Adaptive State Feedback Control for Lipschitz Nonlinear Singular Systems

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Received: August 2013 Revised: October 2013 Accepted: December 2013

ABSTRACT
Singular systems behave more powerfully in terms of dynamical system modeling than ordinary state space systems. Since the algebraic equations in singular models can describe the systems constraints, nonlinear singular systems can present a general method for modeling and controlling constrained dynamical systems. This paper discusses an adaptive control for nonlinear singular systems which satisfy Lipschitz condition. Adaptive methods for singular systems are hardly ever investigated in literatures; however they are very useful methods in practice because the adaptive mechanism during the adaptive control can adjust the controller for a system with the unknown structures and parameters to improve the system performance. The presented controller is composed of a state feedback approach with adaptive gains and a mechanism to adjust the gains based on the Lyapunov stability theorem. First the controller is designed to stabilize the system, as a result, it is extended for the tracking problem. A simulation on a mobile robot singular model is provided to illustrate the effectiveness of the proposed control approach.

KEYWORD: Nonlinear Singular Systems, Adaptive Sontrol, Constrained Robot, Lipschitz Condition, Singular Systems Control.

1. INTRODUCTION

Singular systems (a.k.a, descriptor systems and differential-algebraic systems) which contain both differential and algebraic equations that have attracted a lot of research interests in recent years. This kind of model is a natural and convenient representation of real systems and it can deal with complex and constrained systems, more easily. Many applications of singular systems have been discussed in different systems such as circuit systems[1], power systems[2], economic systems [3], constrained robots [4, 5], chemical process[6], and biological systems [7, 8].

Many control methods are therefore rapidly extended for singular systems. However, the complex nature of this class of systems causes some difficulties in control approaches. In addition to being stable, an acceptable singular system should be regular and impulse free. The singular system index is in charge of the complexity. When the system index increases, the complexity and difficulties grow up. There are several different definitions for singular system index, but the more typical one is the differential index, which is equal to the number of differentiations of algebraic equations needed to change a singular system to an ordinary differential equations' system [9]. Control design for higher index singular systems is more difficult.

The Stability of singular systems for the linear and nonlinear ones has been discussed in several papers [10-12]. Most of the control approaches for singular systems that have been studied so far are in the field of optimal control and robust control [13-18]. Output and state feedback control[18, 19] and intelligent control methods [14] are also extended for singular systems. But few papers discuss the adaptive and model reference control for singular systems [20, 21]. However, the adaptive control is one of the most useful methods in practice and the adaptive mechanism during the adaptive control can adjust the controller for a system with unknown structures and parameters to improve the system performance. So it is necessary to develop the adaptive approaches for singular systems.

As the complex nature of this type of systems causes many difficulties in control strategy, most of the control methods which have been designed so far are for linear systems, while the real systems have mostly nonlinear models. Singular nonlinear control systems are still an open research field.

In this paper, we consider the adaptive control of nonlinear singular systems. Nonlinear singular systems have complex natures, and at the same time, the unknown parameters in the system model make the control design, more complicated. An adjustable state feedback approach is presented for the class of nonlinear systems, in which the nonlinear part of equations satisfies the Lipschitz condition. The update mechanism is directly extracted from the Lyapunov function to guarantee the stability and to decrease the parametric errors. First a simple basic control is designed for stabilizing a single input nonlinear singular system and after that the proposed controller is generalized for tracking problem of multi- input singular systems. For better illustration, the presented control approach is then applied to a mobile robot, which is modeled by singular system.

This paper is organized as follows. The problems of formulations and the theories and assumptions used for control design are presented in section 2. Designing of a basic adaptive control for single input nonlinear singular systems is investigated in section 3. Tracking control design for multi-input singular systems is presented in section 4. Section 5 introduces the singular model of a mobile robot. Simulation results are investigated in section 6 and then section 7 concludes the paper.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following nonlinear singular system:

$$\mathbf{E}\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{f}(\mathbf{x})+\mathbf{B}u(t) \quad (1)$$

where $\mathbf{x}\in R^n$ is the vector of the system's states and $u\in R^l$ is the control input. We have $\mathbf{B}\in R^{n\times l}$ and $\mathbf{f}(\mathbf{x})\in R^{n\times 1}$ is nonlinear function vector. The parameter l is the number of inputs. The matrix $A\in R^{n\times n}$ is the system matrix of linear coefficients. The matrix E can be singular (Rank (E) <n). The parameters **f** and **A** are unknown and **f** satisfies the following Lipschitz condition.

$$\|\mathbf{f}(\mathbf{x}_1)-\mathbf{f}(\mathbf{x}_2)\|\le L\|\mathbf{x}_1-\mathbf{x}_2\| \quad (2)$$

while L is also unknown;. Without loss of generality, it is assumed that **f**(0)=0, so the condition (2) can be rewritten as

$$\|\mathbf{f}(\mathbf{x})\|\le L\|\mathbf{x}\| \quad (3)$$

Now the objective is to design a control input ***u*** to stabilize such kind of systems and then to extend the controller for tracking the problem. First some assumptions should be considered.

Assumption 1.

A. There exists a matrix ***P*** such that

$$\mathbf{E}^{\mathrm{T}}\mathbf{P}=\mathbf{P}^{\mathrm{T}}\mathbf{E}\ge \mathbf{0} \quad (4)$$

B. Knowing the matrix ***P***, it is assumed that there exists a matrix $\boldsymbol{\theta}_1^*\in R^{l\times n}$ which satisfies the following equation

$$\mathbf{P}^{\mathrm{T}}(\mathbf{A}+\mathbf{LI}+\mathbf{B}\boldsymbol{\theta}_1^*)+(\mathbf{A}+\mathbf{LI}+\mathbf{B}\boldsymbol{\theta}_1^*)^{\mathrm{T}}\mathbf{P}=-\mathbf{Q} \quad (5)$$

where **Q** is positive definite and $\boldsymbol{\theta}_1^*$ is also unknown.□

For tracking the problem, we need some more assumptions. Choosing the reference trajectories vector $\mathbf{x_d}$ is an important part of control design. It should be guaranteed that the proposed $\mathbf{x_d}$ is a feasible trajectory for the system. The following assumption is considered in tracking control design.

