

Oscillator Noise Calculation Technique Using Time Varying Model

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ABSTRACT:

All oscillators are periodically time varying systems, so to accurate phase noise calculation and simulation, time varying model should be considered. Phase noise is an important characteristic of oscillator design. It defined as the spectral density of the oscillator spectrum at an offset from the center frequency of the oscillator relative to the power of the oscillator. In this paper, we study linear time invariant (LTI) and linear time variant (LTV) model's to calculate phase noise. Moreover, we propose a simple method for Impulse Sensitivity Function (ISF) calculation. Different oscillators have been selected to evaluate the proposed method. Simulation results show that the proposed method is simpler than other methods, and we can easily simulation ISF.

KEYWORDS: Oscillator, Phase Noise, LTV, LTI, Impulse Sensitivity Function, ISF.

1. INTRODUCTION

With the growth of wireless communication systems and stringent performance requirements, the issue of exact phase noise calculation in oscillators has become an important consideration for oscillators design. Phase noise is closely related to the performance of oscillators. Naturally, the phase noise of a whole complex system can be measured, but it can be closely approximated by adding the phase noises of different oscillators together. Phase noise describes how the frequency of an oscillator varies in short time scale. Effects of phase noise in the time domain, is timing jitter and in the frequency domain, is reciprocal Mixing. Hajimiri and Lee [1] have proposed a time varying model based on the impulse sensitivity function, ISF, to predict phase noise. This technique provides insight to the oscillators design. However, the main problem in their theory is ISF simulation.

In this article, we proposed a simple method for ISF calculation. This method can be applied for all radio frequency oscillators.

This article is including five parts. The oscillator phase noise theories (LTI and LTV models) are examined in section 2. The proposed method is demonstrated in section 3. Simulation results are presented in section 4 and conclusions are presented finally.

2. OSCILLATOR PHASE NOISE THEORIES

The spectrum of an ideal oscillator with no random fluctuations is like a pair of impulses at $\pm\omega_0$. In a practical case, the output is more generally given by:

$$V_{out}(t) = A \cdot f[\omega_0 t + \phi(t)] \quad (1)$$

where $\phi(t)$ is a random phase which is a function of time, and f is a periodic function with period $2\pi/\omega_0$. As a consequence of the fluctuation represented by $\phi(t)$, the spectrum of practical oscillator has sidebands close to the frequency of oscillation. To quantify phase noise, we consider a unit bandwidth at an offset $\Delta\omega$ with respect to ω_0 , calculate the noise power in this bandwidth, and divide the result by the signal power, Fig. 1.

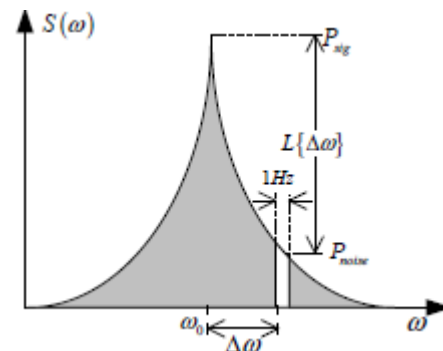


Fig. 1. The definition of phase noise.

$$\mathcal{L}(\Delta\omega) = 10 \log \left[\frac{P_{noise}(\omega_0 + \Delta\omega, 1\text{Hz})}{P_{sig}} \right], \left(\frac{dB_c}{\text{Hz}} \right) \quad (2)$$

where P_{noise} is the noise power and P_{sig} is the signal power.

2.1. LTI Models

In sub-section 2.1 and sub-section 2.2 LTI and LTV

phase noise models have been studied respectively. These models are named with the author's name.

2.1.1. Leeson

Leeson's phase noise model [2] is by far the most cited phase noise model and is based on an LTI assumption for tuned tank oscillators. Phase noise value, $L(\Delta w)$, which is based on lesson modified model is given by:

$$L(\Delta w) = 10 \log \left\{ \frac{2FkT}{P_s} \cdot \left[1 + \left(\frac{w_0}{2Q_L \Delta w} \right)^2 \right] \cdot \left(1 + \frac{\Delta w_{1/f^3}}{|\Delta w|} \right) \right\} \quad (3)$$

where F is an empirical parameter (often called the "device excess noise number"), k is Boltzmann's constant, T is the absolute temperature, P_s is the average power dissipated in the resistive part of the tank circuit, w_0 is the oscillation frequency, Q_L is the effective quality factor of the tank with all the loadings in place, Δw is the offset from the carrier and $\Delta w_{1/f^3}$ is the frequency of the corner between the $1/f^3$ and $1/f^2$ regions. Since lesson's model is based on LTI model, it is not suitable for cyclostationary noise sources. Also this model has an empirical parameter.

