

# Combined Heat and Power (CHP) Allocation and Capacity Determination According to Fuzzy Bus Thermal Coefficient and Nodal Pricing Method using Cooperative Game Theory

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## ABSTRACT:

In this paper a hybrid and practical method is presented to allocate and determine combined heat and power capacity (CHP) generator at a bus. This method consists of two stages. First, the suitable buses for CHP installation will be found by the bus thermal coefficient. This coefficient indicates the possibility of the heat selling around each bus and will be calculated by using the Fuzzy method. Next, for each of the appropriate buses, considering the obtained heat capacity and electrical power ratio to the heat of the CHPs in the market, several CHPs are recommended. Second, on the one hand, the improvement of the technical criteria after the CHPs installation is derived by using the nodal pricing methods as the financial benefits of the distribution companies and on the other hand, the investors' financial benefits from the sold heat output of the CHPs is determined. Finally, using the Game Theory and considering the distribution companies and investors as the players, the suitable location and capacity for CHP installation based on the set Game strategy is obtained. The proposed method is implemented to a sample distribution feeder in the Hamadan city and the results are shown.

**KEYWORDS:** CHP Allocation, Nodal Pricing Method, Bus thermal coefficient, Game Theory

## 1. INTRODUCTION

With the increasing demand for the electrical energy and the electrical energy efficiency of the small units, these units are more likely to be used in the distribution system and near the consumers. These small units that are connected to the distribution system are called "distributed generation" (DG). The privatization of the electricity industry, less environmental pollution, relatively high efficiency and the development of the electricity generation methods through the renewable energies are important factors for the development of these kind of generators. The use of the distributed generation units has significant impact on the technical and economic issues of the power systems [1], [2]. One of these power plants, is electricity and heat co-generation unit (CHP) which supplies the needed heating or cooling energy by consumers through its unnecessary heat output and increases the whole power plant efficiency up to 75% and more. As the gas fuel is available in our country, these power plants are good substitute for the electricity and heat generation.

The location and capacity of the distributed generation resources affects the technical indicators. Some factors such as the reduction of losses, the improvement of voltage profile and voltage regulations at objective functions are usually considered as significant indicators to optimize the location and capacity of these generators [3], [4]. Next these defined functions are optimized by intelligent methods such as GA, PSO and TS and the capacity and location of DG are determined [5], [6].

For allocation and capacity determination of "CHP", in addition to the above technical analysis, the economic analysis is usually considered. In this analysis, the investment criteria are considered to optimize the power, heat, warm water and even cold simultaneous consumption at the objective function [7], [8].

The installation of CHP on the distribution network changes it from passive to active and improves the network losses, voltage regulation and profile [10], [11]. The improvement of these technical indicators can be

considered by the "nodal pricing methods" at the electrical energy price of the buses which CHP is connected to them. In other words, the CHP installation affects the nodal pricing of the buses [9].

In addition to the improvement of the technical indicators that is desirable to the distribution companies, CHP installation will make the opportunity of selling the heat and warm water to the consumers around the bus as the more financial profit is favorable to the CHP investors. Allocation and capacity determination of "CHP" in such a way that simultaneously improves the technical indicators and makes the best financial profits are some practical challenges facing the researchers and highly depend on the strategies and policies of the players of this field of work namely the distribution companies and investors. The use of "Game Theory" has attracted the consideration of the researchers in the recent years. Generally, where a group of individuals or firms compete with each other or they cooperate in a team, the Game Theory can be used to model competition between them. Song Yiqun in [12] using non-cooperative Game Theory and Nash-Stackelberg equilibrium, has presented a new method for determination of power market. Lance B. Cunningham in [13] also using Game Theory and Corn out equilibrium, has presented a way to model the transmission line congestion in the electricity market. In [14] using cooperative Game Theory, the consumers of the heat and power as the members of the coalition are used to achieve higher profits through reducing the investment and increasing the efficiency of the electricity and heat co-generation (CHP).

In this paper a hybrid method has been presented to allocate and determine the CHP capacity on the bus. In this method using the cooperative Game Theory, investors and distribution companies have been used as the coalition members to achieve higher profits and improve the technical indicators of network. The proposed hybrid method has two stages as follows:

In the first stage by investigating the heat consumers around the bus, the thermal coefficient that indicates the heat selling possibility of the bus will be extracted by introduced Fuzzy function. And by this coefficient, the suitable bus for CHP installation regarding the heat selling possibility is identified. Then, at the end of this stage regarding the heat capacity and the ratio of the electrical energy generation to the heat energy of the existed CHPs in the market, several CHPs are suggested for the specified buses, that the installation of each makes different profit for the investors.

Second, the effect of the proposed CHP installation on the technical indicators of the network such as the losses reduction, the profile improvement and the voltage regulation by the nodal pricing method, as the

profits of the distribution companies is calculated. Considering the distribution companies and investors as the players, the CHP capacity and the ratio of its electrical power to heat as the players' strategies, the suitable CHP is determined from among the proposed CHPs by the use of Game Theory. This paper is arranged as follows:

Game theory is described in section 2; the Fuzzy bus thermal coefficient and the nodal pricing method are defined in sections 3 and 4, respectively. The optimization method is described in section 5 and finally the case study results of a sample feeder in the Hamadan city are given.

## 2. GAME THEORY METHOD

In the Game theory, a game is a set of rules known to all players that determines their possible choices and the consequences of each choice. The normal form of game represents the number of players, set strategies and the payoff functions of each player. Assuming  $n$  players, a set of players is:

$$N = \{1, 2, \dots, n\}$$

The decisions set the player  $i$  can get is named "strategy space of the player  $i$ " and is shown as follows:

$$S_i = \{s_{i1}, s_{i2}, \dots, s_{imi}\}$$

$S_{ij}$ : The  $j^{\text{th}}$  strategy for player  $i$ .

$m_i$ : The total number of strategies.

