Sum-of-Sinusoids Based Fading Channel Simulator for Non-Isotropic Scattering Environments

Qiuming Zhu[1] , Dazhuan Xu, Xiaomin Chen, Weihua Lv
College of Electronic Information Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, China
1- Email: zhuqiuming@nuaa.edu.cn

ABSTRACT:
In this paper, we present a modified sum-of-sinusoids (SOS) based simulator for a two-dimensional (2-D) non-isotropic scattering channel. With a new parameter computation method called equal probability area (MEPA), the proposed model can be applied on arbitrary 2-D scattering environments and also can be generalized to multi-path channels with respect to the principle of set partitioning. Simulation results verify that the first and second order statistics of the output channels approximate the reference model with a high precision and when the theoretical results are unknown, it can be used as a reference for unusual distributions.

KEYWORDS: Sum-of-sinusoids, Fading channel, Non-isotropic scattering, Autocorrelation function (ACF), Doppler power spectrum (DPS), Angle of arrival (AOA).

1. INTRODUCTION[1]

The sum-of-sinusoids (SOS) method introduced by Rice and Clarke is widely used in the wireless fading channels simulation [1-2]. Most of the SOS models developed early [1]-[5] had the assumption that the angle of arrival (AOA) was uniformly distributed. However, the realistic scattering scenarios are no-isotropic [6]-[8]. Some modified SOS models have been proposed in [9]-[11], which are suitable for non-isotropic scattering environments. The Lp-norm method introduced in [9] can simulate any channel with minimum mean square error with the given autocorrelation function (ACF), which has good performance but very complicated. Method of equal area [10]-[11] can be used on arbitrary scattering scenarios with given doppler power spectrum (DPS). Obviously, the theoretical ACF or DPS needs to be known before those methods are applied. Many works have been done in [7]-[8] for deriving theoretical ACF and DPS under usual AOA distributions. However, it's still difficult to get the closed-form expressions for real AOA distributions which are complicated.

To overcome the restriction, this paper proposes a modified SOS model and a new parameter computation method, called equal probability area (MEPA) based on [12]. The new method gets simulation parameters directly from realistic AOA distribution which is always available from experiments and analysis [6].

The remainder of this paper is organized as follows. Section II gives the reference model and the modified simulation model. Section III presents a novel parameter computation method. Section IV analyzes the statistical properties of the new model. Section V gives the performance evaluation by extensive numerical results. Section VI concludes the paper.

2. THE REFERENCE MODEL AND NEW MODEL
2.1. The reference model

Consider the normalized frequency nonselective fading channel of 2-D propagation environment given by [1-2],

$$\tilde{h}(t) = \lim_{N\to\infty}\sqrt{\frac{1}{N}}\sum_{n=1}^{N}\exp[j(\omega_d\cos\alpha_n\cdot t + \phi_n)] \qquad (1)$$

where α_n and ϕ_n are, respectively, the angle of arrival signal and initial phase associated with the nth propagation path, N is the number of propagation path, and ω_d is the maximum Doppler radian frequency.

When N approaches infinity, the central limit theorem justifies that $\tilde{h}(t)$ are complex Gaussian random processes, whose statistical properties can be described by envelope distribution, phase distribution, and other second order statistics.

In most cases, the angle of spread is not zero, so the envelope of the reference model is Rayleigh distributed, and phase is uniformly distributed. The ACF and PSD of $\tilde{h}(t)$ are defined respectively as

$$R_{\tilde{h}\tilde{h}}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T}\tilde{h}(t)\tilde{h}^{*}(t+\tau)dt \quad , \tag{2}$$
$$= E[\exp(j\omega_d \tau \cos\alpha_n)]$$

and

$$S_{\tilde{h}\tilde{h}}(\omega) = \int_{-\infty}^{\infty} R_{\tilde{h}\tilde{h}}(\tau)e^{-j\omega\tau}d\tau \quad , \tag{3}$$
$$= 2\pi E[\delta(\omega-\omega_d\cos\alpha_n)]$$

When $\alpha_n \in U(-\pi,\pi]$,(2) and (3) reduce to the results of Clarke's 2-D isotropic scattering model, which are

$$R_{\tilde{h}\tilde{h}}(\tau) = J_0(\omega_d\tau)$$

$$S_{\tilde{h}\tilde{h}}(\omega) = \begin{cases} \dfrac{2}{\omega_d\sqrt{1-(\dfrac{\omega}{\omega_d})^2}} &,|\omega|<\omega_d \\ 0 &,|\omega|\geq\omega_d \end{cases} \tag{4}$$

where $U(-\pi,\pi]$ denotes the uniform distribution over $(-\pi,\pi]$, $J_0(\cdot)$ is the zero-order Bessel function.

