Design a of 3-Dimensional Nonlinear Optimal Robust Guidance Law for Homing Missiles against Maneuvering Targets Using Genetic Algorithm

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ABSTRACT:
Optimal sliding mode guidance (OSMG) law is proposed for tactical missiles which is pursuing maneuvering targets in three-dimensional space. The most important characteristic of sliding mode control (SMC) is high robustness against parameter variations or external disturbances. By using a Lyapunov function, it is demonstrated that the derived guidance law can stabilize the engagement system. Also, coefficients of controller are chosen using genetic algorithm optimally. Compared with traditional augmented proportional navigation guidance (APNG) law, the proposed guidance law not only can increase robustness against external disturbance and eliminate the effect of target maneuvers but can improve tracking performance and reduce interception time and miss distance. 3-D missile-target engagement is simulated for different target maneuvers and for some various scenarios, then results of the OSMG law are compared with conventional APNG law. Simulation Results confirmed the above mentioned pretensions.

KEYWORDS: Missile, guidance, optimal sliding mode, robust, APNG.

1. INTRODUCTION
The kinematic of missile-target is one of the most nonlinear kinematics. Since the basic principles of missile guidance were extensively covered by Locke [1] and Lin [2], many theories have been utilized to improve guidance performance and to overcome environmental disturbances. Many various guidance laws have been exploited with different design concepts over the years. Now a days, the most popular and widely used terminal guidance laws involve line-of-sight (LOS) guidance [1], LOS rate guidance and other advanced guidance such as proportional navigation guidance [1], augmented proportional navigation guidance (APNG) [3] and other proportional navigation strategies, optimal guidance law based on the linear quadratic regulator theory [4] and linear exponential Gaussian theory [5].
In [6] a nonlinear H_∞, robust guidance law for homing missile in two-dimensional space is proposed. A robust state- dependent Riccati equation based guidance/control is investigated in [7]. In this paper, guidance law is designed in two-dimensional space, too. In [8] a guidance law against unpredictable maneuvering targets using tunable H_∞ method was designed in two-dimensional space. A guidance law based on state dependent reccati equation (SDRE) is derived in [9]. In this work, SDRE not only is applied to 2-Dimensional space of missile-target, but also is not robust against maneuvering targets.
In all of above work, either guidance law design is applied to 2D model of missile-target, or guidance law is not robust.
In this paper, a 3D optimal guidance law based on sliding mode theory is designed considering nonlinear missile-target geometry. The main feature of sliding mode control (SMC) theory is robustness against target's maneuvers. A widely used guidance law that's applying to terminal guidance is called augmented proportional navigation (APN) [3]. The way that the SMC guidance law treats maneuvering targets is basically different from that of APN. Target acceleration is regarded as unpredictable disturbance for the OSMG law. Although, target acceleration is neither required, nor estimated in the OSMG law, the robustness of OSMG law guarantees acceptable interceptive performance for any arbitrary target maneuvers, as long as target acceleration is finite. In the circumstance where target acceleration is known or can be estimated, adaptive guidance law such as APN is certainly superior to robust OSMG law; while in the

situation where target acceleration is unknown or is weakly estimated, robust OSMG law could be better than adaptive guidance law. In this research it is supposed that target acceleration is unknown. Main problem in OSMC design is to find the switching surfaces. These switching surfaces are chosen so those satisfy all of the problem desired aims.

This paper is organized as follows. In the next section; 2, we formulate the 3D missile-target engagement and guidance problem. In Section 3, we will design SMG law for missile. In Section 4, we will design optimal sliding mode guidance law, and we will obtain coefficients of the controller by using of genetic algorithm. Simulation results are in section 5. In this section, robustness and tracking of the 3D OSMG law against target maneuvers are illustrated and compared with APNG. Finally, the results of the study are briefly summarized in section 5.