2.1.2. Razavi

B. Razavi proposed a phase noise model for inductorless VCOs and is well suited for CMOS ring oscillators. In [3] and [4], he proposed a new definition for Q factor, which makes Leeson's model applicable to inductorless oscillators. If an oscillator is modeled as in Fig. 2 and open loop transfer function is assumed as $H(jw) = A(w)e^{j\phi(w)}$, an open-loop Q factor is defined as follows:

$$Q = \frac{w_0}{2} \sqrt{\left(\frac{dA}{dw} \right)^2 + \left(\frac{d\phi}{dw} \right)^2} \quad (4)$$

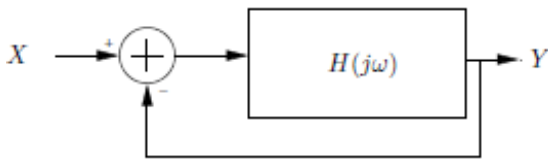


Fig. 2. Linear oscillatory system [3].

Consequently, the phase noise for an N-stage ring oscillator is given by [5]:

$$\mathcal{L}(\Delta w) = \frac{2FkTN}{P_s} \left(\frac{w_0}{2Q\Delta w} \right)^2 \quad (5)$$

The Q factor for a 3-stage ring oscillator is 1.3 and the Q factor for a 4-stage ring oscillator is 1.4. However, Q is not the only factor that determines the phase noise. A 4-stage ring oscillator has more noise

sources than a 3-stage ring oscillator because of more delay stages. It also has to dissipate more power than a 3-stage ring oscillator with the same load capacitance [5].

2.1.3. Jing Zhang

Jing Zhang has applied linear modeling technique to distributed oscillators [6], [7]. A generalized distributed oscillator is shown in Fig. 3.

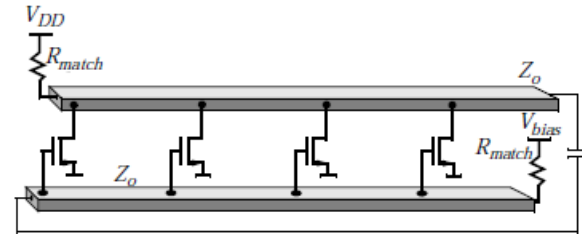


Fig. 3. Generalized distributed oscillator [8].

According to this model, the phase noise is given by:

$$L(\Delta w) = 10 \log \frac{|V_{out,noise,total}|^2}{P_c} \quad (6)$$

$$\begin{aligned} |V_{out,noise,total}|^2 &= \dots \\ &= \frac{2I_n^2 (Z_0/2)^2 (e^{-\alpha_d l} + e^{-\alpha_d 3l} + \dots + e^{-\alpha_d (2n-1)l})}{(\Delta w)^2 \left[\left(\frac{dA(w)}{dw} \right)^2 + \left(\frac{d\phi(w)}{dw} \right)^2 \right]} \end{aligned} \quad (7)$$

where P_c is carrier power, $I_n^2 = 8/3 kT g_m$ is the thermal noise power density of a MOSFET, Z_0 is transmission line characteristic impedance, α_d is attenuation constant of drain line and n is number of stages.

2.2. LTV Models

2.2.1. Hajimiri and Lee

The model of Leeson and Razavi suppose the oscillator circuit as a linear system; therefore, they are not precise. Hajimiri and Lee develop a general theory for phase noise calculation [1]. The advantage in this model is it does not depend on any oscillator topology, and presents a normalized metric, the Impulse Sensitivity Function, with which one can compare relative performance between different oscillators. As an example, consider an ideal parallel LC oscillator as it is shown in Fig. 4 (a). If a current impulse injected to the circuit, the amplitude and phase of the oscillator will be affected similar to Fig. 4 (b) and Fig. 4(c).

