On the Full-Rate Linear Complexity 2×2 Space-Time Block Code: Application in Keyhole Channels

S.Saleh Hosseini Bidaki[1] , Behrouz Bagheri Ranjbar[1] and Siamak Talebi [2]
1- Electrical Eng. Department, University of Kerman, Kerman, Iran.
Email: s.saleh.hosseini@gmail.com, behrouz.bagheri.r@gmail.com
2- Advanced Communications Research Institute, Sharif University, Tehran, Iran.
Email: siamak.talebi@mail.uk.ac.ir

Received: March 2012 Revised: April 2012 Accepted: May 2012

ABSTRACT:
In this paper, a new full rate, full diversity 2×2 STBC with linear complexity in the receiver is introduced for keyhole channels, where the rank-deficiency of the channel matrix degrades system performance. This code is optimized based on the known criteria for these types of the channels. Simulation results demonstrate that the proposed code outperforms some of the well-recognized STBCs when BPSK and 4-QAM constellation are utilized.

KEYWORDS: Keyhole Channels, Multi-Input Multi-Output (MIMO), Space-Time Block Codes (Stbcs).

1. INTRODUCTION

Multi input multi output (MIMO) systems such as space time codes (STCs) encounter a phenomenon, known as keyhole or pinhole, in some real environments that lead to rank-deficiency of the channel matrix. In this phenomenon, the transmitted signal must pass through a keyhole and then propagate to the receiver (Fig.1). An example of a keyhole in a realistic environment arises in a hallway or a tunnel [1].

According to this rank-deficiency, maximum achievable diversity of a STC is the minimum number of transmit and receive antennas, so the system performance will be degraded in comparison with usual i.i.d channels.

We know that various analytical studies of space time block codes (STBCs) under this condition (keyhole) have already taken place. For example, in [2], a general analysis of orthogonal STBCs (OSTBCs) over keyhole channels is undertaken, where error probability expressions involving integration of hyper geometric function is put forward by using orthogonal property of OSTBCs. Furthermore, authors in [3] focused on closed form error probability expressions of OSTBCs based on a systematic analysis. The design criteria for STCs with two transmit antennas and one receive antenna in keyhole channels are presented in [4]. These criteria have been generalized based on the analysis of the pair wise error probability (PEP) in the asymptotic of high signal to noise ratio (SNR) [5].

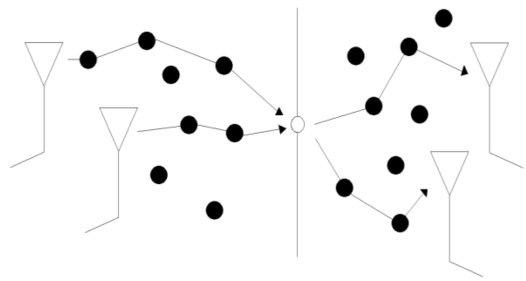

Fig. 1. Keyhole channel

It is worth mentioning that the design of STCs for keyhole channels has not been studied extensively to date. Although some space time trellis codes (STTCs) such as super orthogonal STTC are modified in [4] and [5], there are no STBCs for keyhole channels that could satisfy all the design criteria in [5].

In this paper, the aim is to modify the proposed STBC in [6], which is a full rate and full diversity code with linear complexity in the receiver, for keyhole channels based on the criteria in [5]. Simulation results illustrate that our STBC's performance is far superior to some of well-known STBCs, when BPSK and 4-QAM constellations are utilized. The remainder of this paper is organized as follows: next section describes a wireless communication system and briefly discusses the general design criteria for STCs in keyhole channels. Section 3 details the proposed STBC and also defines its properties. In section 4, simulation results are revealed and compared with results from a couple of big names in this arena to validate our approach. The paper then draws toward a close by outlining its contribution.

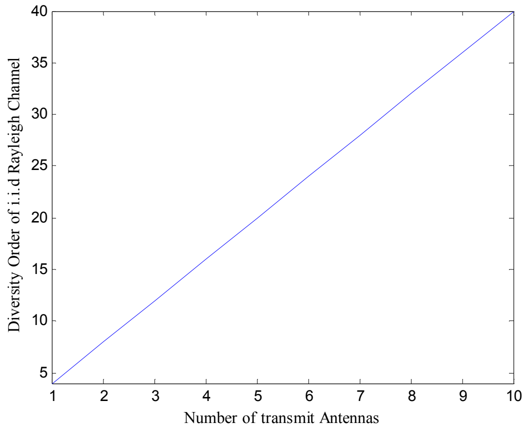

Fig. 2. Diversity order of i.i.d Rayleigh channel when $N_r = 4$.