The typical curves of ACF and PSD for Clarke's 2-D isotropic scattering model with $f_d = 10Hz$ are shown in Fig.1. We observe that the autocorrelation reduces with the increasing of time delay and that the doppler power spectrum is U shaped.

2.2. The new simulation model

As we can see from (1) that the reference model comprises an infinite sum of complex harmonic functions, which is impossible in simulation. Our modified simulator for non-isotropic scattering conditions is defined by in-phase and quadrature components,

$$h_l(t) = h_{l,i}(t) + jh_{l,q}(t) \tag{5}$$

$$\left.\begin{aligned} h_i(t) &= \sqrt{\frac{1}{N}}\sum_{n=1}^{N}\cos(\omega_d\cos\alpha_{i,n}\cdot t+\phi_n) \\ h_q(t) &= \sqrt{\frac{1}{N}}\sum_{n=1}^{N}\cos(\omega_d\cos\alpha_{q,n}\cdot t+\phi_n+\Omega_n) \end{aligned}\right\} \tag{6}$$

where N is the simulation number of propagation path (usually $N = 10\sim20$), ϕ_n indicates the initial phase which uniformly distributed over $[0,2\pi)$, $\alpha_{i|q,n}$ and Ω_n are random variables based on given AOA's distribution and must meet the following conditions:
a). Asymmetric DPS (or $p(\alpha)$ is asymmetrical on $\pi/2$)
$$\Omega_n = \pi/2, n=1,2,\cdots,N$$
$$\alpha_{c,n} = \alpha_{s,n} = \alpha_n, n=1,2,\cdots,N \quad , \tag{7}$$
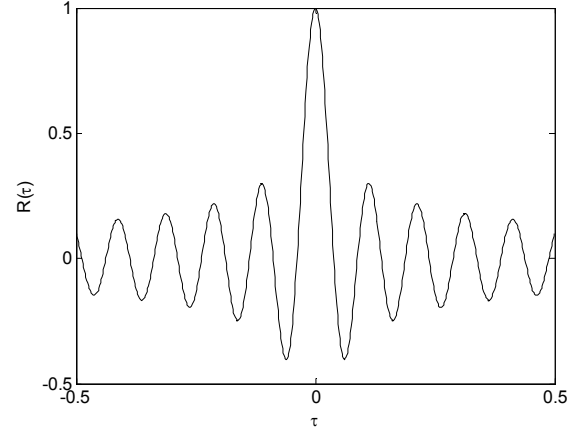$$\alpha_n \neq \pm\alpha_m, m\neq n$$

b). Symmetric DPS (or $p(\alpha)$ is symmetrical on $\pi/2$)
$$\Omega_n \in U(-\pi,\pi], n=1,2,\cdots,N$$
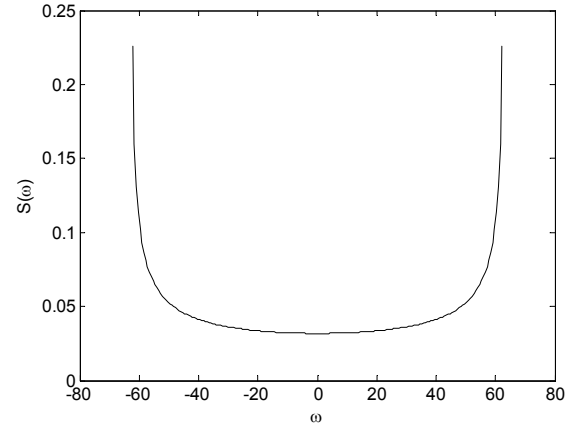$$\alpha_{c,n} \neq \pm\alpha_{s,m}, n,m=1,2,\cdots,N \quad , \tag{8}$$
$$\alpha_{c,n} \neq \pm\alpha_{c,m}, \alpha_{s,n}\neq\pm\alpha_{s,m}, m\neq n$$

where $p(\alpha)$ denotes the probability distribution function(PDF) of AOA.