Assumption 2.

A. The elements of the reference trajectories from vector $\mathbf{x_d}$ are differentiable such that the term $\mathbf{E}\dot{\mathbf{x}}_\mathbf{d}$ exists.

Since ***E*** is a singular matrix and rank(***E***) < n , derivative of some elements of $\mathbf{x_d}$ may not be needed. In other words, as the singular system contains the system states and also the algebraic variables, the derivative of some elements of the vector $\mathbf{x_d}$, which are related to the algebraic variables, may not be needed in the control design; however, the derivative of the elements which are related to the system states are necessary to exist.

B. For a determined reference vector , there exists a matrix $\boldsymbol{\theta}_2^*\in R^{l\times n}$ which satisfies the following equation

$$\mathbf{B}\boldsymbol{\theta}_2^*\mathbf{x_d}=-(\mathbf{A}+L\mathbf{I})\mathbf{x_d}+\mathbf{E}\dot{\mathbf{x}}_\mathbf{d} \quad (6)$$

where $\boldsymbol{\theta}_2^*$ is also unknown. □

First, a stabilizer is designed in the following section as a basic controller for one input of nonlinear singular systems with unknown parameters ***A*** and ***f***. Then the basic controller will be extended for the tracking problem of multi input nonlinear singular systems.

3. BASIC ADAPTIVE CONTROL DESIGN

In this section, a control strategy is designed based on the Lyapunov stability theorem to stabilize a nonlinear singular system with one input (*l=1*). The objective is to find an adaptive state feedback controller like

$$u=\boldsymbol{\theta}_1\mathbf{x} \quad (7)$$

and a mechanism to update the row vector $\boldsymbol{\theta}_1$, such that the closed loop system

$$\mathbf{E}\dot{\mathbf{x}}=(\mathbf{A}+\mathbf{B}\boldsymbol{\theta}_1)\mathbf{x}+\mathbf{f}(\mathbf{x}) \quad (8)$$

is stable. In other words, the goal is that $\boldsymbol{\theta}_1$ tends to $\boldsymbol{\theta}_1^*$

All the work is summarized in the following theorem.

Theorem 1. The nonlinear singular system (1) with unknown ***A*** and ***f***, will be stable, by using the adaptive state feedback control (7) in which θ is an adjustable vector with an update rule as

$$\dot{\boldsymbol{\theta}}_1^{\mathbf{T}} = -\gamma \mathbf{x}\mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{B} \quad (9)$$

where the matrix $\boldsymbol{P}$ satisfies (4) and γ is the adaption rate.

Proof- Consider the closed loop equation (8). After adding and subtracting the term $\mathbf{B}\boldsymbol{\theta}_1^{*}\mathbf{x}$, we have:

$$\mathbf{E}\dot{\mathbf{x}} = (\mathbf{A} + \mathbf{B}\boldsymbol{\theta}_1^{*})\mathbf{x} + \mathbf{f(x)} + \mathbf{B}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})\mathbf{x} \quad (10)$$

The following Lyapunov function is the candidate to stabilize the system and to reduce the parametric error:

$$V = \frac{1}{2}\mathbf{x}^{\mathbf{T}}\mathbf{E}^{\mathbf{T}}\mathbf{P}\mathbf{x} + \frac{1}{2\gamma}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})^{\mathbf{T}} \quad (11)$$

where P satisfies (4). Differentiating V respect to t, one can reach

$$\dot{V} = \frac{1}{2}(\dot{\mathbf{x}}^{\mathbf{T}}\mathbf{E}^{\mathbf{T}}\mathbf{P}\mathbf{x} + \mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{E}\dot{\mathbf{x}}) + \frac{1}{\gamma}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})\dot{\boldsymbol{\theta}}^{\mathbf{T}} \quad (12)$$

Using the system dynamics (10) instead of $\mathbf{E}\dot{\mathbf{x}}$ results in

$$\begin{aligned} \dot{V} &= \mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}(\mathbf{A} + \mathbf{B}\boldsymbol{\theta}_1^{*})\mathbf{x} + \mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{f(x)} + \\ &\mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{B}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})\mathbf{x} + \frac{1}{\gamma}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})\dot{\boldsymbol{\theta}}_1^{\mathbf{T}} \end{aligned} \quad (13)$$

Since $\mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{f(x)}$ is scalar, and according to Lipschitz condition (3), one can get

$$\begin{aligned} \dot{V} &\le \mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}(\mathbf{A} + \mathbf{B}\boldsymbol{\theta}_1^{*})\mathbf{x} + \mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}L\mathbf{I}\mathbf{x} + \\ &\mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{B}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})\mathbf{x} + \frac{1}{\gamma}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})\dot{\boldsymbol{\theta}}_1^{\mathbf{T}} \end{aligned} \quad (14)$$

And it can be said that

$$\begin{aligned} \dot{V} &\le \frac{1}{2}[\mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}(\mathbf{A} + \mathbf{B}\boldsymbol{\theta}_1^{*} + L\mathbf{I})\mathbf{x} + \\ &\mathbf{x}^{\mathbf{T}}(\mathbf{A} + \mathbf{B}\boldsymbol{\theta}_1^{*} + L\mathbf{I})^{\mathbf{T}}\mathbf{P}\mathbf{x}] + (\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})[\mathbf{x}\mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{B} + \frac{1}{\gamma}\dot{\boldsymbol{\theta}}_1^{\mathbf{T}}] \end{aligned} \quad (15)$$

It is clear that if the update rule is chosen as (9) then the derivative of Lyapunov function using (5) will be

$$\dot{V} \le \frac{-1}{2}\mathbf{x}^{\mathbf{T}}\mathbf{Q}\mathbf{x} \quad (16)$$

And based on the Lyapunov stability theorem, the closed loop system will be stable and $\boldsymbol{\theta}_1$ tends to $\boldsymbol{\theta}_1^{*}$.