2. SYSTEM MODEL AND DESIGN CRITERIA

2.1. System Model

Let us consider a MIMO channel with N_t transmit antennas; N_r receive antennas and quasi-static flat fading of block length T. The channel state information (CSI) is assumed to be known at the receiver, but unknown at the transmitter (perfect CSI). Moreover, we assume that the keyhole doesn't lose the energy of captured signal. The received matrix is:

$$Y = \sqrt{\frac{\rho}{N_t}} X(s)H + W \quad (1)$$

where X(s) is the $T \times N_t$ complex matrix of the transmitted signal, ρ is the average SNR at each receive antenna, and W denotes the $T \times N_r$ noise matrix in which all entries are i.i.d $\mathcal{C}N(0,1)$ (zero mean, unit variance, complex Gaussian). Since each transmitted signal has to pass through the keyhole and then propagate to the receiver, H is modelled as:

$$H = \mathbb{t} \times \mathbb{r}^t \quad (2)$$

where $\mathbb{t}$ (multipath gains at the transmitter) and $\mathbb{r}$ (multipath gains at the receiver) are column vectors $(N_t \times 1)$ with entries $\{\mathbb{t}_i\}_{i=1}^{N_t}$ and $(N_r \times 1)$ with entries $\{\mathbb{r}_i\}_{i=1}^{N_r}$, respectively. All the entries of these vectors are also i.i.d $\mathcal{C}N(0,1)$.

2.2 Design Criteria

We now discuss the design criteria for STCs in keyhole channels [5] and differences between these criteria and the well-known design criteria for STCs in common channels [7].

In order to obtain these criteria, moment generating function (MGF) analysis for PEP is used, which leads to the following upper bound for error probability in high SNR [5]:

$$P_{e\leq} \begin{cases} \frac{\Gamma(N_r-N_t)}{\Gamma(N_r)\prod_{i=1}^{N_t}\lambda_i}\frac{1}{\left(\frac{\rho}{4}\right)^{N_t}} & N_r > N_t \\ \frac{1}{\Gamma(N_r)\prod_{i=1}^{N_t}\lambda_i}\frac{\log(\frac{\rho}{4})}{\left(\frac{\rho}{4}\right)^{N_t}} & N_r = N_t \\ \frac{(-1)^{N_r-1}}{\Gamma(N_r)}\sum_{i=1}^{N_t}\frac{\log\lambda_i}{(\lambda_i)^{N_r}}\left(\prod_{i\neq j}\frac{\lambda_i}{\lambda_i-\lambda_j}\right)\frac{1}{\left(\frac{\rho}{4}\right)^{N_r}} & N_r < N_t \end{cases} \quad (3)$$

where λ_i 's are eigenvalues of the codeword difference matrix $(A = (X(s) - X'(s))^H (X(s) - X'(s)), \{X(s) \neq X'(s)\})$. It is also assumed that they are strictly positive and distinct.

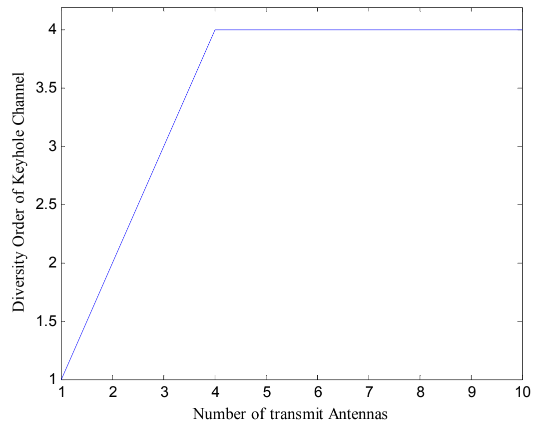

Fig. 3. Diversity order of keyhole Rayleigh channel when $N_r = 4$.