2. STATEMENT OF THE PROBLEM

3D pursuit geometry within the spherical coordinates system (r, θ, ϕ) is shown in Fig. 1, where r is the relative distance between missile and target, and angles θ and ϕ are azimuths of the line of sight (LOS). The missile and target are assumed to be point masses in order to easily analyze the missile guidance. The three dimensional engagement model (Fig. 1) can be represented mathematically by the following equations [12],

$$\begin{aligned} &\ddot{r} - r\dot{\phi}^2 - r\dot{\theta}^2\cos^2\phi = w_r - u_r \\ &r\ddot{\theta}\cos\phi + 2\dot{r}\dot{\theta}\cos\phi - 2r\dot{\phi}\dot{\theta}\sin\phi = w_\theta - u_\theta \\ &r\ddot{\phi} + 2\dot{r}\dot{\phi} + r\dot{\theta}^2\cos\phi\sin\phi = w_\phi - u_\phi \end{aligned} \tag{1}$$

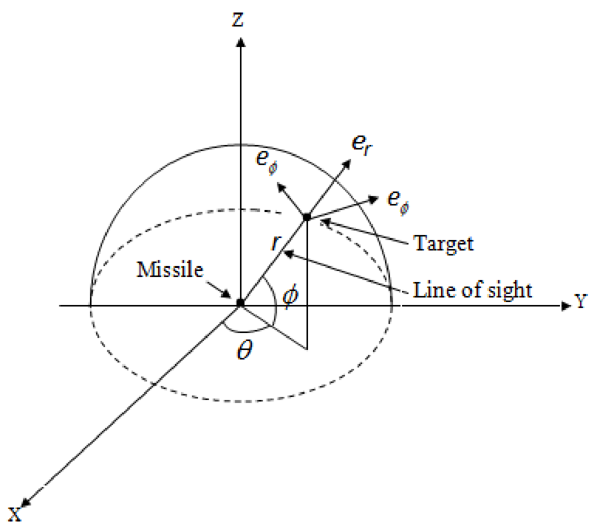


Fig. 1. 3-D pursuit-evasion geometry

w_r, w_θ and w_ϕ are the target's acceleration components. u_r, u_θ and u_ϕ are the missile's acceleration components, which are to be obtained. The kinematics Equation (1) can be rewritten as the following nonlinear state space equation [10]

$$\dot{x}(t) = F(x(t)) + Bu(t) + Dw(t) \tag{2}$$

Where the state vector $x(t)$, the vector field $F(x(t))$, the missile acceleration vector $u(t)$ and the target acceleration vector $w(t)$ are defined, respectively, as follows

$$x(t) = \begin{pmatrix} r \\ \theta \\ \phi \\ v_r \\ v_\theta \\ v_\phi \end{pmatrix}, \quad F(x(t)) = \frac{1}{r}\begin{pmatrix} rv_r \\ \dfrac{v_\theta}{\cos\phi} \\ v_\phi \\ v_\theta^2 + v_\phi^2 \\ -v_r v_\theta + v_\theta v_\phi \tan\phi \\ -v_r v_\phi - v_\theta^2 \tan\phi \end{pmatrix}$$

$$u(t) = \begin{pmatrix} u_r \\ u_\theta \\ u_\phi \end{pmatrix}, \quad w(t) = \begin{pmatrix} w_r \\ w_\theta \\ w_\phi \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{3}$$

The design problem is to derive a SMG law, u, in Equation (2), so that the initial relative distance r_0 is as small as possible under a reduced interception time.

3. DESIGN OF SLIDING MODE GUIDANCE LAW

The main objective here is to derive a guidance law that offers robustness against a variety of target maneuvers, in the terminal phase of a short-range homing missile.

In general, SMC design can be broken into two phases. The first phase is needed to choose a switching surface so that the system restricted to this surface produces the desired behavior. The next step is to choose a type of control that will force the system trajectories move on the switching surface and constrain them to slide along this surface for all subsequent times. Since the desired surface is chosen such that. It is independent of the external disturbances, robustness can be achieved. Like the first step, the selection of the switching surface is crucial because the structure of the guidance law and its robustness properties are a lot dependent on it.