One of the advantages of the proposed controller is that the control law and the update rule are very simple and easy to apply, so in practical systems, the presented controller is more convenient and applicable comparing with the ones which have been already proposed in the literature.

From the proof, the following corollary can be obtained.

Corollary 1. For the nonlinear singular systems in form of

$$\mathbf{E}\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{f(x)} + \mathbf{g(x)}u(t) \quad (17)$$

where the matrix g depends on x and it's not constant, if some constant matrix $\boldsymbol{B}$ with similar dimension is found such that $\|\mathbf{g(x)}\| \le \|\mathbf{B}\|$ for all values of $\mathbf{x}$, then the theorem 1 can be extended for this type of nonlinear systems. Similar to the proof of theorem 1, by using ***g(x)*** instead of $\boldsymbol{B}$, equation (13) will be

$$\begin{aligned} \dot{V} &= \mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{A}\mathbf{x} + \mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{f(x)} + \mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{g(x)}\boldsymbol{\theta}_1^{*}\mathbf{x} + \\ &\mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{g(x)}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})\mathbf{x} + \frac{1}{\gamma}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})\dot{\boldsymbol{\theta}}_1^{\mathbf{T}} \end{aligned} \quad (18)$$

Since the term $\mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{g(x)}\boldsymbol{\theta}_1^{*}\mathbf{x}$ is the scalar and $\mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{g(x)}\boldsymbol{\theta}_1^{*}\mathbf{x} \le \mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{B}\boldsymbol{\theta}_1^{*}\mathbf{x}$, (18) can be rewritten as

$$\begin{aligned} \dot{V} &\le \mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}(\mathbf{A} + \mathbf{B}\boldsymbol{\theta}_1^{*})\mathbf{x} + \mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}L\mathbf{I}\mathbf{x} + \\ &\mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{g(x)}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})\mathbf{x} + \frac{1}{\gamma}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^{*})\dot{\boldsymbol{\theta}}_1^{\mathbf{T}} \end{aligned} \quad (19)$$

which is similar to (14). Therefore, by some changes in the adaption rule of theorem 1 as

$$\dot{\boldsymbol{\theta}}_1^{\mathbf{T}} = -\gamma \mathbf{x}\mathbf{x}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{g(x)} \quad (20)$$

it can be seen that the theorem 1 will be held for such class of nonlinear singular systems.

In the following section, the proposed basic controller is extended for the tracking problem of multi input nonlinear singular systems.

4. TRACKING FOR MULTI-INPUT NONLINEAR SINGULAR SYSTEMS

In this section, the objective is that all the states and also the semi- states of nonlinear singular system (1) track a desired trajectory $\mathbf{x}_{\mathbf{d}}$. The adaptive control input is designed as

$$\mathbf{u} = \boldsymbol{\theta}_1\mathbf{e} + \boldsymbol{\theta}_2\mathbf{x}_{\mathbf{d}} \quad (21)$$

Where, in this investigation, we have l inputs. So, $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are matrices with the dimension $l \times n$. The parameter $\boldsymbol{e}$ is the tracking error as

$$\mathbf{e} = \mathbf{x} - \mathbf{x}_{\mathbf{d}} \quad (22)$$

Now the control problem is to find the update rules for $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ such that the tracking error tends to zero. All the investigations are summarized in the following theorem.

Theorem 2. For nonlinear singular system (1) using the adaptive controller (21) and the adaption rule for $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ as

$$\begin{aligned} vec(\dot{\boldsymbol{\theta}}_1)^T &= -\gamma_1(\mathbf{e}^{\mathbf{T}} \otimes \mathbf{e}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{B}) \\ vec(\dot{\boldsymbol{\theta}}_2)^T &= -\gamma_2(\mathbf{x}_{\mathbf{d}}^{\mathbf{T}} \otimes \mathbf{e}^{\mathbf{T}}\mathbf{P}^{\mathbf{T}}\mathbf{B}), \end{aligned} \quad (23)$$

Then, it can be guaranteed that all the states and also the semi states of the nonlinear singular system asymptotically track the desired reference $\mathbf{x_d}$ and the tracking error tends to be zero, if assumptions 1and 2 are held.

Note that the symbol $\otimes$ denotes Kronecker product and *vec(A)* means vectorization of **A**, which is obtained by stacking the columns of the matrix *A* on top of one another. It is a linear transform which converts the matrix into a column vector.