(a) The autocorrelation function ($f_d = 10\text{Hz}$)

(b) The doppler power spectrum ($f_d = 10\text{Hz}$)
Fig. 1. The ACF and PSD of Clarke's 2-D isotropic scattering model

3. PARAMETER COMPUTATION METHOD

The problem lies in finding a set of $\alpha_{i|q,n}$ which can provide a good approximation to the reference model and must meet the conditions of (7) and (8). The computation procedure based on the method of equal probability area (MEPA) is shown as follows,

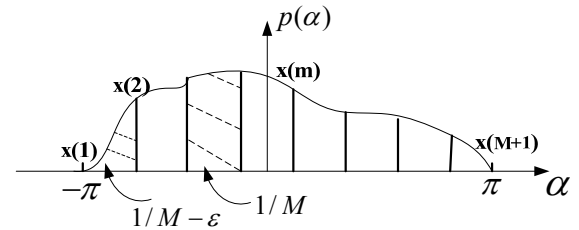1). Divide the area under $p(\alpha)$ into M subspaces equally except the first one (see Fig.2).

Fig. 2. M equal subspaces of arbitrary AOA's PDF

The mth subspace is denoted as

$$\int_{x_1}^{x_2} p(\alpha)d\alpha = \frac{1}{M} + \varepsilon, \varepsilon \in U(-\frac{1}{2M}, +\frac{1}{2M}), \varepsilon \neq 0$$

$$\int_{x_m}^{x_{m+1}} p(\alpha)d\alpha = \frac{1}{M}, m = 2,3,\cdots,M+1 \qquad (9)$$

where, ε is a random phase shift which avoids symmetric $\alpha_{i|q,n}$, when $p(\alpha)$ is symmetrical on $\alpha = 0$.

2). Divide each subspace into two equal areas by points α_m which can be expressed as

$$\int_{x_1}^{\alpha_1} p(\alpha)d\alpha = \frac{1}{2M} + \frac{\varepsilon}{2}$$

$$\int_{x_m}^{\alpha_m} p(\alpha)d\alpha = \frac{1}{2M}, m = 2,3,\cdots,M \qquad (10)$$

If we introduce the cumulative distribution function (CDF) of AOA, (10) can be simplified as

$$G(\alpha_1) = \int_{-\pi}^{\alpha_1} p(\alpha)d\alpha = \frac{1}{2M} + \frac{\varepsilon}{2}$$

$$G(\alpha_m) = \int_{-\pi}^{\alpha_m} p(\alpha)d\alpha = \frac{2m-1}{2M} + \varepsilon, m = 2,\cdots,M \qquad (11)$$

Thus we can obtain a closed form solution of α_m when $G^{-1}(x)$ exists.

$$\alpha_1 = G^{-1}(\frac{1}{2M} + \frac{\varepsilon}{2})$$

$$\alpha_m = G^{-1}(\frac{2m-1}{2M} + \varepsilon), m = 2,\cdots,M \qquad (12)$$

3). Get $\alpha_{i|q,n}$ from random phases set $\{\alpha_m\}$.

For asymmetric DPS, let $M = 2N$ and

$$\alpha_{i,m} = \alpha_{2m-1}, \alpha_{q,m} = \alpha_{2m}, m = 1,2,\cdots,N \qquad (13)$$

For symmetric DPS, let $M = N$ and

$$\alpha_{i,m} = \alpha_{q,m} = \alpha_m, m = 1,2,\cdots,N \qquad (14)$$

The principle of set partitioning allows improving the performance of SOS-based model or simulating multi-path channels. Following the approach proposed in [13], the MPEA method can be modified easily to get better performance. This modification consists in creating L uncorrelated complex Gaussian processes. We can create LM random phases with step (1)-(2) and take out M random phases each time for step (3). The lth random phases are selected as

$$\{\alpha^{(l)}\} = \alpha_{(l-1)M+1}, \alpha_{(l-1)M+2}, \cdots, \alpha_{lM+M}, l = 1 \sim L \qquad (15)$$

4. STATISTICAL PROPERTIES

Invoking the central limit theorem, when the angle of spread is not zero, the envelope PDF of the new model is similar to Rayleigh's distribution:
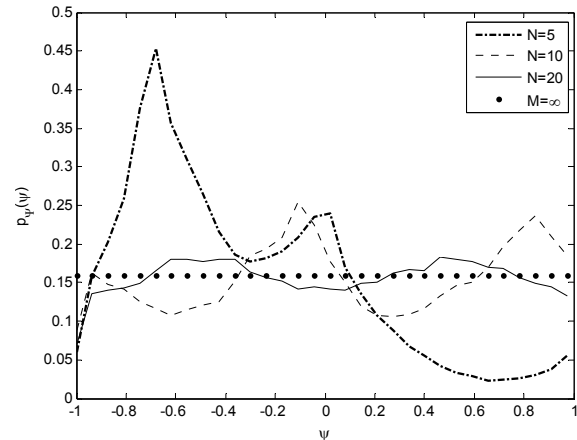
$$p_{|h|}(x) = \frac{x}{2}e^{-x^2} \qquad (16)$$