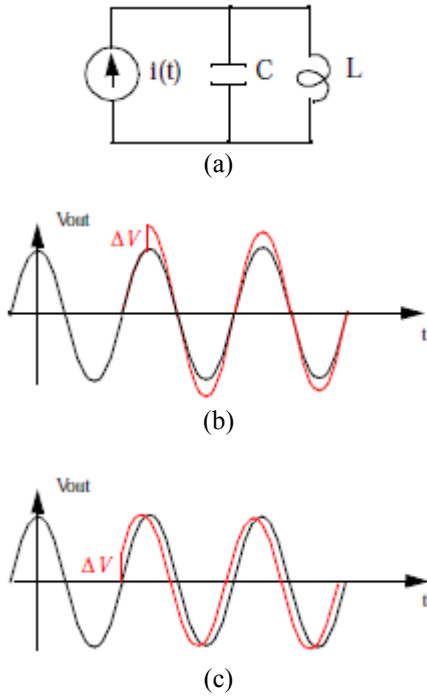


Fig. 4. (a) LC oscillator, (b) Current impulse injected at the peak changes the amplitude and has no effect on the phase, (c) Current impulse injected at zero crossing changes the phase and has minimal effect on the amplitude [10].

The instantaneous voltage change is given by [9]:

$$\Delta V = \frac{\Delta q}{C_{node}} \quad (8)$$

where Δq is the total injected charge due to the current impulse and C_{node} is the effective capacitance on that node at the time of charge injection. Phase shift is proportional to the voltage change ΔV , and hence to the injected charge Δq . Therefore $\Delta\phi$ can be written as [9]:

$$\Delta\phi = \Gamma(w_0 t) \frac{\Delta q}{q_{max}} = \Gamma(w_0 t) \frac{\Delta V}{V_{swing}} \quad (9)$$

where $q_{max} = C_{node} V_{swing}$ and V_{swing} is the voltage swing across the capacitor. The function $\Gamma(w_0 t)$, is the time-varying ‘‘proportionality factor’’. It is called the impulse sensitivity function, since it determines the sensitivity of the oscillator to an impulsive input. It is a dimensionless, frequency- and amplitude-independent periodic function that describes how much phase shift results from applying a unit impulse at any time [1]. Suppose the unit impulse response of the system as the amount of phase shift per unit current impulse, as [1], [10]:

$$h_\phi(t, \tau) = \frac{\Gamma(w_0 t)}{q_{max}} u(t - \tau) \quad (10)$$

Using equation (10) and the superposition integral, we can calculate excess phase $\phi(t)$ by the following equation:

$$\begin{aligned} \phi(t) &= \int_{-\infty}^{+\infty} h_\phi(t, \tau) i(\tau) d\tau \\ &= \frac{1}{q_{max}} \int_{-\infty}^t \Gamma(w_0 t) i(\tau) d\tau \end{aligned} \quad (11)$$

The ISF is a periodic function, so it can be expanded into a Fourier series:

$$\Gamma(w_0 t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(nw_0 t + \theta_n) \quad (12)$$

where the coefficients are real valued, and θ_n is the phase of the n_{th} order harmonic, that can be overlooked. We can consider what happens when we inject a sinusoidal current $i(\tau) = I_m \cos[(mw_0 + \Delta w)t]$, $m = 0, 1, 2, \dots$ into a node. Suppose that $\Delta w \ll w_0$, therefore $\phi(t)$ is given by [1], [10]:

$$\begin{aligned} \phi(t) &= \frac{c_0}{2q_{max}} \int_{-\infty}^t I_m \cos[(mw_0 + \Delta w)t] dt \\ &+ \sum_{n=1}^{\infty} \frac{c_n}{q_{max}} \int_{-\infty}^t I_m \cos[(mw_0 + \Delta w)t] \cos(nw_0 \tau) d\tau \end{aligned} \quad (13)$$

The only component of the integral which is preserved is for $n = m$. Therefore:

$$\begin{aligned} \phi(t) &= \frac{c_0 I_m \sin[(mw_0 + \Delta w)t]}{2q_{max}(mw_0 + \Delta w)} + \dots \\ &+ \frac{c_m I_m \sin[(2mw_0 + \Delta w)t]}{2q_{max}(2mw_0 + \Delta w)} + \dots \\ &+ \frac{c_m I_m \sin(\Delta w)t}{2q_{max}(\Delta w)} \end{aligned} \quad (14)$$

Note that first and second terms are the negligible if $m \neq 0$, and if $m = 0$, the second and third terms are zero since the lowest Fourier coefficient c_n is c_1 . Final value for $\phi(t)$ can be presented as [1], [10]:

$$\phi(t) \approx \frac{c_m I_m \sin(\Delta w)t}{2q_{max}(\Delta w)} \quad (15)$$

