A New Full-Diversity Space-Time-Frequency Block Code for MIMO-OFDM Systems

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Received: June 2012 Revised: X XX Accepted :X XX

ABSTRACT:
Multi-input multi-output-orthogonal frequency-division multiplexing (MIMO-OFDM) is known as a proper solution for the wideband wireless communication. Numerous space-frequency block codes (SFBCs) and space-time-frequency block codes (STFBCs) have been proposed so far for implementing MIMO-OFDM systems. In this paper, a new full-diversity STFBC is proposed for two transmit antennas, which could benefit from the maximum coding advantage when delay and power profiles (DPPs) of the channel are available at the transmitter. Furthermore, simulation results confirm that the proposed STFBC outperforms the other recently proposed STFBCs with the same order of the receiver complexity.

KEYWORDS: Index Terms—Wireless communication, space-time-frequency coding, frequency-selective fading channels, multi-input multi-output-orthogonal frequency-division multiplexing (MIMO-OFDM) systems.

1. INTRODUCTION

As a sort of multi-input multi-output (MIMO) systems, space-time coding is one of the most advanced methods used to deal with ruinous fadingeffect of the wireless channels for the narrowband wireless communication [1]-[4].

High speed data transmission could also turn the flat-fading channels into frequency-selective channels which causes an extreme intersymbol interference (ISI) in addition to fading, Space-time coding can be used in the frequency-selective channels too [5]. But equalizers are needed in the receiver that follows the complexity of the receiver and the loss of the frequency diversity. Indeed,

in the frequency-selective channels, due to L different paths between each pair of the transmitter and the receiver antennas, there are L different replicas of the transmitted signals at the receiver. So the communication system suffers from the ISI. This multipath phenomenon seems to be distasteful at first, but it could be used as a source of diversity [7], [8].

In order to deal with the ISI effect of the channel, orthogonal frequency-division multiplexing (OFDM) have been developed. OFDM spreads symbols over a larger time slot using orthogonal subcarriers for modulating different symbols. In fact, OFDM transforms the wideband channels to a set of narrowband flat-fading sub-channels.

Multi-input multi-output-orthogonal frequency-division multiplexing (MIMO-OFDM) systems take advantage of both OFDM and MIMO for multipath and fading channels, respectively. Space-frequency block codes (SFBCs) and space-time-frequency block codes (STFBCs) are two schemes for implementing MIMO-OFDM systems [6]-[15]. SFBCs use both the multipath and the additional spatial diversity [5]-[8]. The performance criteria of SFBCs are derived in [7]. For a full-diversity SFBC, the highest available diversity is equal to LM_tM_r, where M_t and $M_r$are the number of the transmitter and the receiver antennas, respectively. The STFBCs utilize more than one timeslot by coding across multiple OFDM blocks, which presents the additional time diversity. Design criteria of STFBCs are provided in [8], [10], [16]. The added temporal dimension can be useful from two aspects: First, to reduce the receiver complexity of the STFBCs upon quasi-static channels [10], [15], [17]. Second, to use an additional diversity source when channel varies for different OFDM blocks [8], [9], [15]. When the channel behavior changes for different OFDM blocks, diversity gain of STFBCs is $M_tM_rL \times rank(R_T)$, where $R_T$is temporal correlation matrix of the channel [8].

Numerous SFBCs and STFBCs have been reported to date. By using the existing space-time block codes (STBCs), primary SFBCs were designed by substituting time slots for the frequency subcarriers [5]. Based on the complex orthogonal STBCs (OSTBCs) [2], full-diversity STFBCs are proposed in [6] and [10] which benefit from simple decoder upon quasi-static channels. It could be shown that performances of these codes degrade drastically over channels that change from one OFDM symbol to the next. Rate-M_t STFBCs and SFBCs are put forward in [6] and [13], respectively. In [7], rate-one SFBCs are explored which benefit from the maximum coding advantage, and in [9] linear transform based STFBCs and SFBCs are optimized. In [12], full-diversity SFBCs are attained by repeating each OSTBCs Γ_0 times. Thus, denoting rate of the OSTBCs by R_O, they make a tradeoff between the symbol transmission rate of $\frac{R_O}{\Gamma_0}$ and the diversity advantage equal to $\Gamma_0 M_t M_r$. In [14] and [15], authors presented a new class of full-diversity STFBCs and SFBCs based on the generalized block-diagonal quasi-orthogonal STBCs.

In this paper, we aim to propose a new STFBC. In the new scheme, we first generate a linear combination of symbols by using the Vandermonde matrices. Then, we substitute these combinations of symbols in the Alamouti code. It is shown throughout the paper that the newly proposed scheme attains full-diversity. The proposed STFBC also benefits from the maximum coding advantage when partial channel state information (delay and power profiles) is available at the transmitter. Besides, simulation results depict that newscheme could outperform the best known STFBCs effectively.