According to (3), it is obvious that the maximum achie-vable diversity is minimum (N_t, N_r) for $N_r \neq N_t$, but the upper bound of P_e has the form $\frac{\log s}{s^{N_t}}$ which makes the diversity order to be N_t with a slightly unexpected twist in the case of $N_r = N_t$ [5]. Thus, adding the antenna(s) to the minimum (N_t, N_r) until $N_r \neq N_t$ is the only way to increase diversity order in keyhole channels. As a result, the relation between diversity order and the number of transmit (or receive) antennas is not always linear in the keyhole Rayleigh channels (unlike i.i.d Rayleigh channels) where the number of receive (or transmit) is constant (Fig. 2-3).

According to (3), the coding criteria for designing STCs in the keyhole Rayleigh channels is not different from the well-known rank-determinant criteria in [7] when $N_r \geq N_t$. Hence, the codes reported for the Rayleigh fading channels would also perform well under the keyhole channel if $N_r \geq N_t$ [5].

The main difference between design criteria for these two types of channels appears when $N_t > N_r$, where the coding gain has a different expression. Therefore, the STCs in this case should satisfy this expression. For example, in [4] and [5] some STTCs are modified based on this expression.

3. CODE CONSTRUCTION AND ITS PROPERTIES

The structure of the STBC which is used in this paper is as follows:

$$\mathrm{X(s)} = \begin{bmatrix} a_1 s_1 + a_2 s_2 & b_1 s_3 + b_2 s_4 \\ -a_1 s_3^* - a_2 s_4^* & b_1 s_1^* + b_2 s_2^* \end{bmatrix}. \quad (4)$$

where s_1, s_2, s_3, and s_4 denote the symbols chosen from a constellation such as BPSK or 4-QAM. For the FRLR STBC, coefficients $\{a_1, a_2, b_1, b_2\} \subseteq \mathbb{C}$ have to meet the three constraints that are explained below:

Constraint I. Clearly, coefficients a_1, a_2, b_1, and b_2 must satisfy following equations:

$$a_1 z_1 + a_2 z_2 \neq 0 \quad (5)$$

$$b_1 z_1 + b_2 z_2 \neq 0, \quad (6)$$

except for $z_1 = z_2 = 0$, where z_1 and $z_2 \in \mathbb{Z}(i)$.

Constraint II.

$$|a_1|^2 + |a_2|^2 = |b_1|^2 + |b_2|^2 = 1. \quad (7)$$

The above condition guarantees that in each time slot equal average, power is transmitted.

Constraint III.

$$\boldsymbol{a_1 b_2^* = a_2 b_1^*}. \quad (8)$$

This constraint completely distinguishes the new proposed code from the one introduced in [8], and reduces the complexity of the ML receiver to linear complexity that will be dealt with later.

This STBC transmits four symbols over two-time slots. Therefore, its symbol transmission rate is equal to two, i.e. it benefits from full-rate property. Now, let us describe two other important properties of this code.

3.1. Full-diversity

In order to prove the full-diversity feature of the specified code in (4), the codeword difference matrix (A) must be full rank for any two different codewords $\mathrm{X(s)}$ and $\mathrm{X'(s)}$.

In this case, we have:

$$\left|\det\left(\mathrm{X(s)} - \mathrm{X'(s)}\right)\right|^2 = |a_1 e_1 + a_2 e_2|^2 |b_1 e_1^* + b_2 e_2^*|^2 + |b_1 e_3 + b_2 e_4|^2 |a_1 e_3^* + a_2 e_4^*|^2 + 2\Re\{(a_1 e_1 + a_2 e_2)(b_1 e_1^* + b_2 e_2^*)(b_1^* e_3^* + b_2^* e_4^*)(a_1^* e_3 + a_2^* e_4)\} \quad (9)$$

where $e_k = s_k - s'_k \in \mathbb{Z}(i)$ for *k=1, 2, 3, 4*. Since a_1, a_2, b_1, and b_2 satisfy *Constraint I*, sum of the first and the second terms of (9) is a positive scalar. Using *Constraint III*, the third term of (9) could be rewritten as:

$$2\Re\{[a_1 b_1^* e_1 e_3^* + a_1 b_2^* (e_1 e_4^* + e_2 e_3^*) + a_2 b_2^* e_2 e_4^*] \times [a_1^* b_1 e_1^* e_3 + a_1^* b_2 (e_1^* e_4 + e_2^* e_3) + a_2^* b_2 e_2^* e_4]\}. \quad (10)$$

Clearly, (10) is a non-negative scalar. Consequently, full diversity property is achieved by the specified STBC in (4).

