transmission losses is determined using these equations and the network's impedance matrix. In the fourth section, this method is tested on the IEEE 14-bus system. Finally, the concluding results are presented.

2. THEVENIN'S EQUIVALENT CIRCUIT FROM THE VIEW OF TWO BUSES OF THE NETWORK

It is evident that the relation between the injected currents of the buses, and their voltages is as (1):

$$I_{bus} = V_{bus} \times Y_{bus} \quad (1)$$

If it is desired to attain the reduced order circuit from the view of buses k and j, the injected currents of the other buses should be neutralized. Then, the reduced order impedance matrix of the system from the view of buses k and j can be expressed by the following equation [9]:

$$I_{bus} = \begin{pmatrix} I_1 \\ I_2 \\ I_k \\ \cdots \\ I_j \\ I_n \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n-1} & Y_{1n} \\ Y_{21} & Y_{22} & & Y_{2n-1} & Y_{2n} \\ \vdots & & \ddots & \vdots & \\ Y_{n-11} & Y_{n-12} & \cdots & Y_{n-1n-1} & Y_{n-1n} \\ Y_{n1} & Y_{n2} & & Y_{nn-1} & Y_{nn} \end{pmatrix} \times \begin{pmatrix} V_1 \\ V_2 \\ V_k \\ \cdots \\ V_j \\ V_n \end{pmatrix} \quad (2)$$

The above matrix can be reduced with respect to the k-th and j-th buses. The reduced order admittance matrix or equivalent system is as following (3) and has been shown in Figs. 1 and 2.

$$\begin{pmatrix} \bar{I}_k \\ \bar{I}_j \end{pmatrix} = \begin{pmatrix} \bar{Y}_{kk} & \bar{Y}_{kj} \\ \bar{Y}_{jk} & \bar{Y}_{jj} \end{pmatrix} \times \begin{pmatrix} V_k \\ V_j \end{pmatrix} \quad (3)$$

In the figures, the values of impedances are as follows:

$$\begin{cases} Z1 = Z_{ii} - Z_{ij} \\ Z2 = Z_{jj} - Z_{ij} \\ Z3 = Z_{ij} \end{cases} \quad (4)$$

Using (3) and the values of the parameters of the equivalent system for the k and j buses, the penetration percent of the currents of the k and j buses with respect to each other in the whole network can be attained and actually, the ration of the injected currents of k and j buses in the network can be used for fair loss allocation.

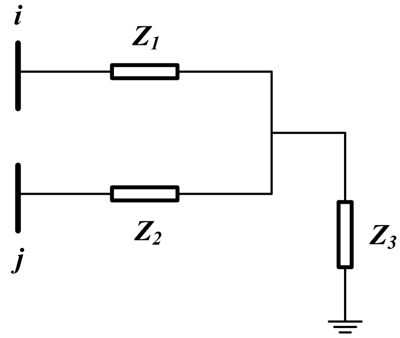

Fig. 1. Network impedance equivalent circuit

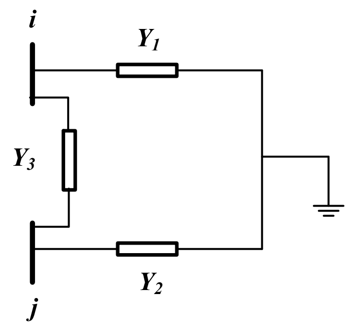

Fig. 2. Network admittance equivalent circuit

3. PROPOSED METHOD

The loss allocation problem is intrinsically different from the loss compensation problem. In a pool-based market, ISO performs an economic load dispatch after the reception of other players' cost suggestions in order to minimize the operational costs of the system. In the loss allocation problem, it is tried to divide the loss costs between all the the parts of the system fairly. This cost allocation is performed after a complete load flow run. Supposing that the economic load dispatch has been done, the total losses of a network with n buses can be expressed as follows [9]:

$$P_{loss} = \sum_{k=1}^{n} P_k = \mathrm{Real}\left\{ \sum_{k=1}^{n} V_k I_k^* \right\} \Rightarrow$$
$$= \mathrm{Real}\left\{ \sum_{k=1}^{n} I_k^* \sum_{j=1}^{n} Z_{kj} I_j \right\} \quad (5)$$

The Z-bus matrix can be written in from of the equation (6):

$$Z_{kj} = R_{kj} + jX_{kj} \quad (6)$$

Replacing this equation in (5) and expressing the values in terms of their magnitude and angle, the total losses can be obtained as:

$$P_{loss} = \sum_{k=1}^{n}\sum_{j=1}^{n} |I_k||I_j| R_{kj} \cos(\delta_k - \delta_j) \quad (7)$$