Notations: We use capital boldface letters for matrices, and boldface letters for vectors. Subscripts $(\cdot)^T$, $(\cdot)^\dagger$ and $(\cdot)^*$ denote transpose, Hermitian and complex conjugation, respectively. $\circ$, and $\otimes$ stand for the Hadamard and the tensor products, respectively. Notation diag$(a_1, a_2, ..., a_n)$ represents a diagonal $n \times n$ matrix whose diagonal entries are $a_1, a_2, \ldots, a_n$. $\mathbb{C}$stands for the complex field, and $\boldsymbol{V}(t_1, t_2, \ldots, t_n)$denotes a Vandermonde matrix with parameters $t_1, t_2, \ldots, t_n$, i.e.

$$\boldsymbol{V}(t_1, t_2, \ldots, t_n) = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ t_1 & t_2 & \cdots & t_n \\ \vdots & \vdots & \ddots & \vdots \\ t_1^{n-1} & t_2^{n-1} & \cdots & t_n^{n-1} \end{bmatrix} \in \mathbb{C}^{n \times n}.$$

$\jmath = \sqrt{-1}$, $\lfloor \cdot \rfloor$stands for the floor operation,$\mathbf{1}_a$ indicates an$a \times a$ matrix of ones, and $\boldsymbol{I}_a$ represents an $a \times a$ identity matrix.

2. System Model

In this section, we describe the system model of a MIMO-OFDM system. Consider a space-time-frequency-coded MIMO-OFDM system with M_t transmit antennas,M_r receiver antennas and N subcarriers with$K$ successive OFDM blocks. Channel impulse response during the k^{th} OFDM block from the transmitter antenna i to the receiver antenna j is given by [8]:

$$h_{i,j}^k(\zeta) = \sum_{l=0}^{L-1} \alpha_{i,j}^k(l)\delta(\zeta - \zeta_l), k = 1,2, \ldots, K, \quad (1)$$

where ζ_l's are delays and $\alpha_{i,j}^k(l)$'s are zero-mean complex Gaussian random variables with variance σ_l^2, indicating the complex amplitude corresponding to the l^{th} path of the i^{th} transmitter and the j^{th} receiver antennas in the k^{th} OFDM block, and $\sum_{l=0}^{L-1} \sigma_l^2 = 1$ for normalization purposes. It is supposed that there is no spatial fading correlation between antennas, and the receiver has the perfect channel state information.

Each codeword of a STFBC can be formed as a $KN \times M_t$ matrix as below:

$$\mathbf{C} = [\mathbf{C}_1^T \quad \mathbf{C}_2^T \quad \cdots \quad \mathbf{C}_K^T]^T, \quad (2)$$

where

$$\mathbf{C}_k = \begin{bmatrix} c_1^k(0) & c_2^k(0) & \ldots & c_{M_t}^k(0) \\ c_1^k(1) & c_2^k(1) & \ldots & c_{M_t}^k(1) \\ \vdots & \vdots & \ddots & \vdots \\ c_1^k(N-1) & c_2^k(N-1) & \ldots & c_{M_t}^k(N-1) \end{bmatrix}, k = 1, 2, \ldots, K. \quad (3)$$

In (3), $c_i^k(n)$'s are symbols or linear combinations of them which are transmitted over the n^{th} subcarrier by the transmit antenna iand the k^{th} OFDM block. After applying an N-point inverse fast Fourier transform to each column of $\mathbf{C}_k$ and adding cyclic prefix, the i^{th}column of $\mathbf{C}_k$ is transmitted by the transmitter antennai.

The received signal at the j^{th} receiver antenna and the k^{th} OFDM block, after crossing from matched filter, removing cyclic prefix and performing fast Fourier transform is given by:

$$r_j^k(n) = \sum_{i=1}^{M_t} c_i^k(n) H_{i,j}^k(n) + \mathcal{N}_j^k(n), \quad n = 0,1, \ldots, N-1, \quad (4)$$

Where $\mathcal{N}_j^k(n)$ denotes the zero-mean additive white complex Gaussian noise corresponding to the n^{th} frequency subcarrier and

$$H_{i,j}^k(n) = \sum_{l=0}^{L-1} \alpha_{i,j}^k(l) w^{n\zeta_l} \quad (5)$$

Represents the channel frequency response at the n^{th} subcarrier between the transmitter antenna i and the receiver antenna j and $w = e^{-\jmath 2\pi \frac{BW}{N}}$, where BW is the total bandwidth of the system

3. Newly Proposed STFBC for MIMO-OFDM Systems

A. Structure of the proposed STFBC:

The initial structure of the proposed STFBC could be considered asbelow:

$$\mathbf{C}_k = [\mathbf{G}_{k,1}^T, \mathbf{G}_{k,2}^T, \ldots, \mathbf{G}_{k,P}^T, \mathbf{Z}^T]^T \in \mathbb{C}^{N \times 2} \quad k = 1, 2, \ldots, K, \quad (6)$$

where $P = \left\lfloor \frac{N}{2L} \right\rfloor$, $G_{k,p}$'s for $p = 1,2,\dots,P$ are matrices of size $2L \times 2$ whose constructions are mutually exclusive, and $\mathbf{Z}$ is an $(N-2PL) \times 2$ matrix of zeros.