Mathematically, the phase noise at a Δf offset from w_0 arising from a white noise source of square magnitude $\overline{i_n^2}/\Delta f$ is equal to:

$$L\{\Delta w\} = 10 \log \left(\frac{\overline{i_n^2}/\Delta f}{8q_{max}^2 \Delta w^2} \cdot \sum_{n=0}^{\infty} C_n^2 \right) \quad (16)$$

Note that I_m represents the peak amplitude, hence $I_n^2/2 = \overline{i_n^2}/\Delta f$, for $\Delta f = 1\text{Hz}$. Using Parseval’s theorem we have:

$$\sum_{n=0}^{\infty} C_n^2 = \frac{1}{\pi} \int_0^{2\pi} |\Gamma(x)|^2 dx = 2\Gamma_{rms}^2 \quad (17)$$

where Γ_{rms} is the root mean square (rms) value of $\Gamma(x)$. Hence phase noise for a current noise is:

$$L\{\Delta w\} = 10 \log \left(\frac{\overline{i_n^2} / \Delta f \cdot \Gamma_{rms}^2}{4q_{max}^2 \Delta w^2} \right) \quad (18)$$

2.2.1.1. Impulse Sensitivity Function Examples

a) Ideal LC Oscillator

Consider the ideal LC oscillator of Fig. 4. The voltage across the capacitor and the current through the inductor can be written as [10]:

$$v(t) = V_{max} \cos(\omega t) \quad (19)$$

$$i(t) = V_{max} \sqrt{\frac{C}{L}} \sin(\omega t) \quad (20)$$

Where V_{max} is the maximum voltage amplitude and $\omega = 1/\sqrt{LC}$ is the angular frequency of oscillation. If a current impulse with an area of Δq is injected into the tank at $t = t_0$, it will induce a voltage change in the capacitor voltage. Therefore, the capacitor voltage at t_0^+ is $V_{max} \cos(\omega t) + \Delta q / C_{node}$ and the inductor current does not change. The capacitor voltage and the inductor current after t_0 will be sinusoids with a phase shift $\Delta\phi$, and an amplitude change ΔV , with respect to the initial sinusoid [10].

$$v(t) = (V_{max} + \Delta V) \cos(\omega t + \Delta\phi) \quad (21)$$

$$i(t) = (V_{max} + \Delta V) \sqrt{\frac{C}{L}} \sin(\omega t + \Delta\phi) \quad (22)$$

The voltage and current given by (21) and (22) should be equal to the initial condition at t_0^+ :

$$\begin{aligned} (V_{max} + \Delta V) \cos(\omega t + \Delta\phi) &= \dots \\ &= V_{max} \cos(\omega t) + \Delta q / C_{node} \end{aligned} \quad (23)$$

$$\begin{aligned} (V_{max} + \Delta V) \sqrt{\frac{C}{L}} \sin(\omega t + \Delta\phi) &= \dots \\ &= V_{max} \sqrt{\frac{C}{L}} \sin(\omega t) \end{aligned} \quad (24)$$

By simplifying the equations (23) and (24) and considering $\cos(\Delta\phi) \approx 1$, $\sin(\Delta\phi) \approx \Delta\phi$ and $V_{max} + \Delta V \approx V_{max}$, the following equation is obtained:

$$\Delta\phi = -\frac{\Delta q}{C_{node} V_{max}} \sin(\omega t) \quad (25)$$

By comparing equations (9) and (25), ISF is $-\sin(\omega t)$. To illustrate its significance, the ISF's together with the oscillation waveforms for a typical LC is shown in Fig. 5.

In the all oscillators, ISF has its maximum value near the zero crossings of the oscillation, and a zero value at maxima of the oscillation waveform [11].

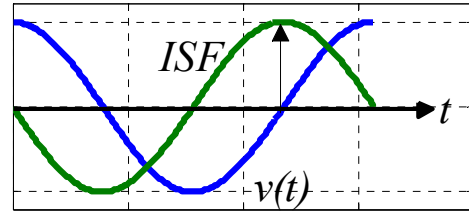


Fig. 5. Waveforms and ISF's for LC oscillator.

b) CMOS Ring Oscillators

To calculate phase noise using (18), needs to know the rms value of the ISF. To estimate Γ_{rms} , suppose that the ISF is triangular in shape and that its rising and falling edges are symmetric [10] as shown in Fig. 6.

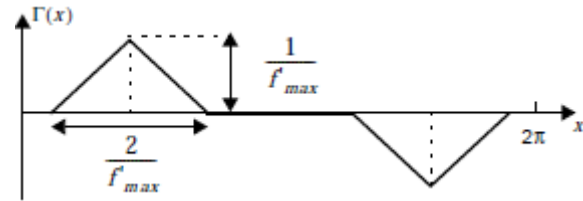


Fig. 6. Approximate ISF for ring oscillators [10].