Since there are  $n$  players, the strategies of all players are:

$$S = \{S_1, S_2, \dots, S_n\}$$

On the other hand, payoff function for player "i" shows the outcome or result (including profit, utility, etc.) the player "i" will obtain at the end of the game. This payoff depends on the chosen strategies by all players and is shown as follows:

$$u_i = u_i(s_{1j}, s_{2j}, \dots, s_{nj})$$

That  $s_{ij} \in S_i$ , shows  $j^{\text{th}}$  strategy of the player "i" in the strategy set ( $S_i$ ).

Also the combination of all players strategy is called Strategy Profile and is shown as follows:

$$S_j = (s_{1j}, s_{2j}, \dots, s_{nj})$$

Thus the normal form of the  $n$ -persons game represents the player's strategy space ( $S_1, \dots, S_n$ ) and their payoff functions ( $u_1, \dots, u_n$ ) that is shown as follows [22].

$$G = \{S_1, \dots, S_n; u_1, \dots, u_n\}$$

Osborne, M.J. and Rubinstein in [21] have shown that the solution of the "Game" is a continuous selection from among the equilibrium strategies. The Nash equilibrium is usually used. In this equilibrium:

$$\forall i, \forall s_{-i} \in S_{-i} \quad U_i(s_i, s_{-i}) \geq U_i(s'_i, s_{-i}) \quad (1)$$

Where :

$s_i \in S_i$ : Nash equilibrium strategy of player  $i$

$s'_i \in S_i$ : None- Nash equilibrium strategy of player  $i$

$s'_i \in S_i$ : Other players' strategy in the Nash equilibrium

That is Nash equilibrium is a condition achieved by a set of strategies, and the players' decision to deviate from such state will reduce the profit. The search to find the equilibrium point includes the following steps:

1. Forming a set of possible strategies, except dominant strategies, namely the  $s'_i$  strategy of player i, so that fulfills the following condition [21]:

$$\forall s_{-i} \in S_{-i} \quad U_i(s_i, s'_{-i}) \geq U_i(s'_i, s'_{-i}) \quad (2)$$

2. Searching the equilibrium point.

The Nash equilibrium point is determined with regard to the 1. In term of theory there will be many equilibrium points which in [21] some methods are presented for reducing the number of them.

3. Considering the rationality and the possibility of organizing the coalition among the players.
4. Selection of the methods to organize the coalitions and the distribution of the excess profit among the coalition participants.

If there is the possibility of a coalition among the players, the possible strategies of this coalition may increase the dimensions of the problem significantly.

Finally, the output of this method is the presentation of a semi-optimal path for all companies and their coalitions regarding the competitors' strategy. In this paper the "Static Game" with complete information is used to allocate and determine CHP capacity. In this method the players are:

- The Electric Power Distribution State Company (player A)
- The Investors (player B)

The possible strategies:

- The ratio of the electrical power to heat of different CHP technologies given in Table1 [20].
- The selection of CHPs capacity that has been considered 0.5 and 1 MW in this paper.

**Table1.** Characteristics of CHP technologies

| technology          | steam turbine | gas engine | gas turbine |           | Fuel cell |
|---------------------|---------------|------------|-------------|-----------|-----------|
| power to heat ratio | 0.1 -0.3      | 0.5 - 1    | 0.5 -2      | 0.4 - 0.7 | 1-2       |

By obtaining the Nash equilibrium point, the suitable location and capacity of the CHP generator can be determined for installing in the bus network.

### 3. BUS THERMAL COEFFICIENT

The power at bus "i" is :

$$P_{T_i} = P_{e_i} + P_{h_i} \quad (3)$$

And

$$P_{h_i} = \sum_{j=1}^n P_{h_{ij}} \quad (4)$$

Where:

$P_{e_i}$  : Active power consumption at the bus "i".

$P_{h_i}$  : The electrical equivalent of the heat selling possibility at the bus " i".

$P_{T_i}$  : The total power

$P_{h_{ij}}$  : the possibility of heat selling (equivalent to the electrical power) consumer j at the bus i

n: the total number of consumers around each bus

In the above equations,  $P_{h_i}$  can be supplied only by CHP source connected to bus "i" and if will be supplied by other buses, heat and cool loss will remove this possibility while  $P_{e_i}$  can be supplied by other buses of network too. The optimization problem can be divided into two parts:

- Optimization with regard to the consumption of  $P_{e_i}$  for each bus of network that can be also supplied by generators at other buses.
- Optimization with regard to  $P_{h_i}$  the sale of heat (equivalent to electric power) for each bus of network that is supplied by generator at the same bus only.

#### 3.1. Bus Thermal Coefficient (BTC)

BTC indicates the possibility of selling steam and warm water to each bus, and with regard to the consumers around the bus is calculated as follows :

$$BTC_i = \frac{P_{h_i}}{1MW} , \quad BTC_i \geq 0.1 \quad (5)$$

Where :

$P_{h_{ij}}$  : The possibility of heat selling (equivalent to electric power) to the consumer "j" at bus " i .

N: Total number of consumers around each bus.

$BTC_i$  : Bus thermal coefficient of bus "i".

$Q_{h_{ij}}$  : The heat consumption (equivalent to electric power) of consumer "j" at bus " i".

$\beta$  : Type of consumer.

d : The distance between the heat consumer and power plant.

x : Coefficient of CHP technology that depends on the conditions of generated heat by CHP.

$\psi$  : Fuel delivery coefficient.

The thermal coefficient of bus will be achieved by normalizing the possibility of heat selling to 1MW.Finally, the buses with higher amount of BTC are eligible for CHP installation that will be considered in the calculations of objective function optimization.  $P_{h_i}$  is the function of effective coefficient phase sharing (minimum) of heat selling and will be expressed by equation (6) :

$$P_{h_i} = \sum_{j=1}^N P_{h_{ij}} = \sum Q_{h_{ij}} \times f_{ij} (\beta \cap d \cap x \cap \Psi) \quad (6)$$

3.1.1. Calculation of  $\beta$

According to the National Building Regulations in Iran [23], there are four group of buildings, A to D. This grouping is based on the three following factors:

- Continuing the use of building during the day and year.
- The temperature difference between the interior and exterior of the building.
- The significance of the temperature stabilization of the indoor spaces.