First, taking the data symbol vector $[s_1^p, s_2^p, \dots, s_{2LK}^p]^T$ from a constellation such as BPSK or QPSK, the precoded vector $[x_1^p, x_2^p, \dots, x_{2LK}^p]^T$ could be derived from equation below:

$$[x_1^p, x_2^p, \dots, x_{2LK}^p]^T = \boldsymbol{V}[s_1^p, s_2^p, \dots, s_{2LK}^p]^T \in \mathbb{C}^{2KL\times 1} \quad (7)$$

where $\boldsymbol{V} \in \mathbb{C}^{2KL\times 2KL}$ stands for the Vandermonde matrix whose parameters are selected in the same way as those of equation (3.16) of [7]. Then, $\boldsymbol{G}_{k,p}$'s are generated as:

$$\boldsymbol{G}_{k,p} = \begin{bmatrix} \boldsymbol{X}^p_{(k-1)L+1} \\ \boldsymbol{X}^p_{(k-1)L+2} \\ \vdots \\ \boldsymbol{X}^p_{kL} \end{bmatrix}, k = 1,2,\dots,K. \quad (8)$$

In (8), each $\boldsymbol{X}_i \in \mathbb{C}^{2\times 2}$ is the Alamouti code constructed by twodistinct x_i^p's.

In the following, we enhance the coding advantage of the proposed STFBC by adding a new parameter, namely γ_{SD}, to its design in order to attain a better performance. The structure of $\boldsymbol{C}_k$when parameter γ_{SD} is added to the code design is changed to $\boldsymbol{C}_k^M$by the following equation:

$$\boldsymbol{C}_k^M = \boldsymbol{P}\boldsymbol{C}_k, k = 1,2,\dots,K, \quad (9)$$

where $\boldsymbol{P} = \mathrm{diag}(\boldsymbol{P}_t, \boldsymbol{Z}_b) \in \mathbb{N}^{N\times N}$, and

$$\boldsymbol{P}_t = \mathrm{diag}\left(\overbrace{\boldsymbol{P}_b, \boldsymbol{P}_b, \dots, \boldsymbol{P}_b}^{\text{Number of} \boldsymbol{P}_b\text{'s} = \left\lfloor \frac{N}{L\gamma_{SD}} \right\rfloor}, \boldsymbol{P}_b^{'} \right) \otimes \boldsymbol{I}_2 \in \mathbb{C}^{L\gamma_{SD}\left\lfloor \frac{N}{L\gamma_{SD}} \right\rfloor \times L\gamma_{SD}\left\lfloor \frac{N}{L\gamma_{SD}} \right\rfloor}. \quad (10)$$

In (10),

$$\boldsymbol{P}_b = [\boldsymbol{P}_1^T \quad \boldsymbol{P}_2^T \quad \dots \quad \boldsymbol{P}_L^T]^T \in \mathbb{N}^{L\frac{\gamma_{SD}}{2}\times L\frac{\gamma_{SD}}{2}}, \quad (11)$$

where

$$\boldsymbol{P}_i = \left[\boldsymbol{e}_i^T \quad \boldsymbol{e}_{L+i}^T \quad \dots \quad \boldsymbol{e}^T_{\left(\frac{\gamma_{SD}}{2}-1\right)L+i}\right]^T \in \mathbb{N}^{\frac{\gamma_{SD}}{2}\times L\frac{\gamma_{SD}}{2}},$$
$$i = 1,2,\dots,L. \quad (12)$$