Guidance law must satisfy a decreasing relative distance r and keep tangential relative velocities as small as possible. It is equivalent to satisfying the following conditions:

1- Radial relative velocity, v_r, converges to negative value,

2- v_θ and v_ϕ, tangential relative velocities, converge to zero

When v_θ and v_ϕ converge to zero, it means that the missile and target are on a convergence course. And when the radial relative velocity, v_r, has decreased to a negative value, the relative distance r between the missile and target decrease to zero. According to that, switching surface is chosen as

$$\begin{aligned} s_1 &= v_r + k, \qquad r_0 < k < 1.1 r_0 \\ s_2 &= v_\theta \\ s_3 &= v_\phi \end{aligned} \tag{4}$$

With the above switching surface, the guidance law is derived such that sliding constraint $(s\dot{s} < 0)$ is satisfied

$$u(t) = \begin{pmatrix} u_r \\ u_\theta \\ u_\phi \end{pmatrix} = \begin{pmatrix} \dfrac{v_\theta^2 + v_\phi^2}{r} - b_1 \operatorname{sgn}(s_1) \\ \dfrac{-v_r v_\theta + v_\theta v_\phi \tan\phi}{r} - b_2 \operatorname{sgn}(s_2) \\ \dfrac{-v_r v_\theta + v_\theta v_\phi \tan\phi}{r} - b_3 \operatorname{sgn}(s_3) \end{pmatrix} \tag{5}$$

Where

$$\begin{aligned} b_1 &> \max\left[w_r\right] \\ b_2 &> \max\left[w_\theta\right] \\ b_3 &> \max\left[w_\phi\right] \end{aligned} \tag{6}$$

To prove the stability and robustness of sliding surface analytically, we introduce a Lyapunov function as follows

$$V = \frac{1}{2} s^2 \tag{7}$$

Taking the derivative of Equation (7) with respect to time, we have

$$\dot{V} = s\dot{s} \tag{8}$$

and

$$\begin{aligned} \dot{s}_1 &= \dot{v}_r = \frac{v_\theta^2 + v_\phi^2}{r} - u_r + w_r \\ \dot{s}_2 &= \dot{v}_\theta = \frac{-v_r v_\theta + v_\phi v_\theta \tan\phi}{r} - u_\theta + w_\theta \\ \dot{s}_3 &= \dot{v}_\phi = \frac{-v_r v_\phi + v_\theta^2 \tan\phi}{r} - u_\phi + w_\phi \end{aligned} \tag{9}$$

Substituting (5) into (9) yields,

$$\begin{aligned} \dot{s}_1 &= \dot{v}_r = -b_1 \operatorname{sgn}(s_1) + w_r \\ \dot{s}_2 &= \dot{v}_\theta = -b_2 \operatorname{sgn}(s_2) + w_\theta \\ \dot{s}_3 &= \dot{v}_\phi = -b_3 \operatorname{sgn}(s_3) + w_\phi \end{aligned} \tag{10}$$

And finally if Equation (10) is substituted into Equation (8) we have,

$$\begin{aligned} s_1\dot{s}_1 &= -b_1 |s_1| + w_r s_1 \le -b_1 |s_1| + |w_r s_1| \\ &= (-b_1 + |w_r|)|s_1| \le 0 \;\Rightarrow b_1 \ge |w_r| \\ s_2\dot{s}_2 &= -b_2 |s_2| + w_\theta s_2 \le -b_2 |s_2| + |w_\theta s_2| \\ &= (-b_2 + |w_\theta|)|s_2| \le 0 \;\Rightarrow b_2 \ge |w_\theta| \\ s_3\dot{s}_3 &= -b_3 |s_3| + w_\phi s_3 \le -b_3 |s_3| + |w_\phi s_3| \\ &= (-b_3 + |w_\phi|)|s_3| \le 0 \;\Rightarrow b_3 \ge |w_\phi| \end{aligned} \tag{11}$$

So with the above conditions Equation (11) we have $\dot{V} < 0$.