$\beta$  is determined based on the user kind in Table 2.

Higher  $\beta$  indicates more possibility of the heat selling to the consumer.

**Table 2.** Building classification according to the National Building Regulations

| sample   | $\beta$ | user type |
|--|---------|-----------|
| Hospitals, hotels(4 and 5stars), industries with the heating consumption for the generation process (cement, steel, melted metals, sugar, food, green houseTown) | 1       | A         |
| Integrated academic and large schools (with dormitory), skyscrapers , large residential complexes (with central heating systems).                                | 0.75    | B         |
| Stores, factories (heating and sanitary use only), international airport   | 0.5     | C         |
| Business Places (shopping centers), offices  | 0.25    | D         |
| Spread consumers who cannot use the central heating systems  | 0       | All cases |

3.1.2. Amount of heat consumption (equivalent to electrical power)  $Q_{hij}$  :

The amount of each user consumption depends on its location, (table 3). Based on Standard National Regulations, buildings in various parts of the country are divided into three groups, based on the need of the annual heating- cooling energy:

- The need of low annual heating -cooling energy
- The need of medium annual heating - cooling energy
- The need of high annual heating - cooling energy

Some examples are given in Table 3.

**Table 3.** The need of heating and cooling energy in different locations of Iran

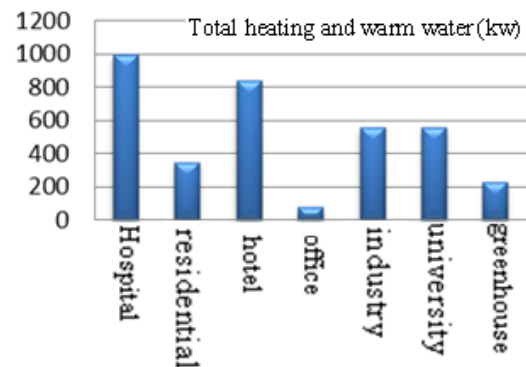
| Warm and humid | High cooling required | High heating required | The energy required | City name    | row |
|----------------|-----------------------|-----------------------|---------------------|--------------|-----|
|                |                       | ×                     | high                | Hamadan      | 211 |
| ×              | ×                     |                       | high                | Bandar abbas | 48  |
|                |                       |                       | low                 | Rasht        | 112 |
|                |                       | ×                     | medium              | Tehran       | 71  |

According to Table 3, it can be seen that Hamadan city requires large amount of heating energy. The calculation of the needed energy of the different loads (various applications) according to references [15], [16], has been done to 1000 m<sup>2</sup> infrastructure and this point indicates that Hamadan city uses of the natural gas of the main pipeline with special heating value of 9434Kcal/m<sup>3</sup> or 1060 Btu / ft<sup>3</sup>.

For example, in multi-unit residential building that use the central heating systems (for 1000 m<sup>2</sup> infrastructure)

- A) The warm water consumption : 231.84 (kw)
- B) The heat consumption for heating : 117.16 (kw)

Total heating and warm water consumption of different buildings are shown in Fig.1 .



**Fig. 1.**  $Q_{hij}$  of different consumers, with infrastructure of 1000 m<sup>2</sup>

3.1.3. The distance between heating consumer and power plant (d) :

The other issue that should be considered at heating distribution is the distance between heating consumer and power plant, so that by increasing the distance, heat selling possibility will be reduced while the transport cost will be increased. In other words, the bus thermal coefficient (fitness) is proportional to the inverse distance :

$$f(d) \approx \frac{k}{d}$$

That, d is the difference between heating consumer and power plant and coefficient k is depends on the heat transferring system that achieved based on the practical results.

The possibility of heat and warm water transferring to the different distances expressed by following fuzzy membership function (Fig.2) :

$$\overline{f(d)} = \begin{cases} 1 & d < 333 \\ \frac{1050 - d}{717} & 333 \leq d \leq 1050 \\ 0 & d \geq 1050 \end{cases}$$

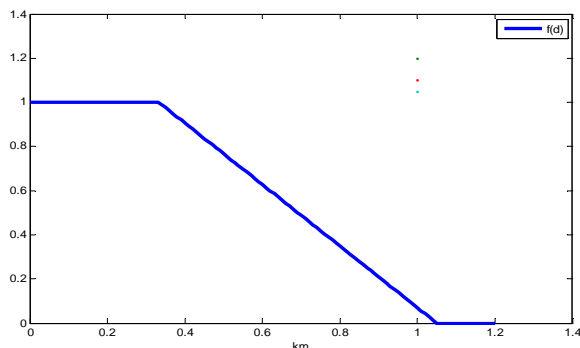


Fig. 2. The fuzzy digit corresponding  $f(d)$

Fuzzy membership function : fuzzy digit  $f(d)$  in parametric mode is the regular pair of  $(\overline{f(d)}, \underline{f(d)})$  which must satisfy the following requirements :

1.  $\underline{f(d)}$  Continuous boundary function from left.
2.  $\overline{f(d)}$  Continuous boundary function from right.
3.  $\underline{f(d)} \leq \overline{f(d)}, 0 \leq f(d) \leq 1$

3.1.4. Determination of Technology Coefficient (x) :

This ratio expresses which technology is used to generate electricity and heat in the CHP (Table 4). Coefficients  $x_1$  to  $x_5$  can be determined according to the CHP thermal output. For example, gas turbine technology, which provides heat, warm water, LP and HP steam, has highest coefficient of x .