The ISF has a maximum of $1/f'_max$, where f'_max is the maximum slope of the normalized waveform f in (1). Therefore Γ_{rms} is given by:

$$\begin{aligned} \Gamma_{rms}^2 &= \frac{1}{2\pi} \int_0^{2\pi} \Gamma^2(x) dx = \dots \\ &= \frac{4}{2\pi} \int_0^{1/f'_max} x^2 dx \\ &= \frac{3}{2\pi} \left(\frac{1}{f'_max} \right)^3 \end{aligned} \quad (26)$$

Stage delay is proportional to the rise time:

$$t_D = \frac{\eta}{f'_max} \quad (27)$$

Where t_D is the stage delay normalized to the period and η is a proportionality constant, which is typically close to unity. The period is $2N$ times longer than a single stage delay:

$$2\pi = 2N t_D = \frac{2N\eta}{f'_max} \quad (28)$$

Using (26) and (28), the following approximate expression for Γ_{rms} is obtained:

$$\Gamma_{rms} = \sqrt{\frac{2\pi^2}{3\eta^3} \cdot \frac{1}{N^{1.5}}} \quad (29)$$

$1/N^{1.5}$ term in above equation is independent of the value of η . For single-ended ring oscillator $\eta = 0.75$ and for differential ring oscillators $\eta = 0.9$.

2.2.1.2. Methods for Calculating ISF

a) The most accurate way of computing the ISF of an oscillator is by using simulations. For a single

noise source and a single output node, a current impulse is injected into the current node from the current noise source and the output excess phase is measured after a few cycles. In each simulation, the moment of injection is varied proportionally to the output signal phase such the simulations will track on whole period of the waveform [12].

- b) ISF can be calculated using the closed-form formula given for an n_{th} order system [1]:

$$\Gamma_i(x) = \frac{f'_i}{|\bar{f}'|^2} = \frac{f'_i}{\sum_{j=1}^n f_j'^2} \quad (30)$$

where f_i is the normalized waveform at node i , f' is the derivative of this waveform. For ring oscillator, the denominator of (30), show little variation, resulting in the following simplification of the closed-form ISF:

$$\Gamma(x) = \frac{f'_i(x)}{f_{max}'^2} \quad (31)$$

Advantages and disadvantages of both methods are described in Table 1.

Table 1. Comparing both methods

	Advantages	Disadvantages
Method (a)	1- Most accurate 2- No limiting assumptions 3- Second order effects is considered (Such as AM-to-PM conversion)	1- Computationally intensive
Method (b)	1- Quick estimation of the ISF	1- Second order effects is ignored

2.2.2. Dai

Based on Hajimiri and Razavi's model, in [13], Dai gives a model of phase noise. If the ring structure VCO has a very sharp transition and is fully switching with rail to rail swing, the amplitude of the oscillator will be clipped by the power supply and the ground, as shown in Fig. 7. Dai also gives the simplified version of ISF and its rms.

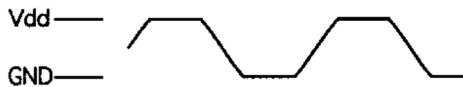


Fig. 7. Sinusoidal waveform clipped by power supplies [13]

Summarizing the Dai's phase noise model as:

$$L\{\Delta\omega\} = \begin{cases} \frac{64FkTR}{9V_{pp}^2} \left(\frac{w_0}{\Delta\omega}\right)^2 & \text{For } V_{pp} \ll V_{dd} \\ \frac{512FkTRV_{dd}}{27\pi V_{pp}^3} \left(\frac{w_0}{\Delta\omega}\right)^2 & \text{For } V_{pp} \gg V_{dd} \end{cases} \quad (32)$$

That, k is the Boltzmann's constant, T is the absolute temperature, F is the noise factor from the passive and active devices in the circuit, V_{pp} is the peak to peak voltage level of the output wave and R is the resistor.

2.2.3. Tohidian

In [14] a new phase noise calculation method is proposed in which noise sources are modeled with single tone sources. This single tone (ST) simulation directly calculates noise frequency contributions and is much faster than Hajimiri's impulse sensitivity function method.