The above equation can be written in a matrix form:

$$P_{loss} = \sum \begin{pmatrix} P_{loss1,1} & P_{loss1,2} & \cdots & P_{loss1,n-1} & P_{loss1,n} \\ P_{loss2,1} & P_{loss2,2} & & P_{loss2,n-1} & P_{loss2,n} \\ \vdots & & \ddots & & \vdots \\ P_{lossn-1,1} & P_{lossn-1,2} & \cdots & P_{lossn-1,n-1} & P_{lossn-1,n} \\ P_{lossn,1} & P_{lossn,2} & & P_{lossn,n-1} & P_{lossn,n} \end{pmatrix} \quad (8)$$

The above loss matrix has diagonal and no diagonal elements, which are as equation (9, 10):

$$P_{loss\,k,k} = R_{kk} |I_k|^2 \quad (9)$$

$$P_{loss\,k,j} = R_{k,j} |I_k||I_j| \cos(\delta_k - \delta_j) \quad (10)$$

Equation (9) which shows the current injection just by the k-th bus shows the self-loss of the k-th bus. On the other hand, equation (10) is a part of the network losses which happen due to the interaction of current injection by the k-th and j-th buses, which is called mutual loss between the k-th and j-th buses. Using equation (3), the share of each of the k-th and j-thbuses from the mutual losses can be expressed as:

$$P_{loss\,k,j}^{\;k} = P_{loss\,k,j} \frac{|\bar{I}_k|^2}{|\bar{I}_k|^2 + |\bar{I}_j|^2} \quad (11)$$

$$P_{loss\,k,j}^{\;j} = P_{loss\,k,j} \frac{|\bar{I}_j|^2}{|\bar{I}_k|^2 + |\bar{I}_j|^2} \quad (12)$$

Where separation of the losses of each bus from the mutual loss element is based on the currents of equation (3). Considering the above equations, the share of thek-th bus from the total network losses can be stated as follows:

$$P_{loss}^{\;k} = P_{loss\,k,k} + 2\sum_{j=1}^{n} P_{loss\,k,j}^{\;k} \quad (13)$$

On the other hand, the total losses of the network are:

$$P_{loss} = \sum_{k=1}^{n} P_{loss}^{\;k} \quad (14)$$

4. WORKED EXAMPLE

A simple example without fixed losses can be worked through to show the application of the proposed allocation method. The three-bus system shows in Fig 3. Transmission line data are shown in Table 1, is used for this purpose. Generator (located at buses 1) supply the power demand (located at buses 2, 3).

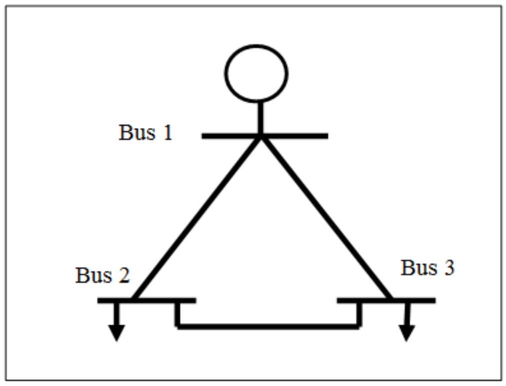

Fig. 3. Three-Bus System

Table 1. Three-bus system: transmission line data

Line From Bus to Bus	R (%)	X (%)	B (%)
1-2	0.0200	0.040	0.025
1-3	0.0100	0.030	0.025
2-3	0.0125	0.025	0.025

Table 2 summaries the power flow solution by the Newton–Raphson method. Columns 2, 3, 4, 5, 6 and 7 shows, respectively, bus magnitude voltages, bus angle voltages, active generated power, reactive generated power, active demand power, reactive demand power.

Loss allocation to each bus of the typical 3-bus network is illustrated in Table 3. As shown in Fig. 3, bus 3 injects the current in the opposite direction with respect to the resultant current of the network in line 2-3. So, the allocated loss of the line 2-3 to the bus 3 has a negative value. The negative allocated loss to the bus 3 is due to its decreasing role in the reduction of the network losses. On the other hand, if this bus increases the network losses, it receives the positive loss allocation cost.

As Table 3 shows, bus 1 has the largest amount of allocated losses while the bus number 3 has the least amount. For further analysis, the losses in the line connecting buses 2 and 3 and the amount of power flow from bus 2 to bus 3 versus load changes in bus 3 has been depicted in Fig 4. The losses of the line connecting bus 2 to bus 3 decrease, when the load of bus 3 is changed from 0 MW to 350 MW, while the power flowing from bus 2 to bus 3 is increasing at the same time. Hence, the allocated losses to bus 3 are decreasing when the load of bus 3 is changed from 0 MW to 350 MW and has the least amount of loss allocation. The losses of the line connecting bus 2 to bus 3 will increase for load increase above 350 MW and so will allocate losses to bus 3.