and the phase uniformly distributed over $[0, 2\pi)$, which is

$$p_{|\Psi|}(\Psi) = 1/2\pi \qquad (17)$$

Fig.3 shows the PDFs of envelope and phase for N=5, 10, 20 and ∞, as well as the reference model's PDFs. As expected $p_{|h|}(x)$ and $p_{|\Psi|}(\Psi)$ converge to the theoretical PDFs with increasing N respectively. It also shows that $N = 20$ has enough precision for most simulation purpose.
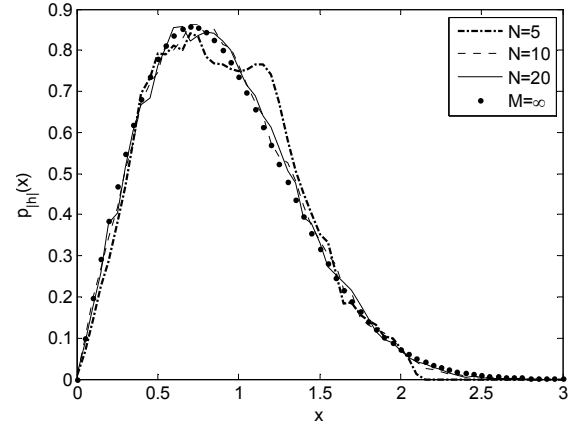
(a) Envelope PDF

(b) Phase PDF

Fig. 3. Envelope and phase PDFs comparison

Based on (2), the ACF can be expressed by

$$R_{hh}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{t=-T}^{T} h(t)h^*(t+\tau)dt$$

$$= R_{h_i h_i}(\tau) + R_{h_q h_q}(\tau) + j[R_{h_i h_q}(\tau) - R_{h_q h_i}(\tau)] \qquad (18)$$

and the in-phase component is

$$R_{h_i h_i}(\tau) = \lim_{T \to \infty} \frac{1}{2TN} \int_{t=-T}^{T} \{\sum_{n=1}^{N} \cos(\omega_d t \cos\alpha_{i,n} + \phi_n) \cdot$$

$$\sum_{m=1}^{N} \cos(\omega_d \cos\alpha_{i,m} \cdot (t+\tau) + \phi_m)\}dt \qquad (19)$$

$$= \frac{1}{2N} \sum_{n=1}^{N} \sum_{m=1}^{N} \overline{A+B}$$

where

$$A = \cos(\omega_d t \cos\alpha_{i,n} + \phi_n + \omega_d(t+\tau)\cos\alpha_{i,m} + \phi_m)$$
$$B = \cos(\omega_d t \cos\alpha_{i,n} + \phi_n - \omega_d(t+\tau)\cos\alpha_{i,m} - \phi_m)$$
$$(20)$$

Assuming that DPS is asymmetrical and $\alpha_{i|q;n}$ meet the conditions of (7), we can rewrite (19) and simplify other items of (18). Finally, we obtain

$$R_{hh}(\tau) = \frac{1}{N}\sum_{n=1}^{N}\exp(j\omega_d\cos\alpha_{i,n}\cdot\tau) \qquad (21)$$

In the limit of $N\to\infty$, (21) matches the ACF of reference model exactly.

Using the same procedure shown above, we can obtain the ACF for symmetric DPS

$$R_{hh}(\tau) = \sum_{n=1}^{N}\frac{\cos(\omega_d\cos\alpha_{i,n}\tau) + \cos(\omega_d\cos\alpha_{q,n}\tau)}{2N} \qquad (22)$$

Since ACF and DPS are Fourier transform pairs, ACF should be real function when DPS is symmetrical. So, formula (22) also approaches the desired ACF as N approaches infinity.

For the set-partitioning model, the L complex waveforms are mutually independent awarding to the phase selection of(15). The ACF is given by

$$\hat{R}_{hh}(\tau) = E[R_{hh}^l(\tau)] \qquad (23)$$

where $E[\cdot]$ denotes the expectation, and $R_{hh}^l(\tau)$ is the ACF of lth waveform defined in (21) or (22).