Proof- From (22), we have

$$\mathbf{x} = \mathbf{e} + \mathbf{x_d} \tag{24}$$

Then by substituting (24) in the system dynamics (1), one can reach

$$\mathbf{E\dot{e}} + \mathbf{E\dot{x}_d} = \mathbf{Ae} + \mathbf{Ax_d} + \mathbf{f(e+x_d)} + \mathbf{Bu} \tag{25}$$

Let's define

$$\mathbf{F(e)} = \mathbf{f(e+x_d)} - \mathbf{f(x_d)} \tag{26}$$

while, according to (2), ***F(e)*** also satisfies Lipschitz condition. By substituting (26) in (25) and by using control input ***u*** from (21), one can get

$$\begin{aligned}\mathbf{E\dot{e}} &= \mathbf{Ae} + \mathbf{F(e)} + \mathbf{B\theta_1 e} + \\ &\mathbf{Ax_d} + \mathbf{f(x_d)} - \mathbf{E\dot{x}_d} + \mathbf{B\theta_2 x_d}\end{aligned} \tag{27}$$

It can be rewritten as

$$\begin{aligned}\mathbf{E\dot{e}} &= (\mathbf{A} + \mathbf{B\theta_1}^*)\mathbf{e} + \mathbf{F(e)} + \\ &\mathbf{B(\theta_1 - \theta_1}^*)\mathbf{e} + \mathbf{Ax_d} + \mathbf{f(x_d)} - \\ &\mathbf{E\dot{x}_d} + \mathbf{B\theta_2}^*\mathbf{x_d} + \mathbf{B(\theta_2 - \theta_2}^*)\mathbf{x_d}\end{aligned} \tag{28}$$

The following Lyapunov function is selected

$$\begin{aligned}V &= \frac{1}{2}\mathbf{e^T E^T P e} + \frac{1}{2\gamma_1} vec(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^*)^T vec(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^*) + \\ &\frac{1}{2\gamma_2} vec(\boldsymbol{\theta}_2 - \boldsymbol{\theta}_2^*)^T vec(\boldsymbol{\theta}_2 - \boldsymbol{\theta}_2^*)\end{aligned} \tag{29}$$

where ***P*** satisfies (4) . Using *vec(A)* and Kronecker product makes the matrix computations more simple.

So, the derivative of *V* will be

$$\begin{aligned}\dot{V} &= \mathbf{e^T P^T (A + B\theta_1^*) e} + \mathbf{x^T P^T F(e)} + \\ &\mathbf{e^T P^T B(\theta_1 - \theta_1^*) e} + \mathbf{e^T P^T B(\theta_2 - \theta_2^*) x_d} + \\ &\mathbf{e^T P^T f(x_d)} + \mathbf{e^T P^T (B\theta_2^* x_d + Ax_d - E\dot{x}_d)} + \\ &\frac{1}{\gamma_1} vec(\dot{\boldsymbol{\theta}}_1)^T vec(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^*) + \\ &\frac{1}{\gamma_2} vec(\dot{\boldsymbol{\theta}}_2)^T vec(\boldsymbol{\theta}_2 - \boldsymbol{\theta}_2^*)\end{aligned}$$

(30)

Then, using Lipschitz condition results in

$$\begin{aligned}\dot{V} &\le \mathbf{e^T P^T (A + B\theta_1^*} + L\mathbf{I)e} + \mathbf{e^T P^T B(\theta_1 - \theta_1^*) e} + \\ &\mathbf{e^T P^T B(\theta_2 - \theta_2^*) x_d} + \frac{1}{\gamma_1} vec(\dot{\boldsymbol{\theta}}_1)^T vec(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^*) \\ &+ \frac{1}{\gamma_2} vec(\dot{\boldsymbol{\theta}}_2)^T vec(\boldsymbol{\theta}_2 - \boldsymbol{\theta}_2^*) + \\ &\mathbf{e^T P^T (B\theta_2^* x_d + Ax_d - E\dot{x}_d} + L\mathbf{Ix_d})\end{aligned} \tag{31}$$

where the last parentheses in (31) are equal to zero according to (6). Since the terms $\mathbf{e^T P^T B(\theta_1 - \theta_1^*) e}$ and $\mathbf{e^T P^T B(\theta_2 - \theta_2^*) x_d}$ are scalar, one can write

$$\mathbf{e^T P^T B(\theta_1 - \theta_1^*) e} = vec(\mathbf{e^T P^T B(\theta_1 - \theta_1^*) e}) \tag{32}$$

$$\mathbf{e^T P^T B(\theta_2 - \theta_2^*) x_d} = vec(\mathbf{e^T P^T B(\theta_2 - \theta_2^*) x_d}) \tag{33}$$

and due to the Kronecker product rule [22], we have

$$\begin{aligned}&vec(\mathbf{e^T P^T B(\theta_1 - \theta_1^*) e}) = \\ &(\mathbf{e^T} \otimes \mathbf{e^T P^T B}) vec(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^*)\end{aligned} \tag{34}$$

$$\begin{aligned}&vec(\mathbf{e^T P^T B(\theta_2 - \theta_2^*) x_d}) = \\ &(\mathbf{x_d}^T \otimes \mathbf{e^T P^T B}) vec(\boldsymbol{\theta}_2 - \boldsymbol{\theta}_2^*)\end{aligned} \tag{35}$$

By substituting (34) and (35) in the derivative of Lyapunov function (31), we can get

$$\begin{aligned}\dot{V} &\le \frac{1}{2}\mathbf{e^T [P^T (A + B\theta_1^*} + L\mathbf{I}) + \\ &(\mathbf{A + B\theta_1^*} + L\mathbf{I})^T \mathbf{P] e} + [(\mathbf{e^T} \otimes \mathbf{e^T P^T B}) + \\ &\frac{1}{\gamma_1} vec(\dot{\boldsymbol{\theta}}_1)^T] vec(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_1^*) + \\ &[(\mathbf{x_d}^T \otimes \mathbf{e^T P^T B}) + \frac{1}{\gamma_2} vec(\dot{\boldsymbol{\theta}}_2)^T] vec(\boldsymbol{\theta}_2 - \boldsymbol{\theta}_2^*)\end{aligned} \tag{36}$$

It is thus clear that by choosing the adaption rules as in (23) and using (5), the derivative of *V* will be negative definite as

$$\dot{V} \le \frac{-1}{2}\mathbf{x^T Q x} \tag{37}$$

and based on the Lyapunov stability theorem, the closed loop system would be stable: **x** tends to $\mathbf{x_d}$ and $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2$ tend to $\boldsymbol{\theta}_1^*, \boldsymbol{\theta}_2^*$. □

It can be seen that the control law and the adaption mechanisms are very simple and easy to apply in tracking problem of singular systems. Similar to the corollary 1, we can obtain similar results as follows.