In (12), $\boldsymbol{e}_i \in \mathbb{C}^{1\times L\frac{\gamma_{SD}}{2}}$ is a vector whose components are all zeros except for the i^{th} element that is one, and

$$\boldsymbol{P}_b^{'} = [\boldsymbol{P}_1^{'T} \quad \boldsymbol{P}_2^{'T} \quad \dots \quad \boldsymbol{P}^{'T}_{\left\lfloor \frac{\gamma_r}{2L} \right\rfloor}]^T \in \mathbb{C}^{L\left\lfloor \frac{\gamma_r}{2L} \right\rfloor \times L\left\lfloor \frac{\gamma_r}{2L} \right\rfloor}, \quad (13)$$

where$\gamma_r = N - L\gamma_{SD}\left\lfloor \frac{N}{L\gamma_{SD}} \right\rfloor$ and

$$\boldsymbol{P'}_i = \left(\boldsymbol{e'}_i^T \quad \boldsymbol{e'}^T_{\left\lfloor \frac{\gamma_r}{2L} \right\rfloor + i} \quad \dots \quad \boldsymbol{e'}^T_{(L-1)\left\lfloor \frac{\gamma_r}{2L} \right\rfloor + i}\right)^T \in \mathbb{C}^{L\times L\left\lfloor \frac{\gamma_r}{2L} \right\rfloor},$$
$$i = 1,2,\dots,\left\lfloor \frac{\gamma_r}{2L} \right\rfloor. \quad (14)$$

In (14),$\boldsymbol{e'}_i \in \mathbb{C}^{1\times L\left\lfloor \frac{\gamma_r}{2L} \right\rfloor}$is a vector whose components are all zeros except for the i^{th} element that is one. And $\boldsymbol{Z}_b$is a $(\gamma_r - 2L\left\lfloor \frac{\gamma_r}{2L} \right\rfloor) \times (\gamma_r - 2L\left\lfloor \frac{\gamma_r}{2L} \right\rfloor)$ matrix of zeros.

Remark: Regarding the structure of the proposed STFBC, itsreceivercomplexity for the ML decoder is the same as the receiver complexity of those of [9]. In other words, when a constellation of size M is employed, the receiver complexity is in the order of $O(M^{2KL})$. Also, by using the well-known sphere decoder this extent of the complexity could be decreased noticeably.

B. Parameter γ_{SD} Indication

In this subsection, we determine parameter γ_{SD} so as to maximize the coding advantage of the proposed STFBC. As discussed in [8], maximizing the coding advantage of a full-diversity STFBC leads to maximizing the minimum determinant of Ξ over all distinct codewords $\boldsymbol{C}$and $\widehat{\boldsymbol{C}}$, where

$$\Xi \triangleq \Delta \circ \boldsymbol{R} \in \mathbb{C}^{KN\times KN}. \quad (15)$$

In (15), $\Delta \triangleq (\boldsymbol{C} - \widehat{\boldsymbol{C}})(\boldsymbol{C} - \widehat{\boldsymbol{C}})^\dagger$and $\boldsymbol{R} \triangleq \boldsymbol{R_T} \otimes \boldsymbol{R_F}$, where$\boldsymbol{R_F}$ and $\boldsymbol{R_T}$are the frequency andtemporal correlation matrices, respectively [8]. For the maximization procedure of the coding advantage associated with our proposed STFBC to be simple, let us suppose that two distinct codewords$\boldsymbol{C}$and $\widehat{\boldsymbol{C}}$are the same except for one symbol. Supposing that this dissimilar symbol is s_1, by manipulation, one can easily verify that maximizing the coding advantage of the proposed STFBC presented in (9), results in maximizing the determinant of the following matrix:

$$\widehat{\Xi} = (\boldsymbol{1}_{LK} \otimes \boldsymbol{I}_2) \circ (\boldsymbol{R_T} \otimes \boldsymbol{R_F}). \quad (16)$$

In (16),$\boldsymbol{R_F} = \boldsymbol{W}\mathrm{diag}(\sigma_0^2, \sigma_1^2, \dots, \sigma_{L-1}^2)\boldsymbol{W}^\dagger$ and$\boldsymbol{W} \in \mathbb{C}^{2L\times L}$is defined below:

$$\boldsymbol{W} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ w^{\zeta_0} & w^{\zeta_1} & \cdots & w^{\zeta_{L-1}} \\ w^{(\gamma_{SD}+1)\zeta_0} & w^{(\gamma_{SD}+1)\zeta_1} & \cdots & w^{(\gamma_{SD}+1)\zeta_{L-1}} \\ w^{(\gamma_{SD}+2)\zeta_0} & w^{(\gamma_{SD}+2)\zeta_1} & \cdots & w^{(\gamma_{SD}+2)\zeta_{L-1}} \\ \vdots & \vdots & \cdots & \vdots \\ w^{((L-1)\gamma_{SD}+1)\zeta_0} & w^{((L-1)\gamma_{SD}+1)\zeta_1} & \cdots & w^{((L-1)\gamma_{SD}+1)\zeta_{L-1}} \\ w^{((L-1)\gamma_{SD}+2)\zeta_0} & w^{((L-1)\gamma_{SD}+2)\zeta_1} & \cdots & w^{((L-1)\gamma_{SD}+2)\zeta_{L-1}} \end{bmatrix} \quad (17)$$

And $\boldsymbol{R_T}$ is the temporal correlation matrix of size $K \times K$.The element associated with the k^{th} row and the p^{th}column of $\boldsymbol{R_T}$is obtained by $R_T(k,p) = v(k-p)$, where $v(k-p) = E\{\alpha_{i,j}^k(l)\alpha_{i,j}^p(l)^*\}$[8].