5. NUMERICAL RESULTS

The parametric Von Mises/Tikhonov distribution plays a prominent role in statistical channel modeling and analysis of angular variables today, which is defined as

$$p(\alpha) = \frac{\exp\{k\cos(\alpha - \alpha_0)\}}{2\pi I_0(k)}, \alpha \in (-\pi, \pi] \qquad (24)$$

Under this scatting environment, the theoretical expression of ACF and PSD are given by [8] as

$$R_{\tilde{h}\tilde{h}}(\tau) = \frac{I_0(\sqrt{k^2 - (2\pi f_d\tau)^2 + j4\pi kf_d\cos(\alpha_0)\tau})}{I_0(k)} \qquad (25)$$

and

$$S_{\tilde{h}\tilde{h}}(\omega) = \frac{2\cdot\exp(k\cos(\alpha_0)\omega/\omega_d)}{\sqrt{\omega_d^2 - \omega^2}} \times$$
$$\frac{\cosh(k\sin(\alpha_0)\sqrt{1-(\omega/\omega_d)^2})}{2} , \qquad (26)$$

where $I_0(\cdot)$ is the zero-order modification Bessel function, α_0 denotes the mean direction of AOA, and $k \geq 0$ controls the angular spread.

Fig.4 gives one set of $\alpha_{i|q,n}$ based on our MEPA with $N=10, k=3, \varepsilon=1/4N$ under symmetrical and asymmetrical AOA distribution. It can been seen from Fig.4(b) that $\alpha_{i|q,n}$ is asymmetrical when $p(\alpha)$ is symmetrical on $\alpha=0$ due to random phase shift ε.
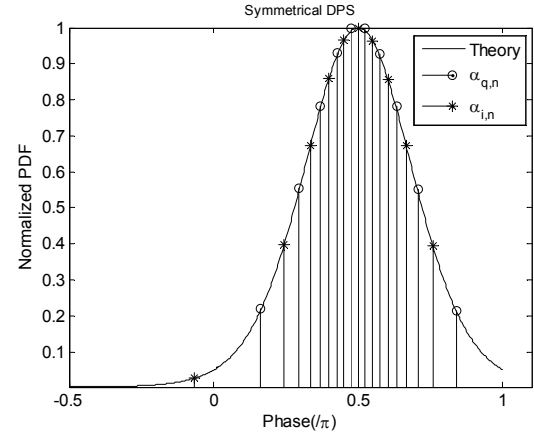
(a) AOA distribution symmetry
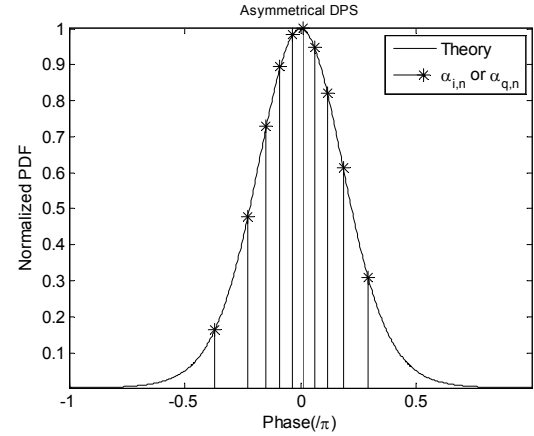
(b) AOA distribution asymmetry
Fig. 4. A set of $\alpha_{i|q,n}$ based on MEPA

We have conducted extensive simulations of our model and MEPA when the AOA's PDF is von Mises distribution ($N=20, f_d=20, f_s=1000$). Fig.5 shows the comparison between the absolute value of the ACF of our model and the reference model for various k, α_0 and L. In the figure, L denotes the number of complex Gaussian processes of set partitioning method and $f_d\tau$ is the normalized time delay. As we can see, the set-partitioning model of $L=4$ has higher precision than $L=1$ to the reference. However, two models are both approximate the reference one and the approximation are excellent, especially when the PDF is a symmetrical ($\alpha_0 = \pi/2$) or uniformly distribution ($k=0$).