Corollary 2. Similar to the corollary 1, if there exists a constant matrix *B* where $\|\mathbf{g(x)}\| \le \|\mathbf{B}\|$ for all values of ***x***, by choosing the adaption mechanisms as

$$vec(\dot{\boldsymbol{\theta}}_1)^T = -\gamma_1(\mathbf{e}^T \otimes \mathbf{e}^T\mathbf{P}^T\mathbf{g}(\mathbf{x}))$$
$$vec(\dot{\boldsymbol{\theta}}_2)^T = -\gamma_2(\mathbf{x_d}^T \otimes \mathbf{e}^T\mathbf{P}^T\mathbf{g}(\mathbf{x})) \quad (38)$$

it can be proved that the nonlinear singular system (17) is stable: $\boldsymbol{x}$ tends to $\mathbf{x_d}$, and $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$ tend to $\boldsymbol{\theta}_1^*$, $\boldsymbol{\theta}_2^*$. The proof is similar to corollary 1, so it is omitted here. □

An adaptive tracking controller is thus designed for multi input nonlinear singular systems in which the control law is very simple and practical. The adaption mechanisms guarantee that all the states and semi states of singular system tend to the desired values. The control structure is summarized in Fig1. For better illustration, the proposed control law is then applied to a nonlinear singular mobile robot system which is investigated in the next section.

5. MOBILE ROBOT SINGULAR SYSTEM

The tracking control of mobile robots could be very difficult due to the nonlinear and complex dynamics of the system; however, this type of robots has a lot of practical applications in industry [23]. Singular model of the mobile robots exhibits a better performance among other modeling methods and it gives more information about the robot movement including the physical and non-dynamic constraints [24]. The simplified model of a mobile robot is displayed in Fig2, where *o* is the location of the center of mass of robot by coordination like *(x,y)* and *v* is the velocity vector. Consider the system states as

$$\mathbf{q} = [x\ y\ \varphi]^T \ ,\ \mathbf{z} = [\mathbf{q}^T\ \dot{\mathbf{q}}^T\ \lambda]^T \quad (39)$$

where λ is the Lagrange multiplier, which is resulted from kinematics constraints; the parameter λ is considered as a semi state or an algebraic variable of the singular system which relates the kinematics of the robot to the system dynamics. Then, the Robot's singular model including dynamics modeled by Lagrange equations and kinematics [19] can be given by

$$\mathbf{E}\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{f}(\mathbf{z}) + \mathbf{g}(\mathbf{z})\mathbf{u}(t) \quad (40)$$

where $\mathbf{u}(\mathrm{t}) = [u_1(t)\ ,\ u_2(t)]^T$ is the control input vector in which u_1 is pushing force in φ direction and u_2 is steering input command. The matrices ***E, A, f, g*** can be given by

$$\mathbf{g}(\mathbf{z}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \cos\varphi & 0 \\ \sin\varphi & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},\ \mathbf{A} = \begin{bmatrix} 0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \end{bmatrix} \quad (41)$$

$$\mathbf{f}(\mathbf{z}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \lambda\sin\varphi \\ -\lambda\cos\varphi \\ 0 \\ \dot{x}\sin\varphi - \dot{y}\cos\varphi \end{bmatrix}, \quad (42)$$

$$\mathbf{E} = diag([1,1,1,m,m,I_o,0])$$

where *m* is the robot's mass, I_o is the robot's rotational inertia and φ is the angle between the velocity vector and *x* axis. It can be easily proved that *f* satisfies Lipschitz condition. Since *rank (**E**) <n*, (40) is a nonlinear singular system which is resulted from considering λ as a semi state of the state space model of the robot. In practical view, λ is the force that opposes against the robot's rotation. By using a singular system, we can model how non holonomic kinematics constraints affect the system dynamics. The last equation in (40) is

$$\dot{x}\sin\varphi - \dot{y}\cos\varphi = 0 \quad (43)$$

It directly arises from robot kinematics which limit the robot movement. During the robot movement in page *(x,y),* the algebraic equation (43) should be satisfied.

After that in the following section, the proposed controller is used to track the singular model of the mobile robot.

6. SIMULATION RESULTS

To illustrate the effectiveness of the proposed control approach, the controller is applied to the mobile robot (40). A circular trajectory is defined as reference trajectory by the following equations:

$$x_d = 5\sin(0.1t)$$
$$y_d = 5\cos(0.1t) - 5 \quad (44)$$

Therefore, the simulation objective is that the robot tracks a circle in page *(x,y)*. The Lagrange multiplier λ should be kept limiting to prevent robot rotation. The system parameters are defined as

$m = 1kg \text{ and } I_0 = 1kgm^2$.Robot desired direction (φ_d) during tracking is defined according to the reference trajectory as

$$\varphi_d = \tan^{-1}(\frac{y_d}{x_d}) \quad (45)$$

The controller (21) is applied to the mobile robot (40) using the adaption mechanisms as in (38). The results are displayed in Figures 3-9. Tracking the desired reference trajectory is shown in Fig3. It is clear that the mobile robot completely tracks the selected trajectory. The time response of states *x* and *y* tracking the desired ones is displayed in Fig4. As illustrated in Fig5, the tracking error is lower than 0.2 m, so the error is acceptable for a mobile robot tracking a circular trajectory with 10 m diameter. Robot direction (φ) is shown in Fig6. The angular velocity of the robot movement is fixed the same as the desired one. The control input *u* is displayed in Fig7 which is acceptable and limited. The frequency of oscillations is less than 1.5 Hz which is not high for the control input for a mobile robot. The convergence coefficient γ is in charge of oscillations. Higher γ increases the oscillations through the responses however it decreases the tracking error faster and improves the convergence rate. Figures 8 and 9 show the convergence of the adjustable parameters $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$. It is clear that the adaptive gains converge to fixed values which can be an estimation of $\boldsymbol{\theta}_1^*$ and $\boldsymbol{\theta}_2^*$. From all result, it can be obtained that the performance of the proposed controller is acceptable while its structure is very simple and easy to apply.