3. PROPOSED METHOD

According to equation (1), ISF can be obtained using the following equation approximately:

$$ISF = \frac{1}{Aw_0} \cdot \frac{dV_{out, norm}(t)}{dt} \quad (33)$$

where A is the maximum voltage amplitude and w_0 is the oscillation frequency. $V_{out, norm}(t)$ is the normalized oscillator output and since $V_{out}(t) = V_{out, norm}(t) * A$, therefore:

$$ISF = \frac{1}{A^2w_0} \cdot \frac{dV_{out}(t)}{dt} \quad (34)$$

Suppose $dV_{out}(t)/dt = \alpha$, so Γ_{rms}^2 is given by:

$$\Gamma_{rms}^2 = \frac{1}{T} \int_0^T \left| \frac{1}{A^2w_0} \cdot \alpha \right|^2 dt \quad (35)$$

where T is period of ISF. For example, consider the output of ideal LC oscillator, equation (19), by using equation (33) impulse sensitivity function is $-\sin(\omega t)$.

4. SIMULATION RESULTS

For compares the prediction and simulation of the phase noise:

1- Consider 5-stage single ended 53.9 MHz ring oscillator that simulated using ADS software, Fig. 8. According to circuit data gate oxide thickness $t_{ox} = 9.5n$ and threshold voltages $V_{t, NMOS} = 0.665V$ and $|V_{t, PMOS}| = 0.9V$. All five inverters are similar with $(W/L)_{N1} = 8\mu m/1\mu m$, $(W/L)_{N2} = 4\mu m/1.5\mu m$ and $(W/L)_P = 8\mu m/1\mu m$. Note that $C_{ox} \approx 13.11 fF/m^2$, $V_{dd} = 2V$ and the total capacitance on each nod $C_{tot} \approx 0.1 pF$, therefore, $q_{max} = 0.2 pC$. Using proposed method $\Gamma_{rms} = 0.281$ while using Hajimiri method $\Gamma_{rms} = 0.3577$.

Also $(i_n^2/\Delta f) = (8/3) \cdot kT\mu C_{ox}(W/L)[(V_{dd}/2) - V_t]$ consequently $(i_n^2/\Delta f)_{tot} = 17.36 * 10^{-24} A^2/Hz$.

With inserting the values in equation (18), $L\{\Delta f\} = 10 \log(0.217/(\Delta f^2))$.

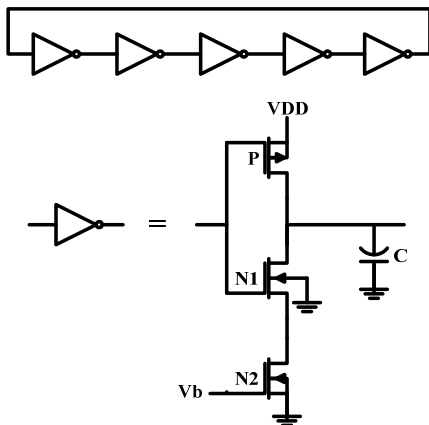


Fig. 8. 5-stage single ended 53.9 MHz ring oscillator.

The simulation results are shown in Fig. 9 and Fig. 10.

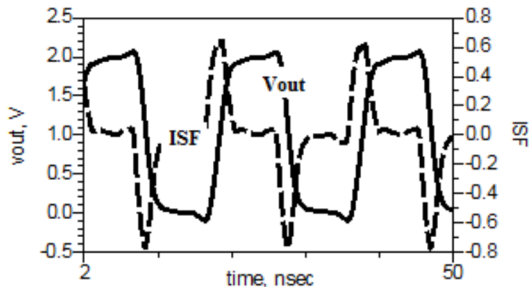


Fig. 9. Output waveform and ISF waveform.

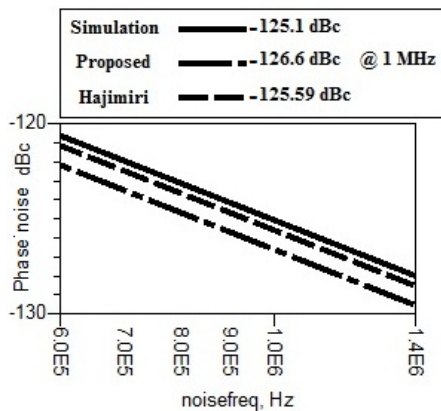


Fig. 10. Simulation result.