Table 4. Various CHP technologies

| Technology                            | steam turbine | reciprocating engine | gas turbine          | micro turbines                | Fuel cell               |
|---------------------------------------|---------------|----------------------|----------------------|-------------------------------|-------------------------|
| Typical ratio of heat to power        | 0.1-0.3       | 0.5-1                | 0.5-2                | 0.4-0.7                       | 1-2                     |
| The Power electrical efficiency (HHV) | 15-38%        | 22-40%               | 22-36%               | 18-27%                        | 30-63%                  |
| Total efficiency (HHV)                | 80%           | 70-80%               | 70-75%               | 65-75%                        | 55-80%                  |
| Using of output heat                  | LP-HP steam   | LP-HP steam          | Warm water, LP steam | Heating, warm water, LP steam | Warm water, LP-HP steam |
| x                                     | 0.25          | 0.5                  | 1                    | 0.5                           | 0.75                    |

3.1.5. Fuel delivery Coefficient ( $\psi$ ) :

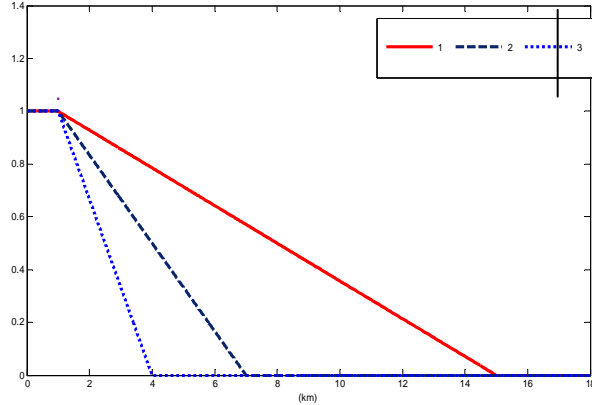
Since the natural gas is used as the main fuel for these power plants and gas lines have three pressures, 1000PSI for gas transmission, 250 PSI and 60 PSI for gas distribution in the cities, therefore considering the consumers distance around each bus from the transmission and distribution gas lines (d), and the experimental results obtained from the gas company, the fuzzy membership functions corresponding to  $\psi$  index, for different gas pressures is presented in the table 5.

Table 5. Fuzzy membership function at different gas pressures

| row | Gas pressure (psi) | Fuzzy membership function   |
|-----|--------------------|---|
| 1   | 1000               | $\overline{\psi} = \begin{cases} 1, 0 < d \leq 1 \\ \frac{15 - d}{14}, 1 \leq d < 15 \\ 0, d \geq 15 \end{cases}$ |
| 2   | 250                | $\overline{\psi} = \begin{cases} 1, 0 < d \leq 1 \\ \frac{7 - d}{6}, 1 \leq d < 7 \\ 0, d > 7 \end{cases}$        |

|   |    |   |
|---|----|---|
| 3 | 60 | $\overline{\psi} = \begin{cases} 1, & d \leq 1 \\ \frac{4-d}{3}, & 1 \leq d \leq 4 \\ 0, & d > 4 \end{cases}$ |
|---|----|---|

The corresponding fuzzy digits with different gas pressures is shown in Fig. 3 .



**Fig. 3.** Fuzzy digit corresponding to  $\psi(d)$  for the pressure of (1)1000PSI, (2) 250PSI and (3) 60PSI

Finally by determination of bus thermal coefficient, the amount of saving the thermal cost of each bus (with regard to government support in this area [19] ) will be obtained after CHP installation as follows :

$$C_{H_i} = BTC_i \times \Delta t_i \times \lambda_H \quad (7)$$

Where:

$C_H$  : saving the thermal cost after CHP installation,  $\frac{\$}{\text{year}}$

$\lambda_H$  : The cost of per "MWh" heating, is equal to 7.2 \$, since the project of "targeted subsidies" is executed.

$\Delta t_i$  : 8760 hour for a year.

#### 4. NODAL PRICING METHOD

The distributed generation resources in the network will change the power flow and losses on two-level transmission and distribution networks. In many tariffs plants in distribution level, use from the equally share of losses cost for consumers, that discourages the consumers for the CHP installation [24]. For solving this problem we can use from the "Nodal Pricing Method". The price of electricity in the nodes indicates the marginal price of electricity in the network buses [9], in this paper the characteristics of formulas are defined as follows:

Marginal losses coefficient (MLC) is the active power losses network change ( $P_L$ ) due to change in production or consumption the active power ( $P_{e_i}$ ) and the reactive power ( $Q_{e_i}$ ) in bus  $i$  is defined as follows [17] :

$$\rho_{P_{e_i}} = \frac{\partial P_L}{\partial P_{e_i}} \quad (8)$$

$$\rho_{Q_{e_i}} = \frac{\partial P_L}{\partial Q_{e_i}} \quad (9)$$

Where :

$\rho_{P_{e_i}}$  : Marginal losses coefficient of active power at the bus "i".

$\rho_{Q_{e_i}}$  : Marginal losses coefficient of reactive power at the bus "i".

The medium point between generation and transmission levels is called "power supply point" (PSP) . If " $\lambda$ " is the price of active power in PSP in  $\frac{\$}{\text{MWh}}$  , and if the active and reactive power consumption at bus "i" change as  $P_i$  and  $Q_i$  respectively and dose not exist any congestion in the distribution network, then we can calculate the nodal pricing for active and reactive power as follows :

$$N_i^a = \lambda + \lambda \cdot \rho_{P_{e_i}} = \lambda(1 + \rho_{P_{e_i}}) \quad (10)$$

$$N_i^r = \lambda \cdot \rho_{Q_{e_i}} \quad (11)$$

The price of electrical bill without CHP installation on the period  $\Delta t$  will be obtained as follows:

$$C_i^{no-CHP}(P_{e_i}, Q_{e_i}) = (N_i^a(P_{e_i}, Q_{e_i}) \times P_{e_i} + N_i^r(P_{e_i}, Q_{e_i}) \times Q_{e_i}) \cdot \Delta t \quad (12)$$

And the total of it for each feeder is equal to :

$$C_{total}^{no-CHP} = \sum_{i=1}^N C_i^{no-CHP}(P_{e_i}, Q_{e_i}) + (\lambda \times P_L) \cdot \Delta t \quad (13)$$

CHP installation decreases the distribution losses, and so the nodal pricing will be reduced [26] .