7. CONCLUSION

An adaptive control for nonlinear singular systems is investigated in this paper. An adjustable linear state feedback is designed for the class of nonlinear systems which satisfy the Lipschitz condition. First a basic control is designed to stabilize a one-input nonlinear system in which some system parameters and dynamics are unknown. After that, the proposed controller is generalized for tracking problem of multi-input nonlinear singular systems. The proposed controller has a very simple structure and it is easy to apply in practice. The controller is then applied to a singular model of mobile robot where an acceptable performance is achieved. The simulation results for tracking of a robot which its movement is limited by its kinematics emphasize the effectiveness of the presented controller.

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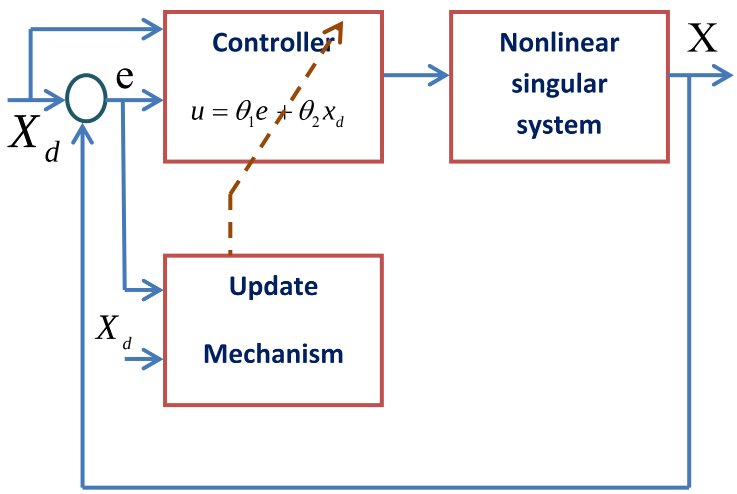

Fig. 1.Adaptive control structure for nonlinear singular systems

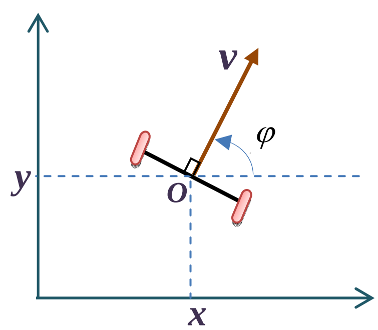

Fig. 2.Simplified model of a mobile robot

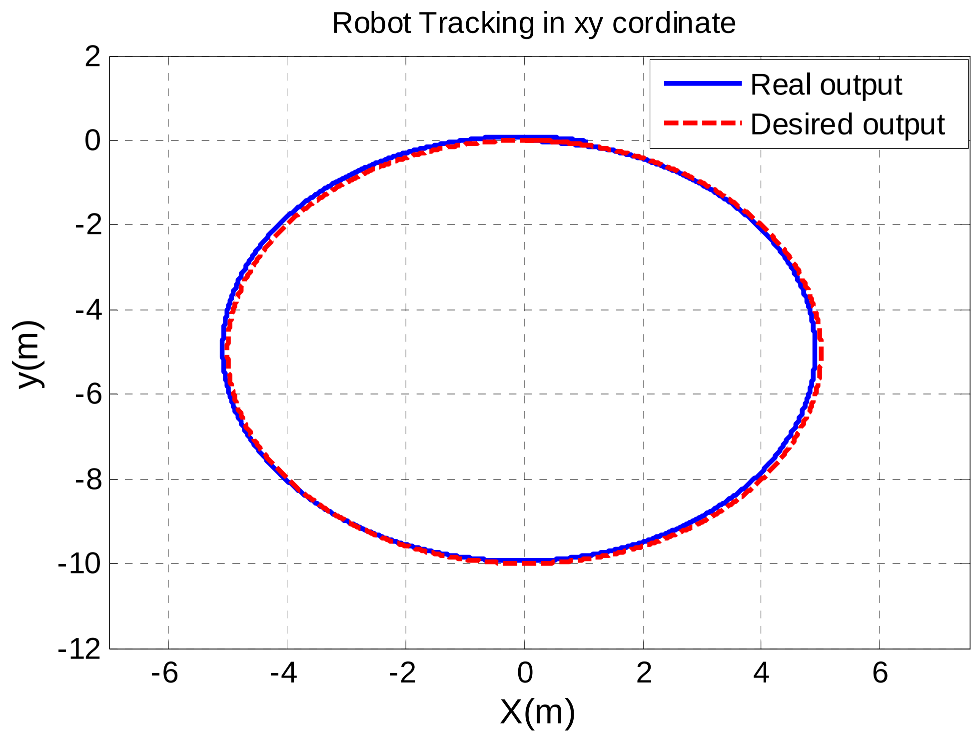

Fig. 3.Tracking the desired reference trajectory in the page (xy)