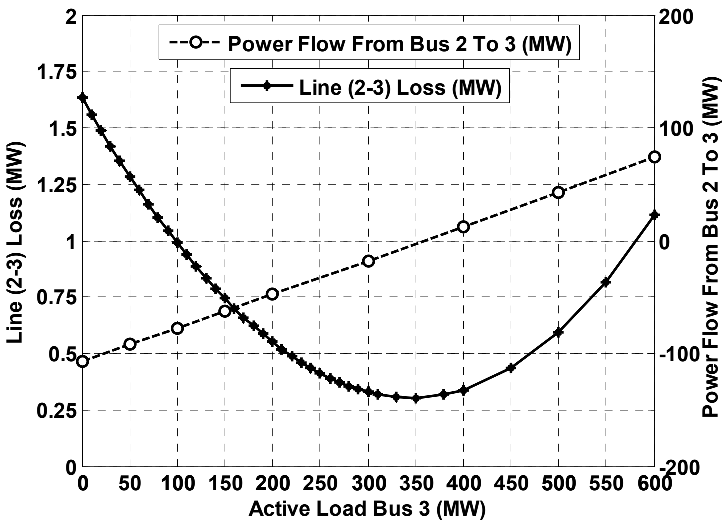

Fig. 4. Line loss and power flow from bus 2 to bus 3 versus load changes in bus 3

Table 2. Three bus system: power flow results

Bus. No	Voltage	Angle	PG(MW)	QG(MVAr)	PD(MW)	QD(MVAr)
1	1.0500	0.0000	409.2289	172.963	0.000	0.000
2	0.9840	-3.539	0.0000	0	256.6	110.2
3	1.0030	-2.892	0.0000	0	138.6	45.20
Total Sum			409.2289	172.963	395.20	155.40

Table 3. The allocated loss of transmission lines to each bus of the typical 3-bus network

Line	Line loss(MW)	Share bus 1	Share bus 2	Share bus 3
1-2	8.3400	5.0339	3.2328	0.0663
1-3	4.8943	3.5211	0.3369	1.0363
2-3	0.7946	0.1895	1.0811	-0.4760

5. NUMERICAL STUDY

The proposed method has been tested on a set of networks with different sizes, and it has been compared to some of the most well-known alternative algorithms described in the literatures. In this paper, the IEEE 14-bus is used to show the result of proposed method in comparison to pro-rata method (PR) and incremental transmission loss method (ITL) as the most referenced loss allocation methods and Z-bus method.

As it can be seen in Fig 5, the IEEE 14-bus system has 5 voltages controlled buses and 2 generators buses, in which bus 1 is considered as the slack bus. According to the power flow results in Table 4, bus 1 provides 13.54 MW that should be divided between market players. Table 5show the contribution of each bus to the transmission-line losses using the proposed method. Based on table 5, bus 8 in proposed method lower loss allocation methods comparison with other method on negative value. Proposed method make minimizednegative loss allocation on the networkbuses.

Using partial derivatives of the equation, the sensitivities of losses to the injected currents of buses are given as (15):

$$\frac{\partial P_{loss}}{\partial I_k} = \left\{\cos\delta_k - j\sin\delta_k\right\}\times \left\{R_{kk} I_k + \sum_{j=1}^{n} R_{kj}\left|I_j\right|\cos(\delta_i - \delta_j)\right\} \qquad (15)$$

The magnitude of the above equation is as (16):

$$\left|\frac{\partial P_{loss}}{\partial I_k}\right| = \left|R_{kk} I_k + \sum_{j=1}^{n} R_{kj}\left|I_j\right|\cos(\delta_i - \delta_j)\right| \qquad (16)$$

Considering this equation and the values of Table 6, it is seen that the bus no.8 has the minimum rate of loss changes in response to current injection, which shows that the corresponding bus acts in the direction of loss reduction. Therefore, fewer shares of losses should be assigned to this bus. On the other hand, Table 5 show that this bus has the least share of allocated losses in the proposed method which proves the fairness of this method in comparison to the other methods. Furthermore, bus no. 1 has the largest rate of loss changes to current injection and in the most of the methods; this bus has the largest share of losses. Anyway, considering the loss change coefficients with respect to current injection, it can be observed that the proposed method gives more general results than the other methods.