3.2 Linear Complexity at the Receiver

The major feature of the proposed code is its linear decoder which is explained below.

For any received signal matrix Y, the coherent ML decoder finds:

$$\arg\min_{\mathrm{X}_\ell} \|\mathrm{Y} - \mathrm{X}_\ell H\|_F \ , \quad \ell = 1, 2, \ldots, M^L \quad (11)$$

where $\|\cdot\|_F$ denotes norm Frobenius norm and M is the constellation size. Considering (4) and (11), should minimize the following expression:

$$\mathcal{F}(s_1, s_2, s_3, s_4) = C + \mathcal{F}_1(s_1, s_2) + \mathcal{F}_2(s_3, s_4) + 2\Re\left\{(a_1 b_2^* - a_2 b_1^*)\left(\sum_{i=1}^{N_r} \hbar_{1i} \hbar_{2i}^*\right) s_1 s_4^*\right\} + 2\Re\left\{(a_2 b_1^* - a_1 b_2^*)\left(\sum_{i=1}^{N_r} \hbar_{1i} \hbar_{2i}^*\right) s_2 s_3^*\right\}. \quad (12)$$

where C is a constant which is independent of the symbols, and $\mathcal{F}_1(s_1, s_2)$ and $\mathcal{F}_2(s_3, s_4)$ are functions of $\{s_1, s_2\}$ and $\{s_3, s_4\}$, respectively. Obviously, the two latter terms of (12) are eliminated by choosing $a_1 b_2^* = a_2 b_1^*$. Therefore, the ML decoding, as in QOSTBC for three and four transmit antennas [11], results in minimizing $\mathcal{F}_1(s_1, s_2)$ for all values of s_1 and s_2 as well as $\mathcal{F}_2(s_3, s_4)$ for all values of s_3 and s_4.

Although this is a considerable reduction of the decoding complexity, it could still be decreased further by using conditional decoding for $\mathcal{F}_1(s_1, s_2)$ and $\mathcal{F}_2(s_3, s_4)$, (see [8]). In what follows, the conditional decoding procedures for $\mathcal{F}_1(s_1, s_2)$ and $\mathcal{F}_2(s_3, s_4)$ are investigated.

For simplicity, we concentrate on two receive antennas. One could imitate similar procedures decoding the received signal for other numbers of receive antennas.

By using the received matrix Y, ω_1, ω_2, ω_3, and ω_4 are prepared for a given value of symbol s_2 as follows:

$$\omega_1 = y_{11} - a_2 s_2 \hbar_{11} \quad (13\text{-a})$$

$$\omega_2 = y_{21} - b_2 s_2^* \hbar_{21} \quad (13\text{-b})$$

$$\omega_3 = y_{12} - a_2 s_2 \hbar_{12} \quad (13\text{-c})$$

$$\omega_4 = y_{22} - b_2 s_2^* \hbar_{22} \quad (13\text{-d})$$

By substituting $(y_{11}, y_{12}, y_{21}, y_{22})$ in the above equations, we have:

$$\omega_1 = a_1 s_1 \hbar_{11} + b_1 s_3 \hbar_{21} + b_2 s_4 \hbar_{21} + n_{11} \quad (14\text{-a})$$

$$\omega_2 = b_1 s_1^* \hbar_{21} - a_1 s_3^* \hbar_{11} - a_2 s_4^* \hbar_{11} + n_{21} \quad (14\text{-b})$$

$$\omega_3 = a_1 s_1 \hbar_{12} + b_1 s_3 \hbar_{22} + b_2 s_4 \hbar_{22} + n_{12} \quad (14\text{-c})$$

$$\omega_4 = b_1 s_1^* \hbar_{22} - a_1 s_3^* \hbar_{12} - a_2 s_4^* \hbar_{12} + n_{22} \quad (14\text{-d})$$