Fig.6 gives some examples of PSD comparison with the same simulation parameters in Fig.5, where f/f_d denotes the normalized doppler frequency. The figure shows that when $\alpha_0 = \pi/2$, which means AOA symmetrical on $\pi/2$, the PSD will be symmetric. Moreover, the PSD curves of two models have minor difference than ACF curves in Fig.5.
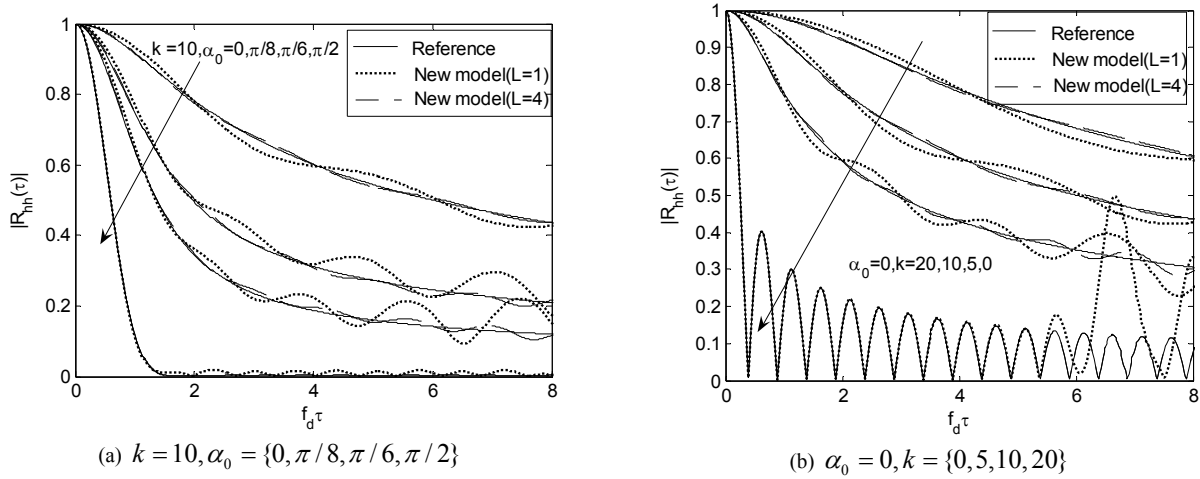
(a) $k = 10, \alpha_0 = \{0, \pi/8, \pi/6, \pi/2\}$

(b) $\alpha_0 = 0, k = \{0, 5, 10, 20\}$

Fig. 5. The absolute value of ACF comparison

(a) $k = 0, \alpha_0 = \pi/2$

(c) $k = 5, \alpha_0 = 0$

(b) $k = 10, \alpha_0 = \pi/2$
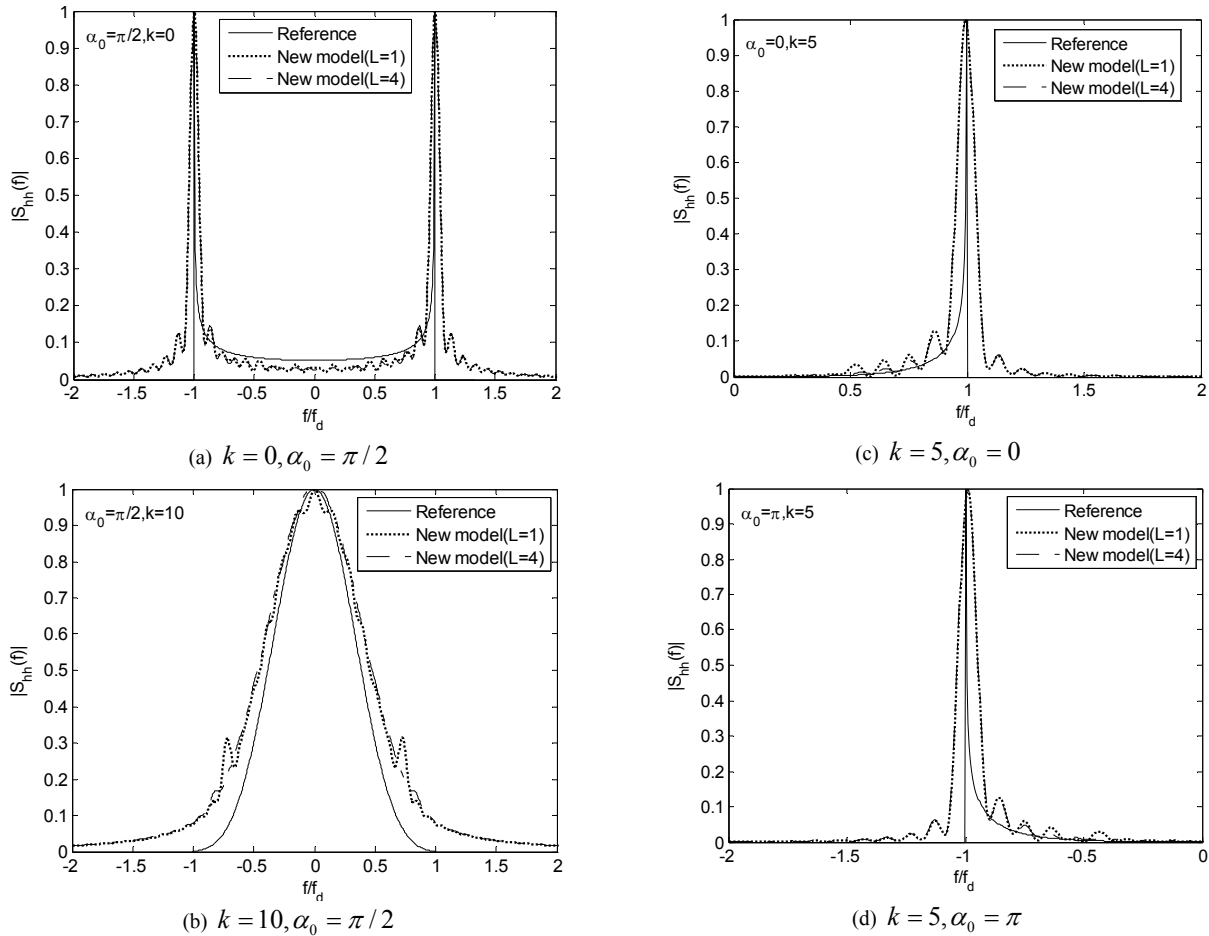
(d) $k = 5, \alpha_0 = \pi$