The price of electrical bill with CHP installation on the period  $\Delta t$  at bus "i" will be obtained as follows :

$$C_i^{CHP}(P_{e_i}, Q_{e_i}) = \{ (N_{i,CHP}^a(P_{e_i}, Q_{e_i}) \times (P_{e_i} - P_{CHP_i}) + N_{i,CHP}^r(P_{e_i}, Q_{e_i}) \times (Q_{e_i} - Q_{CHP_i}) \} \cdot \Delta t + \{ C_{(CHP)} \times P_{CHP_i} \} \cdot \Delta t \quad (14)$$

And the total of it for each feeder is equal to:

$$C_{total}^{CHP} = \sum_{i=1}^N C_i^{CHP}(P_{e_i}, Q_{e_i}) + (\lambda \times P_{L,(CHP)}) \cdot \Delta t \quad (15)$$

Where :

$N_i^a$  : Nodal pricing of active power without CHP

$N_{i,CHP}^a$  : Nodal pricing of active power with CHP

$N_i^r$  : Nodal pricing of reactive power without CHP

$N_{i,CHP}^r$  : Nodal pricing of reactive power with CHP

- $Q_{ei}$  : Reactive power consumption at bus  $i$
- $P_{CHPi}$  : Active power supplied by the CHP at bus  $i$
- $Q_{CHPi}$  : Reactive power supplied by the CHP at bus  $i$
- $C_{total}^{no-CHP}$  : Price of electricity supplied by the network without CHP
- $C_{total}^{CHP}$  : Price of electricity supplied by the network with CHP
- $C_{(CHP)}$  : Price of electricity supplied by CHP
- $P_{r(CHP)}$  : Active power losses by considering CHP.
- $P_r$  : Active power losses without CHP.

CHP is intended as a negative load at its bus and to simplify the calculations assume that  $Q_{CHPi}$  and  $P_{CHPi}$  are zero at all buses except that DG is installed .

$$P_{CHPi} = \begin{cases} 0, & i \neq i_{best} \\ P_{CHPi}, & i = i_{best} \end{cases}$$

And

$$Q_{CHPi} = \begin{cases} 0, & i \neq i_{best} \\ Q_{CHPi}, & i = i_{best} \end{cases} \quad (16)$$

The larger difference " $C_{total}^{no-CHP} - C_{total}^{CHP}$ " leads to the distribution company profit increases by DG installation, and its formulation will be as follows:

$$T = C_{total}^{no-CHP} - (C_{total}^{CHP} + C_{total}^{CHP})$$

Where:

$T$  : Benefits of technical indexes improvement (for the distribution company)

$C_{total}^{no-CHP}$  : Price of electricity supplied by the network without CHP

$C_{total}^{CHP}$  : Price of electricity supplied by the network with CHP

$C_{total}^{CHP}$  : Price of CHP electricity

The voltage increasing at the CHP connected to bus and its effect on the voltage profile should be considered [18].

Also the voltage of each bus should be limited within the minimum and maximum defined permissible range in the distribution network, therefore CHP should be installed with the voltage condition in accordance relation (18), so that the bus voltage will be limited within its permitted range.

$$V_i^{min} \leq V_i \leq V_i^{max}, \quad i = 1, \dots, N_n \quad (18)$$

Where:

$V_i$  : Voltage at bus "i"

$V_i^{min}$  : Minimum permitted voltage at bus "i"

$V_i^{max}$  : Maximum permitted voltage at bus "i"

$N_n$  : Number of network buses

### 5. PROPOSED ALGORITHM

Block diagram of the proposed algorithm for optimal allocation and capacity determination of CHP is as follows (Fig.4):

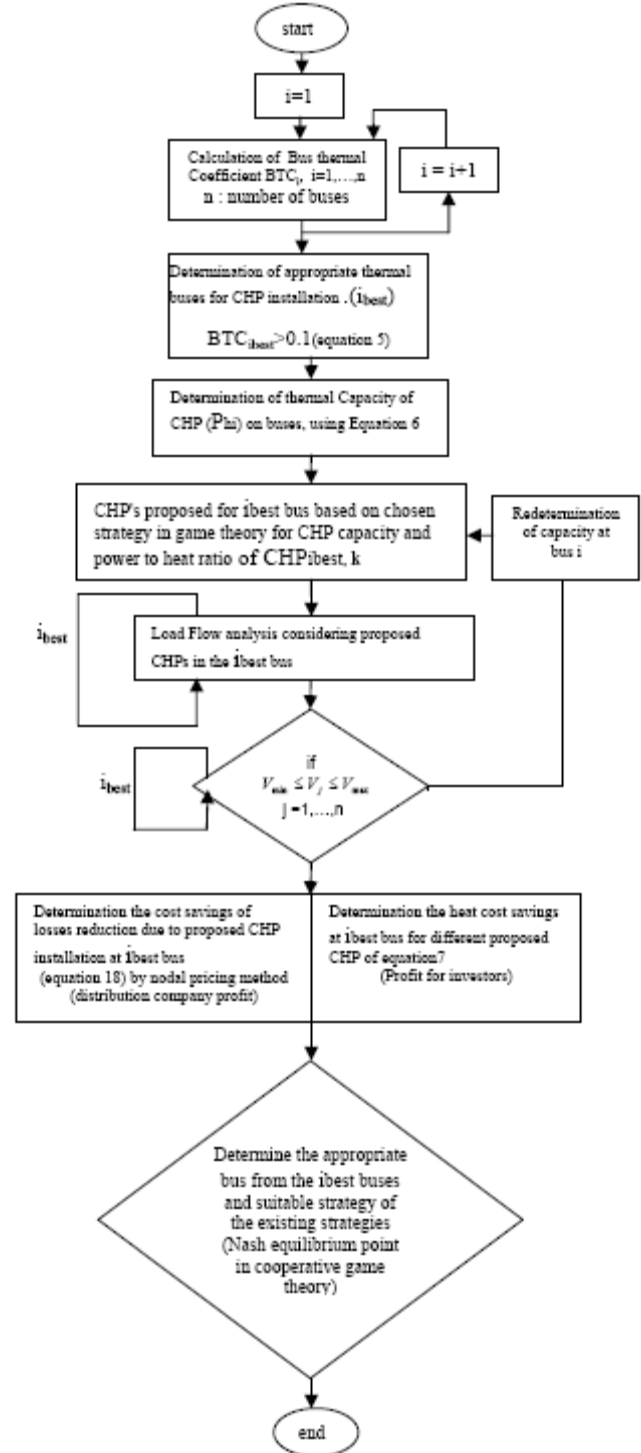


Fig.4. Block diagram of CHP Placement algorithm

Where :



CHP<sub>best,k</sub> : The CHP installed at bus  $i_{best}$  that follows the k strategy, ( $k : 1, \dots, k_{max}$ ).