Now, $\alpha|_{s_2}$ is computed from (14-a) to (14-d) as:

$$\begin{aligned} \alpha|_{s_2} &= a_2^* \hbar_{11}^* \omega_1 + b_2 \hbar_{21} \omega_2^* + a_2^* \hbar_{12}^* \omega_3 + b_2 \hbar_{22} \omega_4^* \\ &= \{a_1 a_2^* (|\hbar_{11}|^2 + |\hbar_{12}|^2) + b_1^* b_2 (|\hbar_{21}|^2 + |\hbar_{22}|^2)\} s_1 \\ &\quad + \{(a_2^* b_1 - a_1^* b_2)(\hbar_{11}^* \hbar_{21} + \hbar_{12}^* \hbar_{22})\} s_3 \\ &\quad + \{(a_2^* b_2 - a_2^* b_2)(\hbar_{11}^* \hbar_{21} + \hbar_{12}^* \hbar_{22})\} s_4 \\ &\quad + \{a_2^* \hbar_{11}^* n_{11} + b_2 \hbar_{21} n_{21}^* + a_2^* \hbar_{12}^* n_{12} + b_2 \hbar_{22} n_{22}^*\}. \end{aligned} \quad (15)$$

By applying *Constraint III* to (15) and dividing the result by coefficient of s_1, say θ, we have:

$$s_1^{ML}|_{s_2} = s_1 + \frac{\mathcal{N}}{\theta} \quad (16)$$

where

$$\mathcal{N} = a_2^* \hbar_{11}^* n_{11} + b_2 \hbar_{21} n_{21}^* + a_2^* \hbar_{12}^* n_{12} + b_2 \hbar_{22} n_{22}^*.$$

Therefore, we could obtain the ML estimate of symbol s_1 conditional on symbol s_2, say $s_1^{ML}|_{s_2}$. Hence, instead of minimizing $\mathcal{F}_1(s_1, s_2)$ for all possible values of s_1 and s_2, we just need to minimize the metric $\mathcal{F}_1(s_1^{ML}|_{s_2}, s_2)$ for all values of s_2.

In a similar way, to minimize $\mathcal{F}_2(s_3, s_4)$ conditionally, we have:

$$\beta|_{s_2} = \{a_1^* a_2 (|\hbar_{11}|^2 + |\hbar_{12}|^2) + b_1 b_2^* (|\hbar_{21}|^2 + |\hbar_{22}|^2)\} s_3 + \{b_2^* \hbar_{21}^* n_{11} - a_2 \hbar_{11} n_{21}^* + b_2^* \hbar_{22}^* n_{12} - a_2 \hbar_{12} n_{22}^*\} \quad (17)$$

and therefore:

$$s_3^{ML}|_{s_4} = s_3 + \frac{\mathcal{N}'}{\theta'} \quad (18)$$

where

$$\theta' = a_1^* a_2 (|\hbar_{11}|^2 + |\hbar_{12}|^2) + b_1 b_2^* (|\hbar_{21}|^2 + |\hbar_{22}|^2)$$

$$\mathcal{N}' = b_2^* \hbar_{21}^* n_{11} - a_2 \hbar_{11} n_{21}^* + b_2^* \hbar_{22}^* n_{12} - a_2 \hbar_{12} n_{22}^*$$

Since $s_3^{ML}|_{s_4}$ only depends on symbol s_3 and noise terms, minimizing procedure of $\mathcal{F}_2(s_3, s_4)$ is limited to minimizing the cost function $\mathcal{F}_2(s_3^{ML}|_{s_4}, s_4)$ for all possible values of symbol s_4. Now, by minimizing $\mathcal{F}_1(s_1^{ML}|_{s_2}, s_2)$ for all possible values of s_2, estimations of symbols s_1 and s_2 and minimizing $\mathcal{F}_2(s_3^{ML}|_{s_4}, s_4)$ for all possible values of s_4, estimations of symbols s_3 and s_4 are obtained. As a result, the suggested STBC in (4) leads to a linear complexity at the receiver for an optimum decoder.

4. SIMULATION RESULTS

In this section, we first choose parameters of the proposed code according to criteria in section 2 by computer search and then compare its performance against two well-known STBCs, Alamouti code [9], and Golden code [10] in the keyhole Rayleigh fading channel.

For BPSK and 4-QAM constellations, parameters a_1, a_2,b_1, and b_2 are tabulated in Table 1 and 2. As these tables show, however, the allocated powers for different symbols are different, *Constraint II* being still observed. That means the total average power at each time slot per transmit antenna is equal.

We compare the symbol error probability (SER) performances of the new proposed code with Golden and Alamouti codes in Fig. 4 and Fig. 5, where the bandwidth efficiencies are 2 bits/sec/Hz and 4 bits/sec/Hz, respectively. Note that two transmit antennas and one receive antenna are used in both figures.