Equations(16) and(17) show that the coding advantage of the proposed STFBC depends on γ_{SD} as well as on the system bandwidth (BW), DPPs and the number of subcarriers. If DPPs are known to the transmitter, we can find γ_{SD} so as to maximize the coding advantage of the intended STFBC. On the other hand, if DPPs are unknown, we design the proposed STFBC based on the permutation method introduced for the block circular delay diversity (BCDD) codes [9]. It should be also noted that γ_{SD} must be a multiple of two regarding size of $\boldsymbol{P}$.

In the appendix, it is shown that the proposed STFBC achieve the maximum attainable diversity gain.

4. SIMULATION RESULTS

In the simulations, we considered a MIMO-OFDM system with two transmitter antennas, N=128 subcarriers, BW = 1 MHz and the length of cyclic prefix of 20 µs. We evaluated the performances of the new scheme by sketching average bit-error-rate (BER) versus average signal-to-noise-ratio (SNR). We compared the performance of the proposed code against those of the BCDD codes and optimum STFBCs, presented in [9] for the unknown and the known DPPs cases, respectively. These codes are the best STFBCs in the literature to the best of authors' knowledge.

In the unknown DPPs case, 2-ray equal power channel model with delay profile {0, 5} μsecis considered. In the known DPPs case, the same channel model as [9] was investigated, i.e. a 2-ray equal power channel model with delay profile {0, 1} μsec.

As can be seen in Fig. 1 and Fig. 2, regarding the BER values, our proposed STFBC in both cases outperforms the BCDD STFBCs in [9]. For example, Fig. 1 demonstrates that at BER= 10^{-4}, the proposed code achieves about 0.5 dB gain over the BCDD codes for the 2-ray channel with delay spread of 5 µs. Furthermore, Fig. 2 depictsperformances of the BCDD code and the proposed STFBC when two antennas are employed at the receiver. As this figure shows, our proposed STFBC outdoes the BCDD code by almost 0.75 dB at BER=10^{-6}.Fig. 3 also demonstrates that our proposed optimized STFBC outperforms the optimum STFBC in [9] by nearly 3 dB at BER=10^{-4}.

APPENDIX

In this appendix, we show that our proposed codeachieves the maximum attainable diversity advantage for two transmitter antennas.Regarding Appendix I of [6], it could be shown that the proposed STFBC is full-diversity if$\prod_{k=1}^{K}\det(\boldsymbol{\Gamma}_k)\neq 0$for $x_i \neq x'_i$.Now, for $K=2$, we have:

$$\boldsymbol{\Gamma_1} \triangleq \boldsymbol{D_1} \circ \boldsymbol{T} \in \mathbb{C}^{2L\times 2L}, \quad (18)$$

Where

$$\mathbf{D_1}=\begin{bmatrix} \sigma_0\Delta_1 & \sigma_0\Delta_2 & \cdots & \sigma_{L-1}\Delta_1 & \sigma_{L-1}\Delta_2\\ \sigma_0\Delta_2^* & \sigma_0\Delta_1^* & \cdots & \sigma_{L-1}\Delta_2^* & \sigma_{L-1}\Delta_1^*\\ \sigma_0\Delta_3 & \sigma_0\Delta_4 & \cdots & \sigma_{L-1}\Delta_3 & \sigma_{L-1}\Delta_4\\ \sigma_0\Delta_4^* & \sigma_0\Delta_3^* & \cdots & \sigma_{L-1}\Delta_4^* & \sigma_{L-1}\Delta_3^*\\ \vdots & \vdots & \cdots & \vdots & \vdots\\ \sigma_0\Delta_{2L-1} & \sigma_0\Delta_{2L} & \cdots & \sigma_{L-1}\Delta_{2L-1} & \sigma_{L-1}\Delta_{2L}\\ \sigma_0\Delta_{2L}^* & \sigma_0\Delta_{2L-1}^* & \cdots & \sigma_{L-1}\Delta_{2L}^* & \sigma_{L-1}\Delta_{2L-1}^* \end{bmatrix} \in \mathbb{C}^{2L\times 2L} \quad (19)$$

and

$$\boldsymbol{\Gamma_2} \triangleq \boldsymbol{D_2} \circ \boldsymbol{T} \in \mathbb{C}^{2L\times 2L} \quad (20)$$