Fig. 6. The absolute value of PSD comparison

It is worth mentioning that the residual errors of ACF or PSD, as well as other statistical parameters, can be further reduced by a larger N or L. The results of Fig.5 and Fig.6 show that the statistical properties of the new simulator are very close to the theory even

with $N = 20, L = 4$. Moreover, when the realistic AOA distribution is too complicated and the theoretical formulations are not easy to derive, the simulation results can be used as references. Some results of ACF under Gaussian and Laplace's distribution [7] are

shown in Fig7, and they can help us to validate the analytical results such as (25).
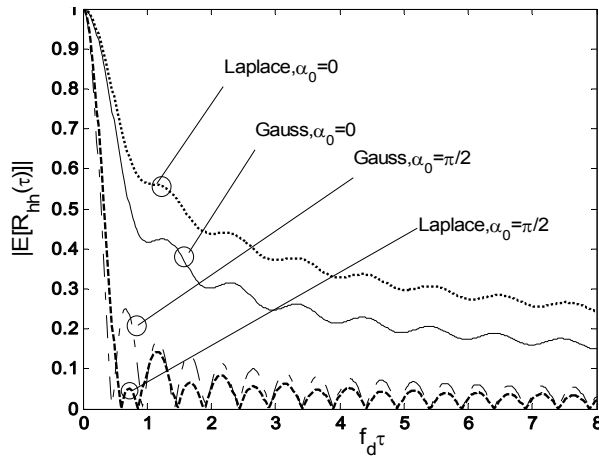

Fig. 7. The normalized absolute value of ACF under other distributions

The level crossing rate(LCR) and average fade duration(AFD)[14-15] are two other important second-order statistical properties of wireless channel. The exact solution of LCR and AFD for any AOA distribution are unknown today due to its complexity. Some simulation results of von Mises distribution are shown in Fig.8, where ρ_{rms} is the normalized fading envelope level given by $|h|/|h|_{rms}$, with $|h|_{rms}$ being the root mean square envelope level. In the simulation, 10^5 samples are generated and f_d is set to 20Hz. It's interesting to see that three curves of $k=0$, which mean isotropic scattering situation or AOA uniformly distributed, coincide with each other completely and agree with the result of Clarke's 2-D isotropic scattering model.
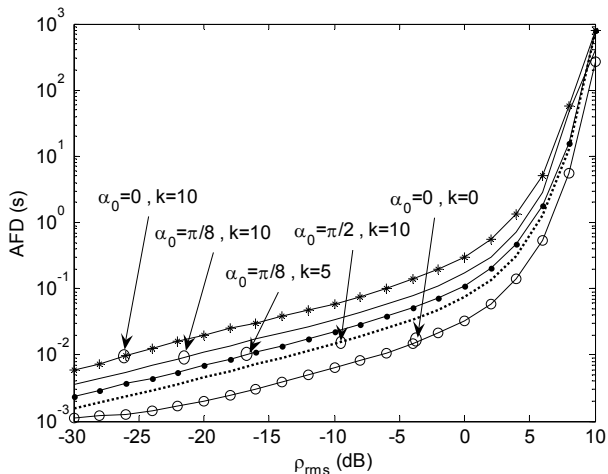