According to Fig. 4, our proposed code outperforms both Alamouti and Golden codes. For instance, the new proposed STBC has contributed almost 2dB reduction in SNR at a SER of 10^{-3} when throughput is 2 bits/sec/Hz. A similar scenario occurs in Fig. 5, i.e. nearly 2 dB reduction in SNR at a SER of 2×10^{-3} with throughput equal to 4 bits/sec/Hz.

It can also be shown that similar results are obtained for two transmit and two receive antennas, as in Fig. 6-7. As it can be seen from Fig. 6, with SER at 10^{-5}, the proposed code performs better than the Alamouti code by almost 1.5dB, when bandwidth efficiency is 2 bits/sec/Hz. Moreover, the difference in SNR between this code and the Alamouti code, is approximately 1dB with throughput equal to 4 bits/sec/Hz, when SER is 10^{-5}, as shown in Fig. 7.

5. CONCULTIONS

In this paper, we have introduced an innovative full rate, full diversity 2×2 STBC with linear complexity in the receiver for the keyhole channels. This code is based on the special criteria of keyhole condition. Simulation results also confirm that the proposed code performance is outstanding for 4-QAM and BPSK constellations.

Table 1. Optimum parameters for the proposed code ($N_r = 1$).

	a_1	a_2	b_1	b_2
BPSK	$.7071e^{j3.1416}$	$.7071e^{-j1.9416}$	$.7071e^{-j1.9416}$	$.7071e^{j3.1416}$
4-QAM	$.4472e^{j3.1416}$	$.8944e^{j3.1416}$	$.4472e^{-j3.0416}$	$.8944e^{-j3.0416}$

Table 2. Optimum parameters for the proposed code ($N_r = 2$).

	a_1	a_2	b_1	b_2
BPSK	$.6708e^{j.5}$	$.7416e^{j2}$	$.6708e^{-j1.5}$	$.7416e^{-j3}$
4-QAM	$.4472e^{j.1}$	$.8944e^{j.3}$	$.4472e^{-j.4}$	$.8944e^{-j.6}$

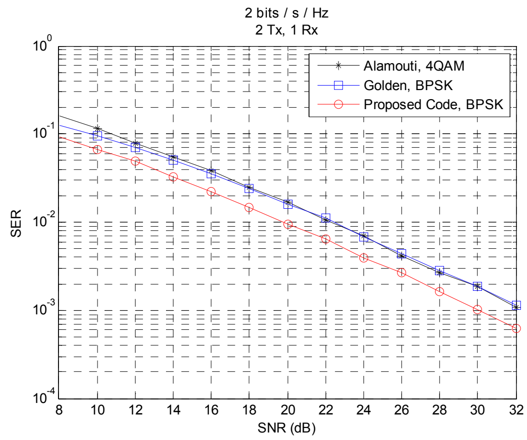

Fig. 4. Performance comparison of the proposed code with Golden and Alamouti codes with the same bandwidth efficiency 2bits/s/Hz.

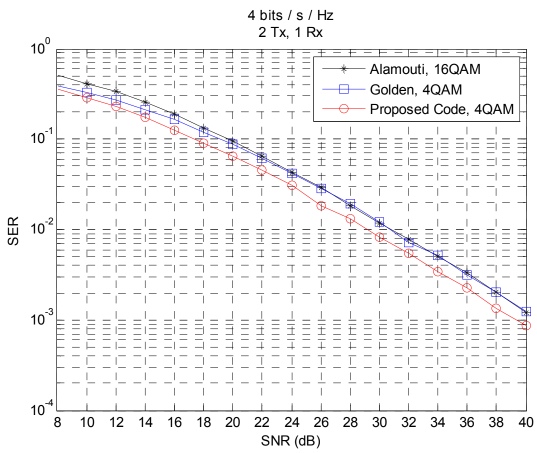

Fig. 5. Performance comparison of the proposed code with Golden and Alamouti codes with the same bandwidth efficiency 4bits/s/Hz.