**6. CASE STUDY**

In this part, one of the 20 KV Hamadan distribution feeders with 63 buses has been studied. This feeder is fed by Hamadan 63/20 kv station 1 (Fig.5) . Specifications of this feeder are presented in table 6:

**Table 6.** Specifications of feeder has been studied

| Length (KM) | Peak load of current (A) | P <sub>max</sub> (MW) | Price of electricity supplied by the network (US \$ / MWh[25]) |
|-------------|--------------------------|-----------------------|--|
| 12          | 80                       | 2.3                   | 50   |

The system has been simulated for a fixed time in this paper. With regard to the reciprocating engines CHP type, and based on cost of CHP in table 7, and assuming 75% efficiency achieved through the placement method in this paper, the cost of electricity supplied by CHP is equal to 53 \$ for a megawatt hour .

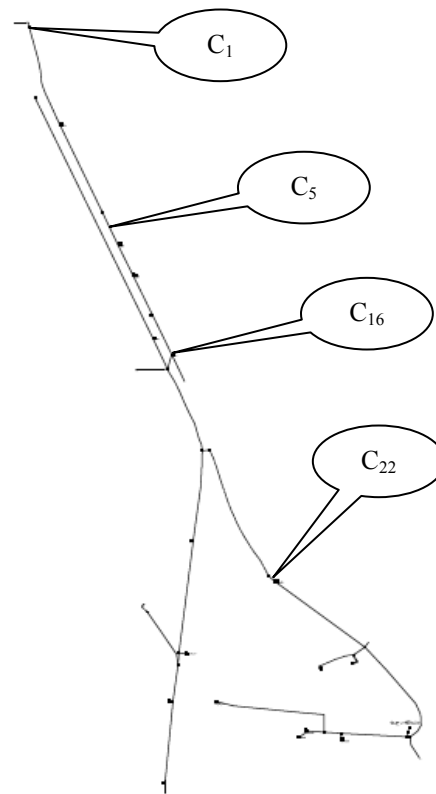
**Table 7.** Cost of used CHP

| the investment of installation price (US \$ / KW) | maintenance and operation cost (US \$ / KW/h) | operation time (h / year) | equipment life (year) |
|---|---|---------------------------|-----------------------|
| 900-1500  | 0.5-2   | 8760                      | 50                    |

According to consumers information, the large thermal loads of feeder are installed on buses : 1, 5, 16 and 22 . That their specifications are given in table 8.

**Table 8.** Thermal specifications of the major buses consumers

| Heat and warm water consumption (KW) (Pis) | consumer infrastructure (m <sup>2</sup> ) | Type of Consumption located around each bus | Bus Number |
|--|---|---|------------|
| 3040                                       | 37840                                     | Load 1,(office) C <sub>1</sub>              | 1          |
| 5619                                       | 27825                                     | Load 2, (university)C <sub>5</sub>          | 5          |
| 890  | 11110                                     | Load 16, (office) C <sub>16</sub>           | 16         |
| 1000                                       | 13300                                     | Load 22,(Residential) C <sub>22</sub>       | 22         |



**Fig. 5.**The major heating consumers (on feeder)

The buses that heat selling possibility are available and  $BTC_i \geq 0.1$  are suitable for CHP installation. In these buses the CHP capacities are calculated using Fuzzy method (Table 9).

**Table 9.** Determination of CHP thermal capacity for candidate buses

| CHP capacity based on buses thermal capacity (MW) | BT C | Thermal capacity of bus (kw) $P_{Sij} = P_i \times f(\beta, d, x, \Psi)$ | Bus number |
|---|------|--|------------|
| 0.7   | 0.7  | $3040 \times (0.25 \cap 1 \cap 0.75 \cap 1) = 7$                         | 1          |
| 1.4   | 1.4  | $5619 \times (0.75 \cap 1 \cap 0.5 \cap 0.25) =$                         | 5          |
| 0.22  | 0.22 | $890 \times (0.25 \cap 1 \cap 0.75 \cap 1) = 22$                         | 16         |
| 0.25  | 0.25 | $1000 \times (0.75 \cap 1 \cap 0.75 \cap 0.25) =$                        | 22         |

**6.1. Thermal benefit calculation**

In this stage we assume that CHPs will be installed on the every proposed buses(1, 5, 16, 22) have 0.5



&1MW capacities and the electrical power to heat ratios is 0.7 and 1. Then for each cases the heating cost savings is calculated using equation 7 that is shown in table 10 .

**Table 10.** Benefit of the heat consumers in the different game strategies

| Power / Heat Ratio = 1   |                        |                       |  |
|--------------------------|------------------------|-----------------------|--|
| Bus number               | Electric capacity (MW) | supplied Heating (MW) | Heat cost saving at each bus (investor profit) $\frac{\$}{year}$ |
| 1                        | 1                      | 0.7                   | 44150  |
|                          | 0.5                    | 0.5                   | 31536  |
| 5                        | 1                      | 1                     | 63072  |
|                          | 0.5                    | 0.5                   | 31536  |
| 16                       | 1                      | 0.22                  | 13875  |
|                          | 0.5                    | 0.22                  | 13875  |
| 22                       | 1                      | 0.25                  | 15768  |
|                          | 0.5                    | 0.25                  | 15768  |
| Power / Heat Ratio = 0.7 |                        |                       |  |
| 1                        | 1                      | 0.7                   | 44150  |
|                          | 0.5                    | 0.7                   | 44150  |
| 5                        | 1                      | 1.4                   | 88300  |
|                          | 0.5                    | 0.71                  | 44781  |
| 16                       | 1                      | 0.22                  | 13875  |
|                          | 0.5                    | 0.22                  | 13875  |
| 22                       | 1                      | 0.25                  | 15768  |
|                          | 0.5                    | 0.25                  | 15768  |

**6.2. Technical indicators benefit calculation**

CHP installation will improve the network technical indicators, and this improvement is considered as benefit for electrical distribution company. At first with doing Load Flow and then we use the nodal pricing for candidate buses. These prices are available for the CHP candidate buses before and after installation (for 0.5MW and 1 MW) in table XI, also it is assumed that CHP works with "unit power factor", means it will produce the (real)active power only. As it is shown in table 11 the active nodal price of each bus will be reduced essentially, when CHP is present.