where

$$\mathbf{D_2}=\begin{bmatrix} \sigma_0\Delta_{2L+1} & \sigma_0\Delta_{2L+2} & \cdots & \sigma_{L-1}\Delta_{2L+1} & \sigma_{L-1}\Delta_{2L+2}\\ \sigma_0\Delta_{2L+2}^* & \sigma_0\Delta_{2L+1}^* & \cdots & \sigma_{L-1}\Delta_{2L+2}^* & \sigma_{L-1}\Delta_{2L+1}^*\\ \sigma_0\Delta_{2L+3} & \sigma_0\Delta_{2L+4} & \cdots & \sigma_{L-1}\Delta_{2L+3} & \sigma_{L-1}\Delta_{2L+4}\\ \sigma_0\Delta_{2L+4}^* & \sigma_0\Delta_{2L+3}^* & \cdots & \sigma_{L-1}\Delta_{2L+4}^* & \sigma_{L-1}\Delta_{2L+3}^*\\ \vdots & \vdots & \cdots & \vdots & \vdots\\ \sigma_0\Delta_{4L-1} & \sigma_0\Delta_{4L} & \cdots & \sigma_{L-1}\Delta_{4L-1} & \sigma_{L-1}\Delta_{4L}\\ \sigma_0\Delta_{4L}^* & \sigma_0\Delta_{4L-1}^* & \cdots & \sigma_{L-1}\Delta_{4L}^* & \sigma_{L-1}\Delta_{4L-1}^* \end{bmatrix} \in \mathbb{C}^{2L\times 2L} \quad (21)$$

In (21), $\Delta_i = x_i - x'_i$, for $i = 1, 2, \dots, 4L$, where x_i's and x'_i's are symbols associated with two distinct codewords $\mathbf{C}$ and $\hat{\mathbf{C}}$. As we mentioned in section III-B of the paper, we choose γ_{SD} so that it increases the coding advantage of the proposed STFBC. Therefore, in order to obtain the diversity order of the proposed STFBC, without loss of generality and for the sake of simplicity, let us suppose that we have no permutation in the code. Thus, $\mathbf{T} \in \mathbb{C}^{2L\times 2L}$ could be presented as follows:

$$\mathbf{T}=\begin{bmatrix} 1 & 1 & \cdots & 1 & 1\\ -w^{\zeta_0} & w^{\zeta_0} & \cdots & -w^{\zeta_{L-1}} & w^{\zeta_{L-1}}\\ w^{2\zeta_0} & w^{2\zeta_0} & \cdots & w^{2\zeta_{L-1}} & w^{2\zeta_{L-1}}\\ -w^{3\zeta_0} & w^{3\zeta_0} & \cdots & -w^{3\zeta_{L-1}} & w^{3\zeta_{L-1}}\\ \vdots & \vdots & \cdots & \vdots & \vdots\\ -w^{(2L-1)\zeta_0} & w^{(2L-1)\zeta_0} & \cdots & -w^{(2L-1)\zeta_{L-1}} & -w^{(2L-1)\zeta_{L-1}} \end{bmatrix}, \quad (22)$$

Therefore, $\mathbf{T}$ could be rewritten as below:

$$\mathbf{T}=\mathbf{V}\left(-w^{\zeta_0}, w^{\zeta_0}, -w^{\zeta_1}, w^{\zeta_1}, \dots, -w^{\zeta_{L-1}}, w^{\zeta_{L-1}}\right) \in \mathbb{C}^{2L\times 2L}. \quad (23)$$

It can be numerically shown that the minimum value of$\prod_{k=1}^{K}\det(\boldsymbol{\Gamma}_k)$is obtained when all s_i's and s'_i's are the same except for one symbol. Without loss of generality, let us suppose that s_1 and s'_1 are not the same, i.e. $\Delta_1 \neq 0$. In this case, we have:

$$\hat{\boldsymbol{\Gamma}}_1 \triangleq \hat{\mathbf{D}}_1 \circ \mathbf{T} \in \mathbb{C}^{2L\times 2L}, \quad (24)$$

$$\hat{\boldsymbol{\Gamma}}_2 \triangleq \hat{\mathbf{D}}_2 \circ \mathbf{T} \in \mathbb{C}^{2L\times 2L}, \quad (25)$$