4. OPTIMAL SLIDING MODE GUIDANCE LAW

In equation (11), coefficients of the controllers b_1, b_2, b_3 are variable parameters. So, SMG law is not optimal. To optimize these parameters, first we define a cost function as follows:

$$J = \sum (x^T R x + u^T Q u)$$

Where R and Q are defined as follows:

$$\begin{aligned} R &= diag(0.001, 10^{-10}, 10^{-10}, 10^{-10}, 10^{-10}, 1, 1) \\ Q &= diag(10, 10^{-10}, 10^{-10}) \end{aligned}$$

Now to limit control input, constrains are regarded as follows:

$$u = \begin{cases} u_r = \begin{cases} u_{r(SMC)} & u_r < u_{r(\max)} \\ u_{r(\max)} & u_r > u_{r(\max)} \end{cases} \\ u_\theta = \begin{cases} u_{\theta(SMC)} & u_\theta < u_{\theta(\max)} \\ u_{\theta(\max)} & u_\theta > u_{\theta(\max)} \end{cases} \\ u_\phi = \begin{cases} u_{\phi(SMC)} & u_\phi < u_{\phi(\max)} \\ u_{\phi(\max)} & u_\phi > u_{\phi(\max)} \end{cases} \end{cases}$$

Where $u_{r(\max)}$, $u_{\theta(\max)}$ and $u_{\phi(\max)}$ are definite values that for different scenarios which are different. Moreover, all states are not important for us. So, cost function is improved as follows:

$$\mathrm{ic}_r = \frac{1}{2}(1+\mathrm{sgn}(\mathrm{u}_r\text{-}\mathrm{u}_{r(\max)})$$
$$\mathrm{ic}_\theta = \frac{1}{2}(1+\mathrm{sgn}(\mathrm{u}_\theta\text{-}\mathrm{u}_{r(\max)})$$
$$\mathrm{ic}_\phi = \frac{1}{2}(1+\mathrm{sgn}(\mathrm{u}_\phi\text{-}\mathrm{u}_{r(\max)})$$
$$S = \begin{bmatrix} v_r + k \\ v_\theta \\ v_\phi \end{bmatrix}$$
$$U = [ic_r(u_r - u_{r(\max)}) \quad ic_\theta(u_\theta - u_{\theta(\max)}) \quad ic_\phi(u_\phi - u_{\phi(\max)})]$$
$$J = \sum (S^T RS + U^T QU)$$

Coefficients of controller b_1, b_2, b_3 are obtained by using genetic algorithm optimally.

5. SIMULATION RESULTS

In this Section, a numerical simulation is presented to justify the use of our proposed method.

Engagement performance and robustness of the SMG law and the APNG [4] against different types of targets and different scenarios are compared. Three maneuvering strategies of the target in 3-Dimensional space [2] are employed to investigate the robustness and tracking performance of the guidance laws, i.e.,w_r, w_θ and w_ϕ are generated by the following maneuvering targets,

1- Step target

$$w_r = \lambda_T \vec{e}_r$$
$$w_\theta = \lambda_T \frac{-\dot{\phi}}{\sqrt{\dot{\phi}^2 + \dot{\theta}\cos^2\phi}}\vec{e}_\theta$$
$$w_\phi = \lambda_T \frac{\dot{\theta}\cos\phi}{\sqrt{\dot{\phi}^2 + \dot{\theta}\cos^2\phi}}\vec{e}_\phi$$

2- Ramp target

$$w_r = \lambda_T t\vec{e}_r$$
$$w_\theta = \lambda_T t \frac{-\dot{\phi}}{\sqrt{\dot{\phi}^2 + \dot{\theta}\cos^2\phi}}\vec{e}_\theta$$
$$w_\phi = \lambda_T t \frac{\dot{\theta}\cos\phi}{\sqrt{\dot{\phi}^2 + \dot{\theta}\cos^2\phi}}\vec{e}_\phi$$

3- Sinusoidal target

$$w_r = \lambda_T \sin(\Omega t)\vec{e}_r$$
$$w_\theta = \lambda_T \sin(\Omega t)\frac{-\dot{\phi}}{\sqrt{\dot{\phi}^2 + \dot{\theta}\cos^2\phi}}\vec{e}_\theta$$
$$w_\phi = \lambda_T \sin(\Omega t)\frac{\dot{\theta}\cos\phi}{\sqrt{\dot{\phi}^2 + \dot{\theta}\cos^2\phi}}\vec{e}_\phi$$

Where λ_T is the target's navigation gain with random value within $0-4\,g$ and $\Omega = 20$ rad/s.