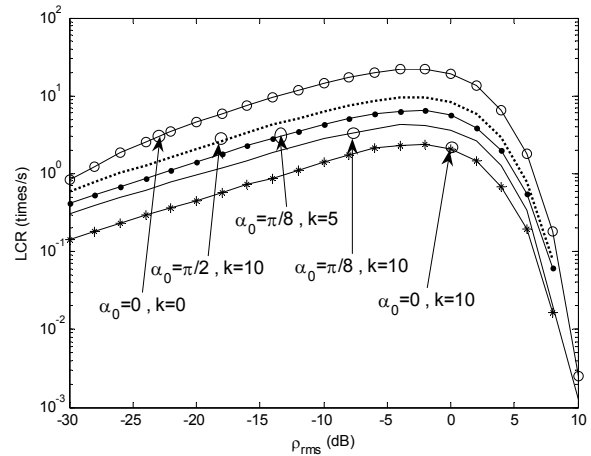**Fig. 8.** Simulation results of AFD with different α, k

Fig. 9. Simulation results of LCR with different α, k

6. CONCLUDE

In this paper, we present a modified SOS based model and a new parameter computation method which can simulate any 2-D non-isotropic channel directly with given AOA distribution. The first and second order statistics of this model converge to the reference model with increasing number of simulating the propagation path. In addition, we apply the set partition method into practice to improve the performance and generate it for multiple channels situation. Simulation results show that the statistical properties of the modified model are very close to the theory model and can be used as a reference when the theoretical results are unknown in unusual and complicated AOA distributions.

REFERENCES

[1]   S. O. Rice, **"Mathematical analysis of random noise,"** *Bell System Technology Journal*, vol.24, pp.46-156, 1945.

[2]   R. H. Clarke, **"A statistical theory of mobile radio reception,"** *Bell System Technology Journal*, vol.47, no.6, pp.957-1000, 1968.

[3]   P. DENT, G. E. BOTTOMLEY, T. CROFT. **"Jakes fading model revisited,"** *Electronics Letters*, vol.29, no.6, pp.1162-1163, 1993

[4]   S. P. Chirag, L. S. Gordon, G. P. Thomas, **"Comparative analysis of statistical models for the simulation of Rayleigh faded cellular channels,"** *IEEE Trans. on Communs.*,vol.3,no.6, pp.1017-1027,2005

[5]   C. S. XIAO, Y. R. Zheng, N. C. Beaulieu, **"Novel Sum-of-Sinusoids Simulation Models for Rayleigh and Rician Fading Channels,"** *IEEE Trans Wireless Commun*, vol.5, no.12, pp.3667-3679,2006.

[6]   W. C. Y. Lee, **"Finding the approximate angular probability density function of wave arrival by using a directional antenna,"** *IEEE Trans., Antennas Propagation*, vol.21, pp.328-334, 1973.

[7]   P. D. Teal, T. D. Abhayapala, R.A. Kennedy. **"Spatial correlation for general distributions of scatterers,"** *IEEE Signal Processing Letters*, vol. 9, no. 10, pp.

305–308, 2002

[8] A. Abdi, J. A. Barger, M. Kaveh, **"A parametric model for the distribution of the angle of arrival and the associated correlation function and power spectrum at the mobile station,"** *IEEE Trans. on VT.*, vol.51, no.3, pp.425-434, 2002.

[9] M. Patzold, **"***Mobile Fading Channels***,"** *Chichester, England, John Wiley and Sons*, 2002.

[10] M. Patzold, U. Killat, F. Laue, **"On the statistical properties of deterministic simulation models for mobile fading channels,"** *IEEE Trans.on VT.*, vol.47, no.1, pp.254-269, 1998.

[11] A. Carlos, Gutierrez-Dıaz-de-Leon, M. Patzold, **"Efficient sum-of-sinusoids-based simulation of mobile fading channels with asymmetric Doppler power spectra,"** *The 4th IEEE International Symposium on Wireless Communication Systems*, Trondheim, Norway, Oct, 2007, pp.246-251

[12] A. Carlos, Gutierrez-Dıaz-de-Leon, M. Patzold, **"Sum-of-Sinusoids-Based Simulation of Flat Fading Wireless Propagation Channels under Non-Isotropic Scattering Conditions,"** *Global Telecommunications Conference*, Washington, America, Nov, 2007, pp.3842-3846.

[13] M. Patzold, B. O. Hogstad, D. Kim, **"A new design concept for high-performance fading channel simulators using set partitioning,"** *Journal of Wireless Personal Communications*, vol.40, no.3, pp.267-279, 2007.

[14] Y. R. Zheng, C. S. Xiao, **"Simulation Models with Correct Statistical Properties for Rayleigh Fading Channels,"** *IEEE Trans on Commun*, vol.51, no.6, pp.920-928, 2003.

[15] M. Patzold, C. A. Gutierrez. **"Level-Crossing Rate and Average Duration of Fades of the Envelope of a Sum-of-Cisoids,"** *Proceeding of IEEE 66th Vehicular Technology Conference*, Singapore, May, 2008, pp.488-494