**Table 11.** Nodal pricing of active power obtained by fuzzy bus thermal coefficient for fixed loads without and with CHP

| Bus number | CHP capacity based on bus thermal coefficient (MW) | Nodal pricing of active power at buses without CHP (US \$ / MWh) | Nodal pricing of activepower at buses with CHP (US \$ / MWh) |
|------------|--|--|--|
| r          |  |  |  |

|    |     |        |        |
|----|-----|--------|--------|
| 1  | 1   | 51.445 | 50.945 |
|    | 0.5 | 51.475 | 51.175 |
| 5  | 1   | 51.015 | 50.965 |
|    | 0.5 | 51.44  | 51.24  |
| 16 | 1   | 51.14  | 50.99  |
|    | 0.5 | 51.41  | 51.31  |
| 22 | 1   | 51.485 | 51.035 |
|    | 0.5 | 51.505 | 51.4   |

By CHPs installation with the capacities mentioned, using formulas 8 to 17,and table XI, the profits of losses reduction for the CHP buses candidates will be calculated .

By considering CHP installed at bus 1 and doing load flow analysis, the new calculated losses, the amount of electrical energy supplied by the CHP and network will be determined and the cost of CHP and network electricity will be calculated (columns 5 and 6, Table XII). The CHP installation benefits is obtained from the equation {a-(b + c)} of column 7 in the table XII. The column 7 indicates the benefits of CHP installation which is desirable for Distribution Company.

**Table 12.** Distribution company profit produced by the generator installed at each bus using the nodal pricing method

| Bus number | cost of network electricity without CHP <sup>1</sup> (a) $\frac{\$}{year}$ | CHP capacity (MW) (c) | losses (b) (MW) | cost of network electricity (b) $\frac{\$}{year}$ | cost of CHP electricity (c) $\frac{\$}{year}$ | Distribution company profit a-(b+c) $\frac{\$}{year}$ |
|------------|--|-----------------------|-----------------|---|---|---|
| 1          | 100740   | 1                     | 0.189           | 341640  | 464280  | 201480  |
|            |  | 0.5                   | 0.235           | 754236  | 232140  | 21024   |
| 5          |  | 1                     | 0.193           | 516840  | 464280  | 26280   |
|            |  | 0.5                   | 0.248           | 759930  | 232140  | 15330   |
| 16         |  | 1                     | 0.198           | 519030  | 464280  | 24090   |
|            |  | 0.5                   | 0.262           | 766062  | 232140  | 9198  |
| 22         |  | 1                     | 0.207           | 522972  | 464280  | 20148   |
|            |  | 0.5                   | 0.281           | 774384  | 232140  | 876   |

1.The total losses of network will be 0.313 MW without CHP installation.

**6.3. Game theory for Optimal selection**

In the proposed method, the distribution company and investors are players A and B respectively,the

strategies which these two players can choose them, are electrical power to heat ratio (0.7 or 1) and electrical capacity (0.5 MW or 1 MW) of CHP.

By installation of specified CHPs at the candidate buses through the above strategies, the benefit of consumers and distribution companies (payoff (winning) for each player) will be determined from table X and XII that is shown in Table 13.

We can specify the Nash equilibrium point in static game with above complete information from table XIII. This point is chosen indicates that benefits of both players are maximum and each player attempts to change these selection will lead to detriment of other players and the whole set.

According to Table 13, can be seen that the choice of strategy  $A_3$  (CHP installed capacity of 1MW and power to heat ratio of 0.7) at bus 5, the Nash equilibrium of this game is obtained that in this point the player gains A and B are respectively 26,280 and 88,300 dollars per year.

**Table 13.** The payoff (winning) amount for players with different strategies

|          |                | Player B                       |  |                               |                               |
|----------|----------------|--------------------------------|--|-------------------------------|-------------------------------|
|          |                | B <sub>1</sub>                 | B <sub>5</sub>                             | B <sub>16</sub>               | B <sub>22</sub>               |
| Player A | A <sub>1</sub> | 201480 <sup>+</sup> ,<br>44150 | 26280 ,<br>63072                           | 24090 <sup>+</sup> ,<br>13875 | 20148 <sup>+</sup> ,<br>15768 |
|          | A <sub>2</sub> | 21024 ,<br>31536 <sup>+</sup>  | 15330 ,<br>31536 <sup>+</sup>              | 9198,<br>13875                | 876 ,<br>15768                |
|          | A <sub>3</sub> | 201480 <sup>+</sup> ,<br>44150 | 26280 <sup>+</sup> ,<br>88300 <sup>+</sup> | 24090 <sup>+</sup> ,<br>13875 | 20148 <sup>+</sup> ,<br>15768 |
|          | A <sub>4</sub> | 21024 ,<br>44150               | 15330 ,<br>44781 <sup>+</sup>              | 9198 ,<br>13875               | 876 ,<br>15768                |

## 7. CONCLUSION

In this paper, a new method was provided for allocation and capacity determination of Combined Heat and Power (CHP) on the bus.

CHP installations in the distribution network in addition to improving technical indicators such as reduced losses, improved voltage profile and voltage regulation for the distribution company's profit ability, creates possibility of heat selling around the bus and profit ability for the investor.