Two scenarios are proposed to illustrate the performance, robustness and tracking of the SMG law.

Case 1. Target escapes from missile ($w_r > 0$)

$$r = 4\,km,\ \ \theta = \frac{\pi}{3},\ \ \phi = \frac{\pi}{3}$$
$$v_r = -500\,m/s,\ v_\theta = 200\,m/s,\ v_\phi = 300\,m/s$$

Case 2. Target is toward the missile ($w_r < 0$)

$$r = 10\,km,\ \ \theta = \frac{\pi}{3},\ \ \phi = \frac{\pi}{3}$$
$$v_r = -1000\,m/s,\ v_\theta = 200\,m/s,\ v_\phi = 300\,m/s$$

Parameters b , ε and N are regarded as follows,

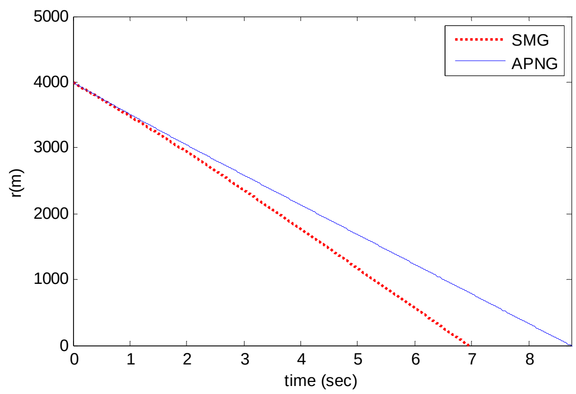
$$\begin{cases} b = [30, 45, 70]^T \\ \varepsilon = [5, 2, 1]^T \\ N = 3.8 \end{cases}$$

Where N is the navigation gain in APNG law. Chattering phenomena is one of the undesirable effects of sliding mode control. The main reason of chattering phenomena is the existence of sign function in control inputs u_r, u_θ and u_ϕ. In order to overcome this problem, $\tanh(s/\varepsilon)$ function is used instead of $sign(s)$ function.

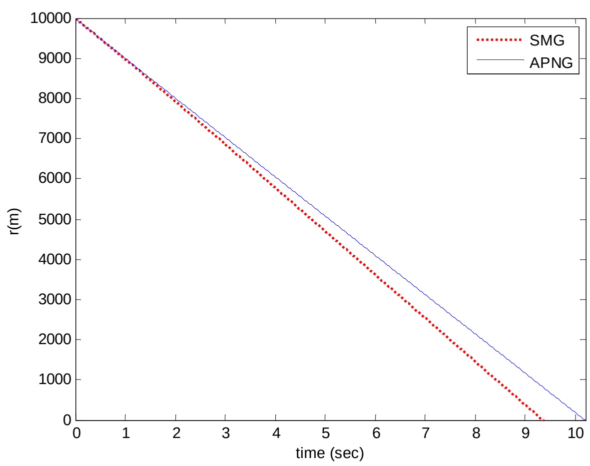
Fig. 2 illustrates convergence of the relative distance for the initial conditions of cases 1 and 2. Results show that the interception time in SMG law is less than APNG law. Our designing objective is to develop an effective guidance law to keep the pitch LOS angular rate, yaw LOS angular rate, and relative distance as small as possible under uncertain target accelerations. From Fig. 3(a) and (b), it is obvious that tangential relative velocities of the proposed guidance law converge to zero, quickly, more than those of the APNG law. This finding reveals that the SMG law has admirable target tracking ability, and it is possible to get smaller miss distances than that of the conventional one. Control commands for both guidance laws are shown in Fig. 4(a) and (b).