2- Consider 0.5 to 15GHz bipolar Clapp oscillator in ADS software, Fig. 11. According to circuit data $q_{max} = 13.1318 pC$. Using proposed method $\Gamma_{rms} = 0.225$ and $(i_n^2/\Delta f)_{tot} = 2.1 * 10^{-21} A^2/Hz$. With inserting the values in equation (18), $L\{\Delta f\} = 10 \log(0.0647/(\Delta f^2))$. Results of circuit simulation are shown in Fig. 12 and Fig. 13.

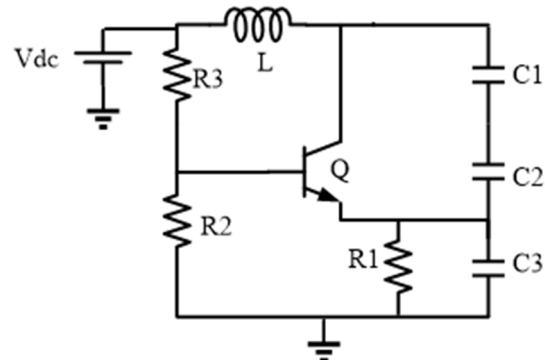


Fig. 11. Clapp oscillator.

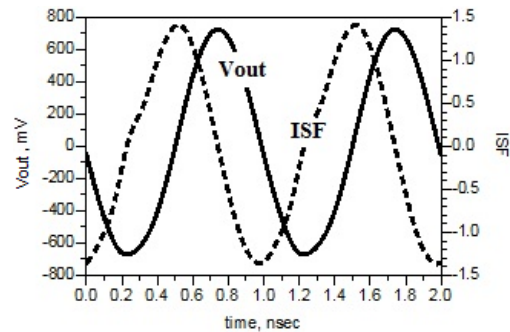


Fig. 12. Output waveform and ISF waveform.

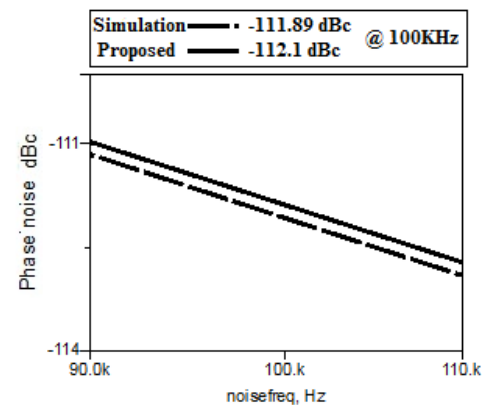


Fig. 13. Simulation result.

3- Consider 2.45 GHz LC oscillator [15, Figure 13] that TSMC 0.18 μm CMOS Process is used for simulation, Fig. 14. According to circuit data $q_{max} = 0.1484 pC$ ($V_{dd} = 2 V$). Using proposed method $\Gamma_{rms} = 0.33$ and $(i_n^2/\Delta f)_{tot} = 39.234 * 10^{-24} A^2/Hz$. With inserting the values in equation (18), $L\{\Delta f\} = 10 \log(\frac{0.03135}{\Delta f^2})$. Results of circuit simulation are shown in Fig. 15 and Fig. 16.

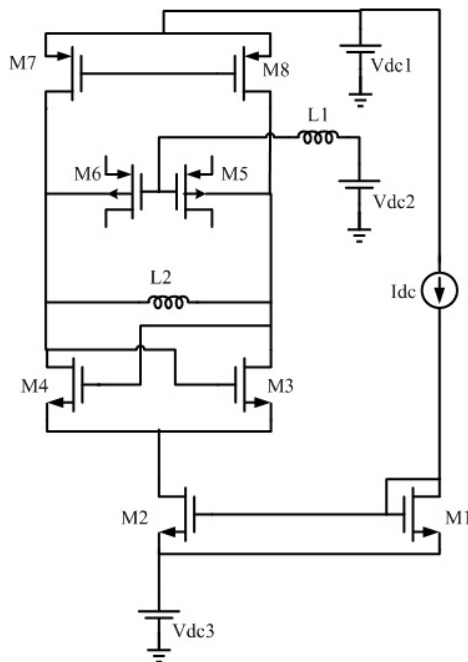


Fig. 14. LC oscillator.

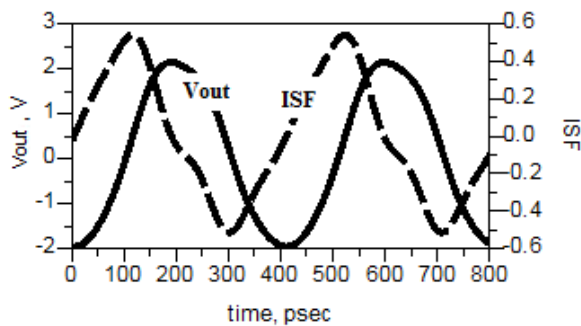


Fig. 15. Output waveform and ISF waveform.