In this paper, the distribution companies and investors were considered as players and capacity and power to heat ratio as the strategies of the players . Then using the Nash equilibrium, the equilibrium point is determined by two players so that at this point each player gains are maximum and changing this point by one of the players causing detriment is another players .

Investors benefit create from the selling of heat generated around the bus and profits of distribution company due to improved technical indicators using

price change of a node before and after installation of CHP has been calculated.

At the end of paper, the proposed method is applied on a sample feeder in HAMADAN city and the optimal location and capacity of CHP is determined at the distribution buses system . The results show that proposed method is effective and efficient.

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## REFERENCES

- [1] M. H. Moradi, M. Abedini, "A Combination of Genetic Algorithm and Particle Swarm Optimization for Optimal DG location and Sizing in Distribution Systems", *Int. Journal of Electrical Power and Energy Systems* (34), pp 66-74, 2012.
- [2] T. Ackermann, G. Andersson and L. Soder. "Distributed generation a definition, *Electr. Power Syst*", Res. Vol. 57, No. 3, pp 195-204, 2001.
- [3] N. Mithulanathan, Oo Than, Van Phu Le. "Distributed generator placement in power distribution system using genetic algorithm to reduce losses. *TIJSAT*", Vol. 9, No. 3, pp. 55-62, 2004.
- [4] T. Griffin, K. Tomosovic, D. Secrest and A. Law "Placement of dispersed generations systems for reduced losses", *In: Proceedings of the 33rd Hawaii international conference on sciences, Hawaii , 2000.*
- [5] M. H. Moradi and M. Abedini, "A Combination of GA and PSO for Optimal DG location and Sizing in Distribution Systems with Fuzzy Optimal Theory", *International Journal of Green Energy, In press*, 2011.
- [6] K. Nara, Y. Hayashi, K. Ikeda, and T. Ashizawa, "Application of Tabu Search to optimal placement of distributed generators", *Proceedings of the IEEE Power Engineering Society*, Vol. 2, pp. 918-923, February. 2001.
- [7] G. Bidini, U. Desideri, S. Saetta and P. ProiettiBocchini, "Internal combustion engine combined heat and power plants: case study of the university of Perugia power plant", *Appl. Therm. Eng.* Vol. 18, No. 6, pp. 401-412, 1998.
- [8] A. C. Caputo, M. Palumbo and F. Scacchia, "Perspectives of RDF use in decentralized areas: comparing power and co-generation solutions", *Appl. Therm. Eng.*, Vol. 24, No. (14-15), pp. 2171-2187, 2004
- [9] R. K. Singh and S.K. Goswami. "Optimum allocation of distributed generations based on nodal pricing for profit, loss reduction, and voltage improvement including voltage rise issue", *Electrical Power and Energy Systems*, Vol. 32, pp. 637-644, 2010.
- [10] A. Viawan Ferry, S. Ambra, and D. Jaap, "Voltage control with on-load tap changers in medium

- voltage feeders in presence of distributed generation. *Electr Power Syst Res*", Vol. 77, pp. 1314–22, 2007.
- [11] S. Repo, H. Laaksonen, J. Pertti, H. Osmo, and M. Mikael, "A case study of a voltage rise problem due to a large amount of distributed generation on a weak distribution network", In: *Proceedings of IEEE Bologna power tech conference*, Vol. 4. Bologna, Italy, 2003.
- [12] S. Yiqun, H. Zhijian, W. Fushuan, and N. Yixin, "Wu F.F. Analysis of marketpower in oligopolistic electricity market based on game theory", *power systems and communications infrastructures for the future, Beijing*, September .2002.
- [13] L. Bunningham, R. Baldick and L. Martin, "Baughman. Anempiricalstudy of applied game theory: Transmission constrained cournot behavior", *IEEE transactions on power systems*, Vol. 17, No. 1, February. 2002.
- [14] V. Neimane ,A. Sauhats, G. Vempers, I. Tereskina and G. Bockarjova, "Allocating Production Cost at CHP Plant to Heat and Power using Cooperative Game Theory", *IEEE Bucharest Power Tech Conference, June 28th – July 2nd, Bucharest, Romania*, 2010.
- [15] ASHREA handbook of fundamental. the American society of heating, refrigerating and air –conditioning engineers, inc, 2005.
- [16] S. M. Tabatabaie, "Computing facility construction", 2008.
- [17] J. Mutale, G. Strbac, S. Crucis and N. Jenkins "Allocation of losses in distribution systems with embedded generation", *IEEE Proc Gen TransmDistribut*; Vol. 147(1), pp. 7–12, 2000.
- [18] A. Viawan Ferry, S. Ambra and D. Jaap, "Voltage control with on-load tap changers in medium voltage feeders in presence of distributed generation. *Electr Power Syst Res*", Vol. 77, pp. 1314–22, 2007.
- [19] D. W. Wu and R. Z. Wang. "Combined cooling, heating and power: A review", *Progress in Energy and Combustion Science*, Vol. 32, pp. 459- 495, 2006.
- [20] Catalog of CHP Technologies. U.S. Environmental Protection Agency(EPA), Combined Heat and Power Partnership, Arlington, Virginia 22209, 2008.
- [21] M. J. Osborne, and A. Rubinstein, "A Course in Game Theory , MIT Press(Chapters 13, 14, 15)", 1994.
- [22] A. Souri, "Game Theory and Economic Applications", *Department of Economic Science, Tehran, Iran, Nore Elm Entesharat*, 2008.
- [23] 19th section. National Building Regulations In Iran, Tehran, Iran .
- [24] A. Jalali and H. zekri. "Allocation of losses costs in distribution networks in the presence of distributed generation using nodal pricing method", *the second electrical energy saving conference, Ahvaz ,Iran*, 2011.
- [25] M. H. Moradi and F.Samaie. "Optimal Allocation of Combined Heat and Power (CHP) in Hamedan City .Research Project", *Research Committee of HAMADAN power Distribution Company*, 2011.
- [26] M. Sotkiewicz Paul and J. Mario Vignolo, "Nodal pricing for distribution networks: efficient pricing for efficiency enhancing DG", *IEEE Trans power Syst ;21:1013–4*, 2006.