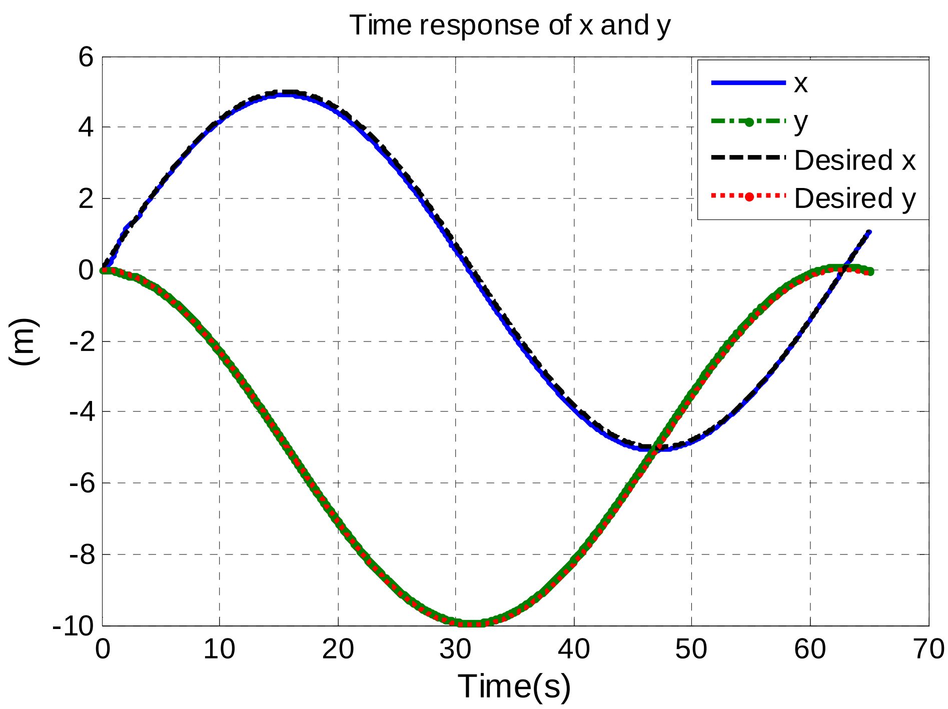

Fig. 4.Tracking performance for position states x and y

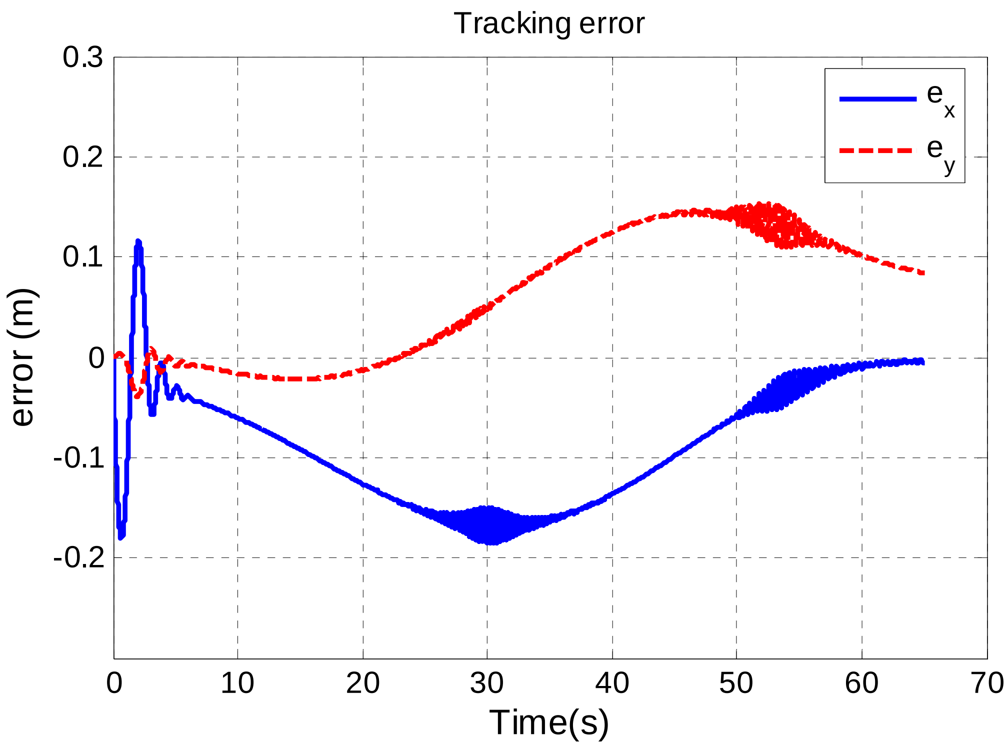

Fig. 5.Tracking error for x and y

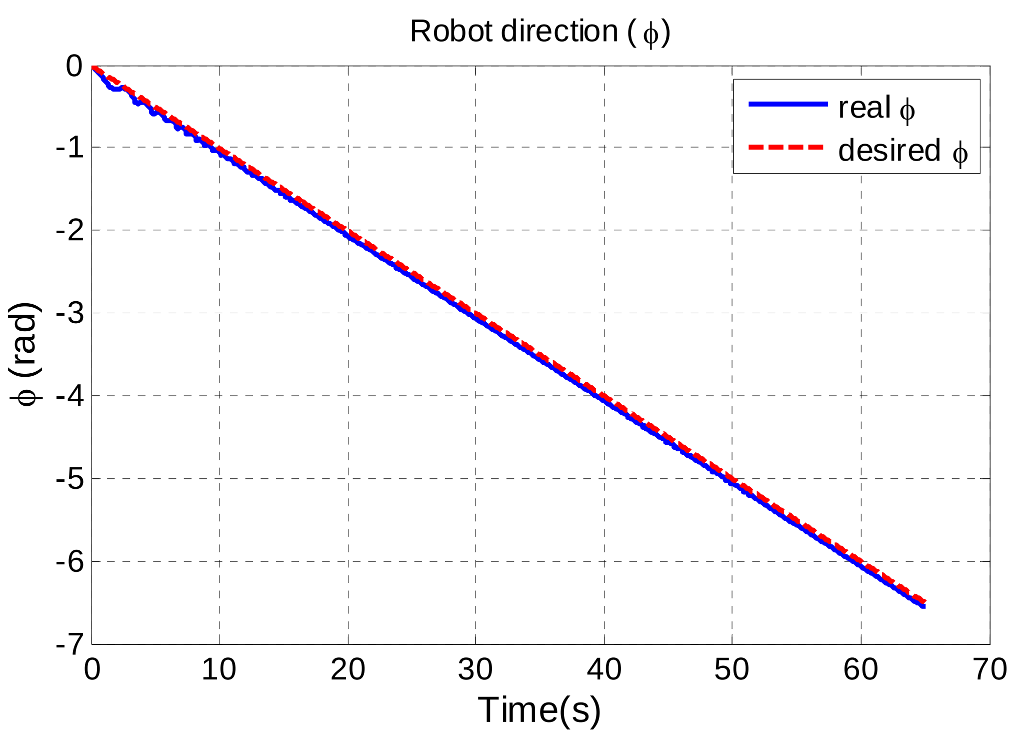

Fig. 6.Tracking of Robot direction (φ)

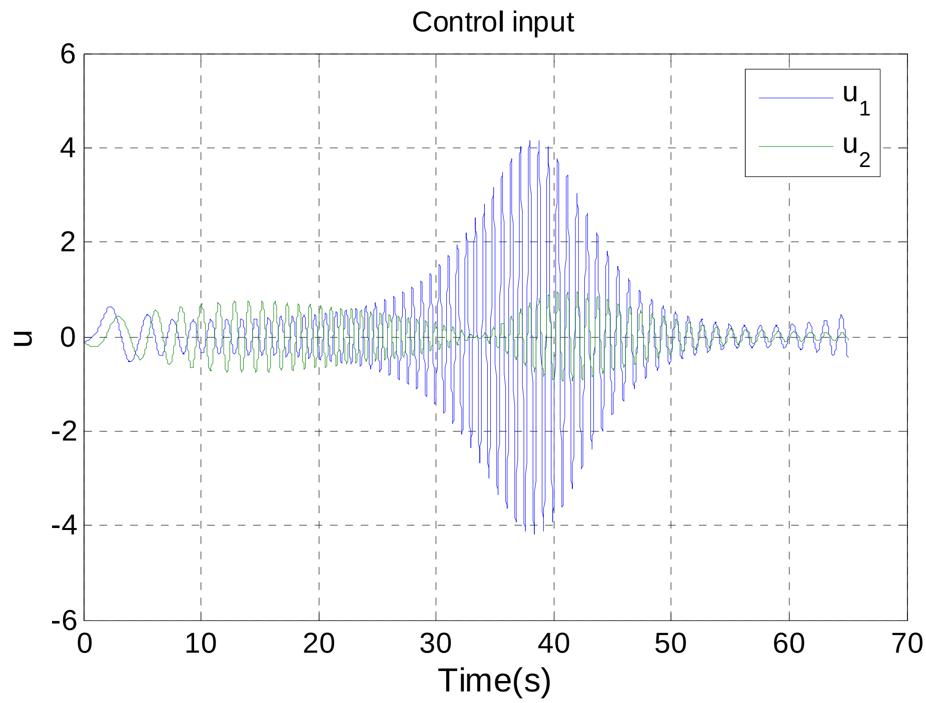