where

$$\hat{\mathbf{D}}_1=\hat{\mathbf{D}}_2=\begin{bmatrix} \sigma_0\Delta_1 & \sigma_0\Delta_1 & \cdots & \sigma_{L-1}\Delta_1 & \sigma_{L-1}\Delta_1\\ \sigma_0\Delta_1^* & \sigma_0\Delta_1^* & \cdots & \sigma_{L-1}\Delta_1^* & \sigma_{L-1}\Delta_1^*\\ \sigma_0\Delta_1 & \sigma_0\Delta_1 & \cdots & \sigma_{L-1}\Delta_1 & \sigma_{L-1}\Delta_1\\ \sigma_0\Delta_1^* & \sigma_0\Delta_1^* & \cdots & \sigma_{L-1}\Delta_1^* & \sigma_{L-1}\Delta_1^*\\ \vdots & \vdots & \cdots & \vdots & \vdots\\ \sigma_0\Delta_1 & \sigma_0\Delta_1 & \cdots & \sigma_{L-1}\Delta_1 & \sigma_{L-1}\Delta_1\\ \sigma_0\Delta_1^* & \sigma_0\Delta_1^* & \cdots & \sigma_{L-1}\Delta_1^* & \sigma_{L-1}\Delta_1^* \end{bmatrix} \in \mathbb{C}^{2L\times 2L}. \quad (26)$$

By utilizing basic determinant properties, we have:

$$\det(\hat{\boldsymbol{\Gamma}}_2)=\det(\hat{\boldsymbol{\Gamma}}_1)=\prod_{i=0}^{L-1}\sigma_i^2 \times |\Delta_1|^{2L} \times \det(\mathbf{T}). \quad (27)$$

Therefore,

$$\prod_{k=1}^{2}\det(\boldsymbol{\Gamma}_k)=\left(\prod_{i=0}^{L-1}\sigma_i^2 \times |\Delta_1|^{2L} \times \det(\mathbf{T})\right)^2. \quad (28)$$

Clearly, since

$$\det(\mathbf{V}(t_1, t_2, \dots, t_n)) = \prod_{\substack{\mu,\nu=1 \\ \mu>\nu}}^{n} (t_\mu - t_\nu) \quad (29)$$

and $\zeta_0 < \cdots < \zeta_{L-2} < \zeta_{L-1}$, $\det(\mathbf{T})$ is non-zero, $\hat{\boldsymbol{\Gamma}}_1$ and $\hat{\boldsymbol{\Gamma}}_2$ are of rank $2L$ and consequently the proposed code attain a diversity advantage of $4LM_r$.

5. CONCLUSION

In this paper, we introduced a novel full-diversity space-time-frequency block codes that can achieve the maximum coding advantage when partial channel side information is available at the transmitter. It was also shown by simulation results that the proposed space-time-frequency block code model outperforms the recently introduced space-time-frequency block codes.

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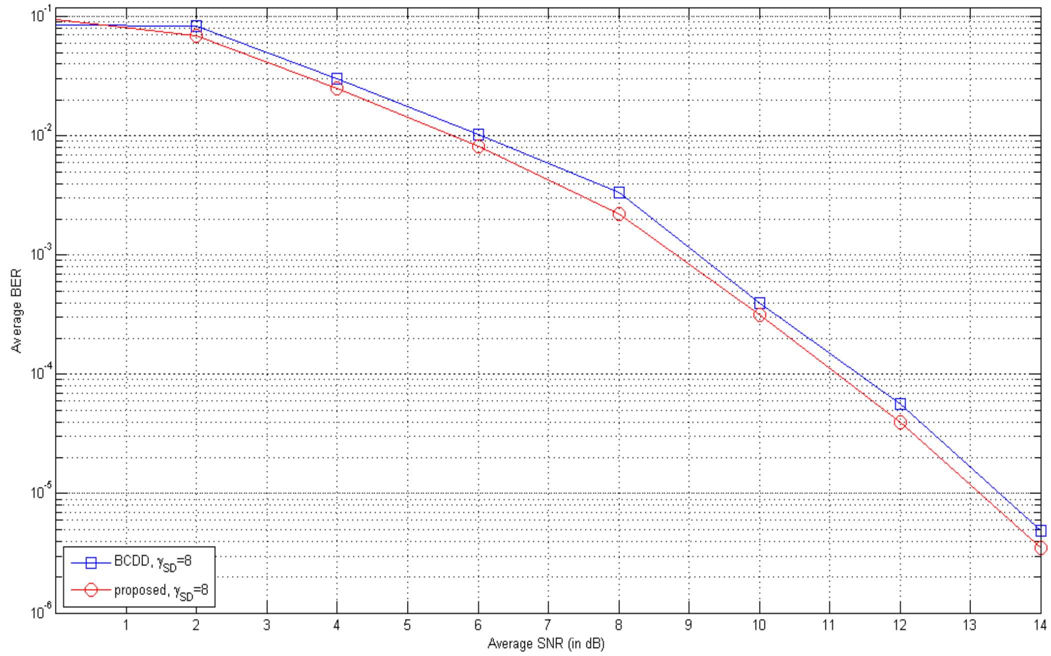

Fig.1. BER performance for frequency-selective 2-ray equal power channel, BPSK constellation, □□ =1, 1 bit/s/Hz.