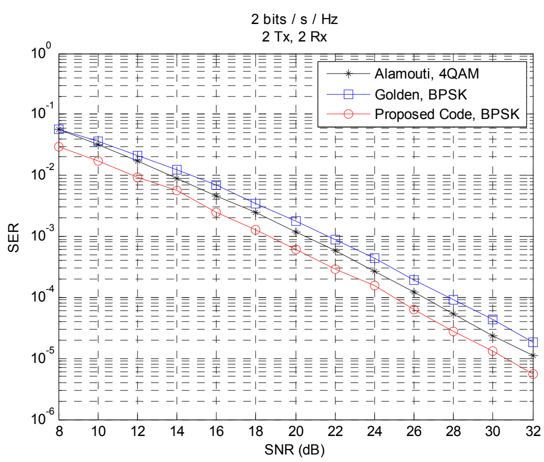

Fig. 6. Performance comparison of the proposed code with Golden and Alamouti codes with the same bandwidth efficiency 2bits/s/Hz.

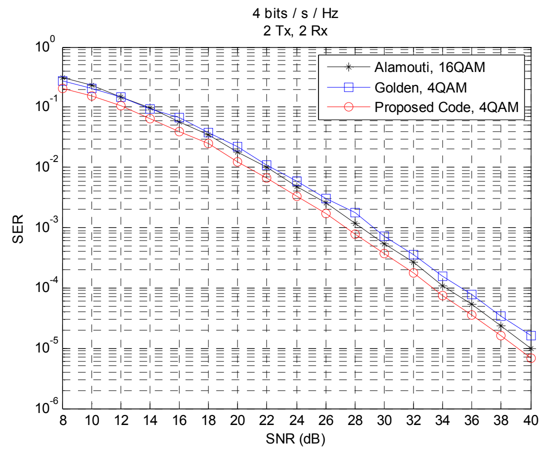

Fig. 7. Performance comparison of the proposed code with Golden and Alamouti codes with the same bandwidth efficiency 4bits/s/Hz.

REFERENCES

[1] D. Chizhik, G. J. Foschini, M. J. Gans, and R. A. Valenzuela, **"Keyholes, correlations, and capacities of multi-element transmit and receive antennas,"** *IEEE Trans. Wireless Commun.*, vol. 1, no. 2, pp. 361–368, Apr. 2002.

[2] H. Shin and J. H. Lee, **"Performance analysis of space–time block codes over keyhole Nakagami-m fading channels,"** *IEEE Trans. Veh. Technol.*, vol. 53, no. 2, pp. 351–362, Mar. 2004.

[3] Y. Gong and K. B. Letaief, **"On the error probability of orthogonal space-time block codes over keyhole MIMO channels,"** *IEEE Trans. Wireless Commun.*, vol. 6, no. 9, pp. 3402–3409, Sept. 2007.

[4] A. M. Nasri Nasrabadi, H. R. Bahrami, and S. H. Jamali, **"Design of space-time trellis codes for keyhole channels: Performance criterion and code design,"** *IEEE Electron. Lett.*, vol. 40, no. 1, pp. 53–55, Jan. 2004.

[5] S. Sanayei, A. Hedayat, and A. Nosratinia, **"Space-time codes in keyhole channels: Analysis and design,"** *IEEE Trans. Wireless Commun.*, vol. 6, no.6, pp. 2006–2011, Jun. 2007.

[6] S. S. Hosseini B., S. Talebi, and M. Shahabinejad, **"A full-rate full-diversity 2×2 space-time block code with linear complexity for the maximum likelihood receiver,"** *IEEE Commun. Lett.*, vol. 15, no.8, pp. 842-844, Aug. 2011.

[7] V. Tarokh, N. Seshadri, and A. R. Calderbank, **"Space-time codes for high data rate wireless communications: Performance criterion and code construction,"** *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.

[8] S. Sezginer, H. Sari, and E. Biglieri, **"On high-rate full-diversity 2×2 space-time codes with low-complexity optimum detection,"** *IEEE Trans. Commun.*, vol. 57, no. 5, pp. 1532-1541, May 2009.

[9] S. M. Alamouti, **"A simple transmit diversity technique for wireless communications,"** *IEEE*

J.Sel. Areas Commun. vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

[10] J.-C. Belfiore, G. Rekaya, and E. Viterbo, **"The Golden code: a 2×2full- rate space-time code with non-vanishing determinants,"** *IEEE Trans. Inf. Theory*, vol. 51, no. 4, pp. 1432–1436, Apr. 2005.

[11] H. Jafarkhani, **"A quasi-orthogonal space–time block code,"** *IEEE Trans. Commun.*, vol. 49, no. 1, pp. 1–4, Jan. 2001.