Table 4. The power flow results

Bus. No	Voltage	ANGLE	PG(MW)	QG(MVAr)	PD(MW)	QD(MVAr)
1	1.060	0.000	232.54	-16.50	0.000	0.000
2	1.045	-4.983	40.00	30.86	21.70	12.70
3	1.010	-12.72	0.000	6.000	94.20	19.00
4	1.018	-10.31	0.000	-3.900	47.80	-3.900
5	1.020	-8.774	0.000	-1.600	7.600	1.600
6	1.070	-14.22	0.000	5.000	11.20	7.500
7	1.062	-13.36	0.000	0.000	0.000	0.000
8	1.090	-13.36	0.000	31.6443	0.000	0.000
9	1.056	-14.93	0.000	-16.600	29.50	16.60
10	1.051	-15.09	0.000	-5.800	9.00	5.800
11	1.057	-14.79	0.000	-1.800	3.500	1.800
12	1.055	-15.07	0.000	-1.600	6.100	1.600
13	1.050	-15.15	0.000	-5.800	13.50	5.800
14	1.036	-16.03	0.000	-5.000	14.90	5.000
Total Sum			272.54	14.9043	259.0	73.50

Table 5. The results of proposed method beside the other methods

Bus. No	Z-bus Method	ITL Method	Pro-Rata Method	Proposed Method
1	7.3848	6.140	6.40	8.5130
2	0.2850	1.130	0.50	0.3730
3	2.6980	2.920	2.62	2.2054
4	0.9056	1.260	1.36	0.8369
5	0.0903	0.180	0.22	0.0359
6	0.6783	0.320	0.32	0.0899
7	0.0000	0.000	0.00	0.0000
8	-0.130	-0.170	0.00	-0.0644
9	0.4484	0.660	0.82	0.4740
10	0.1690	0.200	0.24	0.1141
11	0.0620	0.080	0.10	0.0221
12	0.1385	0.180	0.16	0.0542
13	0.3412	0.320	0.38	0.3310
14	0.4689	0.320	0.42	0.5549
Total Sum	13.54	13.54	13.54	13.54

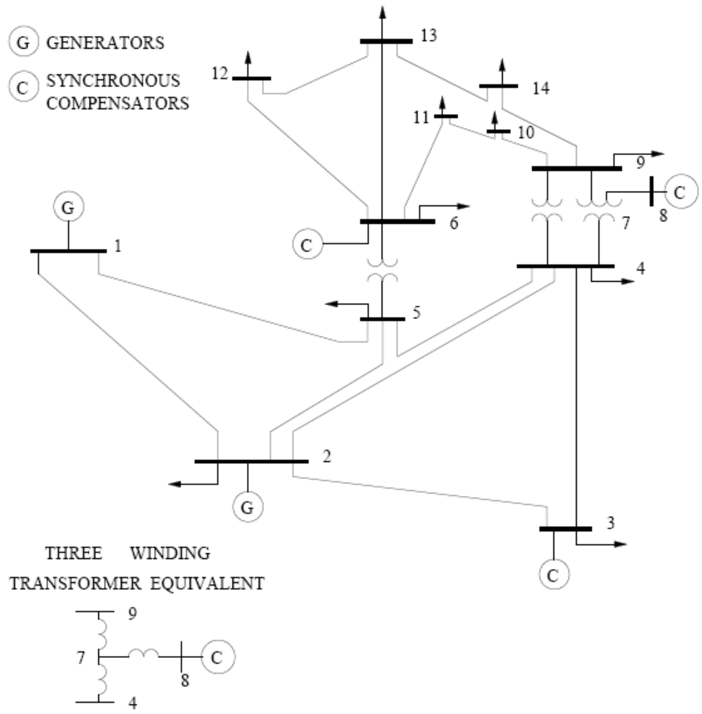

Fig. 5. 14-bus IEEE test system

Table 6. Partial derivatives of the active power losses with respect to the bus currents coefficients

Bus number	$\left\|\frac{\partial P_{loss}}{\partial I_k}\right\|$
1	0.0025
2	0.0076
3	0.0244
4	0.0273
5	0.0045
6	0.0358
7	0.000
8	0.0007
9	0.0156
10	0.0120
11	0.0066
12	0.0176
13	0.0294
14	0.0374

6. CONCLUSION

In this paper, a fair method has been proposed to allocate the losses in a power system using its circuit equations and simplifying them. This method divided the losses between the players of a pool-based market using the network impedance matrix and the reduced admittance matrix in from the view of two of the buses of the network. The method is based on load flow and the following principles:

1) Incorporates the main equations of the power system in conjunction with the network impedance matrix and the vectors of the injected currents of the buses.

2) Uses the reduced admittance matrix of the system for fair allocation of losses between the network customers.

3) It is a simple and easily understandable method.

The proposed method is this paper doesn’t consider any bus or buses compensating the network total losses. It is actually independent of the slack bus and divides the losses between the market players considering the penetration percent of them in the network. The method separates the self and mutual losses and is therefore, applicable in other forms of the power system such as multi-transaction contract markets. It can actually be used to compensate the losses by the buses, themselves. Finally, the proposed method has been tested on the IEEE 14-bus system and fair results have been achieved in comparison to the other methods.

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