Robustness of the guidance design is investigated by three types of target acceleration command. According to the definition of performance robustness index, a robust guidance law should keep the engagement

performance with less sensitivity to the external disturbances, i.e., the target acceleration commands.
In APNG the successful engagement is based on the assumption that information about the target acceleration profiles is precisely measured but in the SMG law we only need the maximum target acceleration.
Simulation results in Fig. 5(a) and (b) have indicated it is hard for the APNG to track the step and ramp targets with initial conditions of case 1, but the proposed SMG laws still can accomplish the missions. Hence, the optimal robust proposed guidance law is more robust to uncertain target accelerations than the conventional one in different initial conditions.

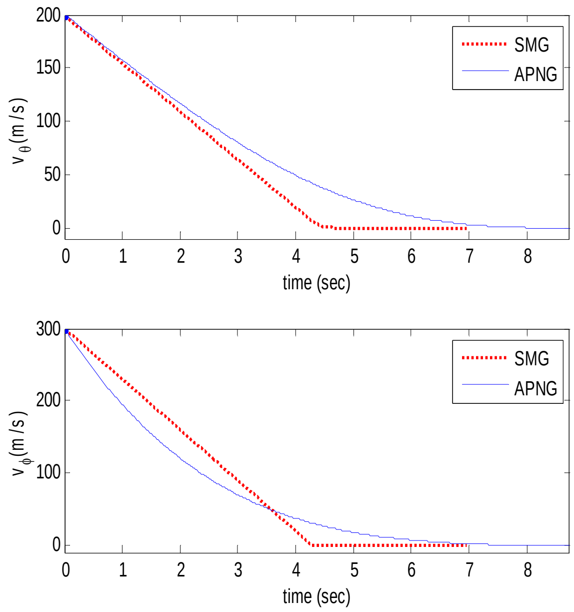


(a)

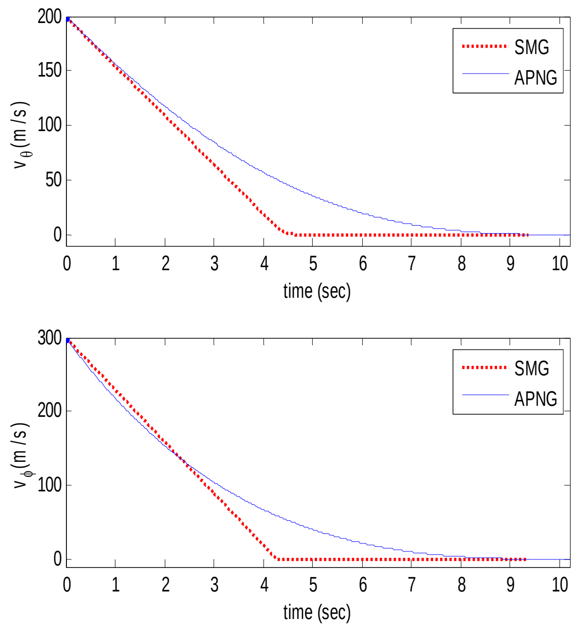


(b)

Fig. 2. Trajectories of relative distances between missile and target for OSMG and APNG versus sinusoidal target with different initial conditions. (a): Case 1. (b): Case 2

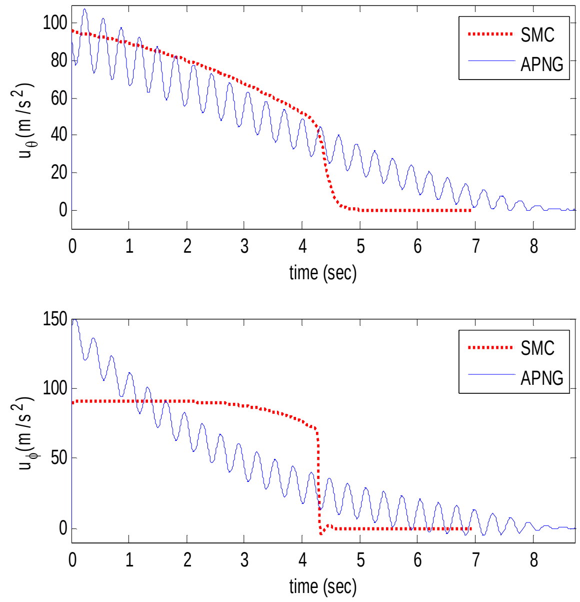


(a)