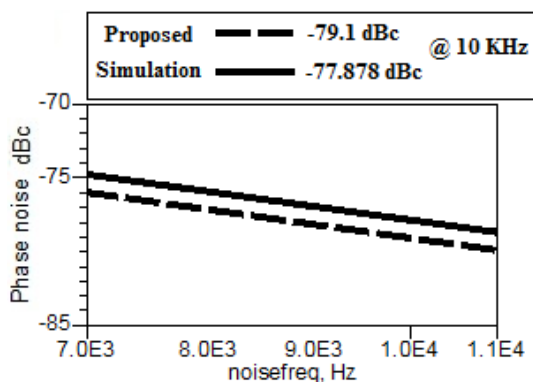


Fig. 16. Simulation result.

phase noise. According to Fig. 9 to Fig. 14, proposed method in this paper is almost exact and for simulating and calculating ISF is easier and faster than other methods. Note that proposed method, derived from a Hajimiri method and this method is suitable for simulating the ISF.

REFERENCES

- [1] A. Hajimiri, T.H. Lee. "A general theory of phase noise in electrical oscillators." *IEEE Journal Solid State Circuits*, vol. 33, no. 2, pp. 179-194, Feb. 1998.
- [2] D. B. Leeson. "A simple model of feedback oscillator noise spectrum," in *Proceeding of IEEE*, vol. 54, pp. 329-330, Feb. 1966.
- [3] B. Razavi. "A study of phase noise in CMOS oscillators," *IEEE Journal Solid State Circuits*, vol. 31, no. 3, pp. 331-343, Mar. 1996.
- [4] B. Razavi. "Analysis, modeling, and simulation of phase noise in monolithic voltage-controlled oscillators," in *Proceeding IEEE Custom Integrated Circuits Conf.*, pp. 323-326, 1995.
- [5] Liang Dai, Ramesh Harjani. *Design of high performance CMOS voltage-controlled oscillators*. Kluwer Academic Publishers, pp. 46-48, 2003.
- [6] J. Zhang, H. Mei, T. Kwasniewski. "Prediction of phase noise in CMOS distributed oscillators," in *Proceeding SBMO/IEEE MTT-S International Microwave and Optoelectronics Conf.*, vol. 1, pp. 157-162, Sep. 2003.
- [7] J. Zhang, T. Kwasniewski, H. Mei. "A computational method in predicting distributed oscillator phase noise," in *Proceeding 5th International Conference on ASIC*, vol.2, pp. 1025-1028, Oct 2003.
- [8] H. Wu and A. Hajimiri, "A 10 GHz CMOS distributed VCO," *IEEE Custom Integrated Circuits Conf.*, pp.581-584, Sept. 2000.
- [9] A. Hajimiri, S. Limotyrakis and T. H. Lee. "Phase Noise in Multi- Gigahertz CMOS Ring Oscillators," in *Proceeding Custom Integrated Circuits Conference*, May 1998.
- [10] A. Hajimiri, T.H. Lee. *The Design of Low Noise Oscillators*. Kluwer Academic Publishers, 1999, pp. 79-86 and 153-155.
- [11] T.H. Lee, A. Hajimiri. "Oscillator phase noise: a tutorial," *IEEE Journal Solid State Circuits*, vol. 35, no. 3, pp. 326-336, Mar. 2000.
- [12] Eken, Yalcin Alper. "High frequency voltage controlled ring oscillators in standard CMOS." PHD Thesis, Georgia Institute of Technology, Jun 2004.
- [13] L. Dai and R. Harjani. "Design of low-phase-noise CMOS ring-oscillators," *IEEE Trans. Circuits Syst. II*, vol. 49, pp. 328-338, May 2002.
- [14] M. Tohidian, A. Fotowat Ahmady, and M. Kamarei. "A Simplified Method for Phase Noise Calculation," in *Proceedings of the IEEE Custom Integrated Circuit Conference*, San Jose, CA, September 2009.
- [15] J. P. Silver. "LC oscillator Tutorial", Available at: http://www.odysseus.nildram.co.uk/RFIC_Circuits_Page.htm

5. CONCLUSION

The Hajimiri phase noise analysis employs a linear time variant model for the oscillator. It gives physical insight into how device noise contributes to the overall