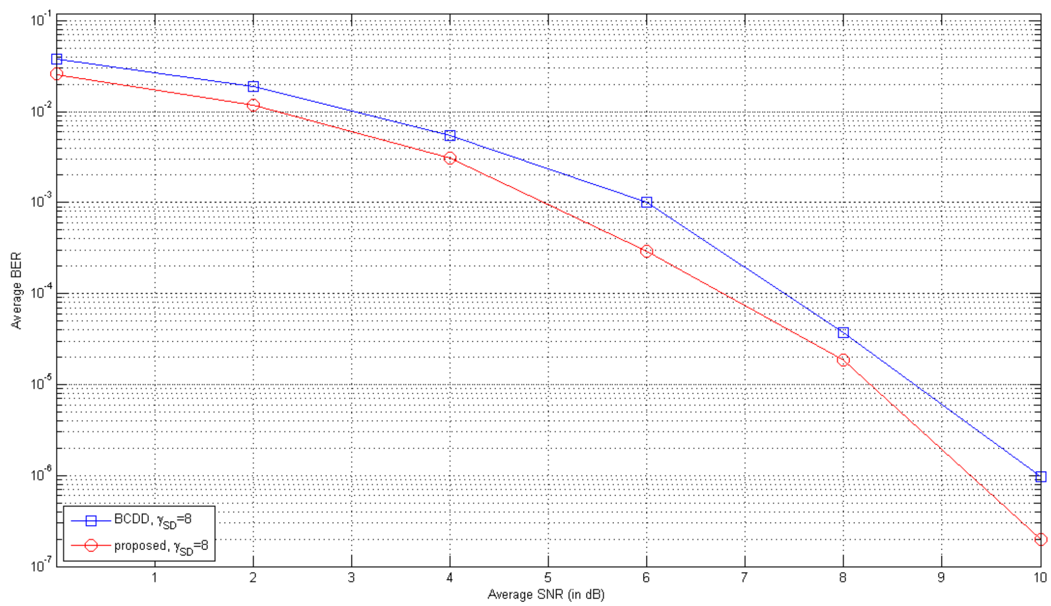

Fig. 2. BER performance for frequency-selective 2-ray equal power channel with delay spread of 5 □sec, BPSK constellation, □□ =2, 1 bit/s/Hz.

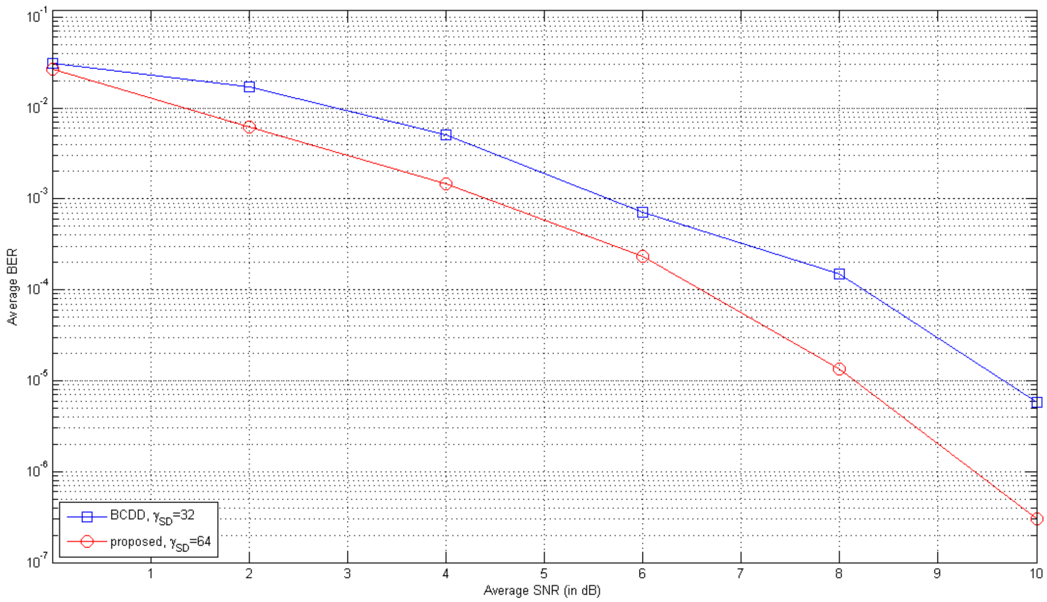

Fig. 3.
BER performance for frequency-selective 2-ray equal power channel with delay spread of 1 □sec, BPSK constellation, □□ =2, 1 bit/s/Hz.