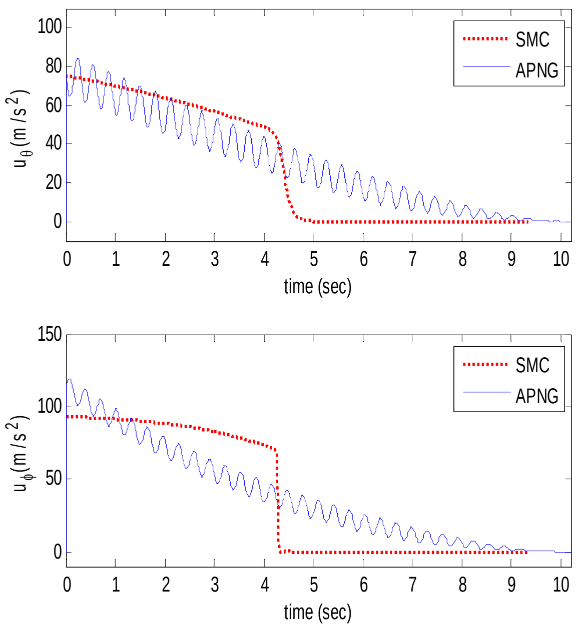


(b)

Fig. 3. Tangential relative velocities of OSMG and APNG versus sinusoidal target with different initial conditions. (a): Case 1, (b): Case 2

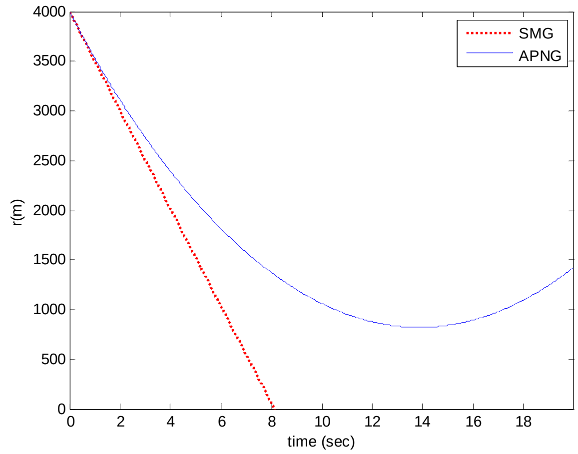


(a)

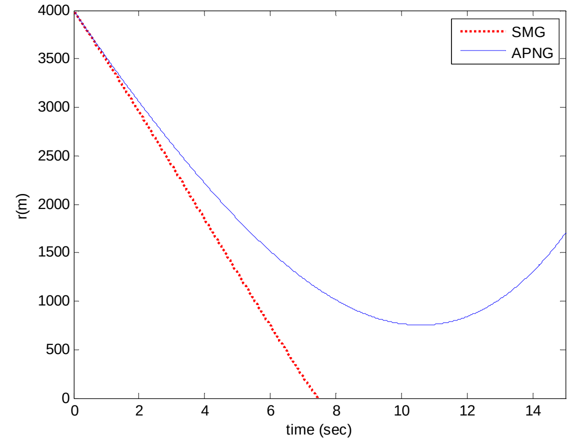


(b)

Fig. 4. Control commands for OSMG and APNG versus sinusoidal target. (a): Case 1, (b): Case 2



(a)



(b)

Fig. 5. Trajectories of relative distances between missile and target for OSMG and APNG versus step (a) and ramp target (b) with initial condition of case1

6. CONCLUSION

In this paper, an optimal sliding mode guidance law within using genetic algorithm was proposed for tactical missiles pursuing maneuvering targets in 3-Dimensional space that achieved the designing goal with less interception time. The proposed guidance law has higher maneuverability and results in miniature LOS angular rates than the traditional APNG. It also offers better performance against uncertain target accelerations. Simulation results show that SMG law is better than APNG in following cases: interruption time, robustness, tracking performance.

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