Fig. 7.Control input applied to the robot. It is limited and the frequency of oscillations is acceptable. u_1 is pushing force in Newton unit (N) and u_2 is the steering input command in Newton. Meter unit (N.m)

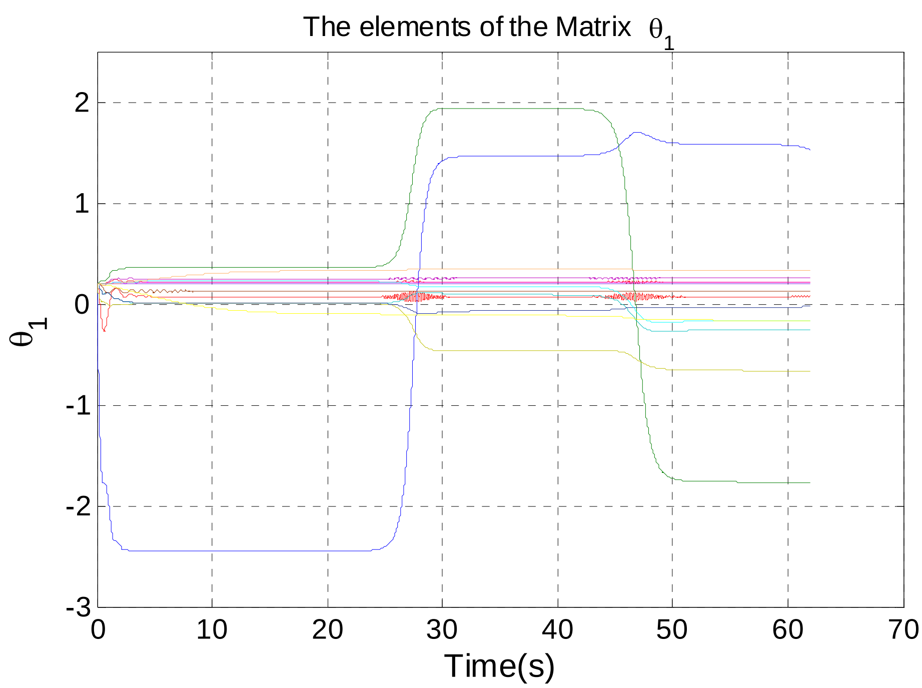

Fig. 8.Convergence of adaptive gains, Matrix θ_1

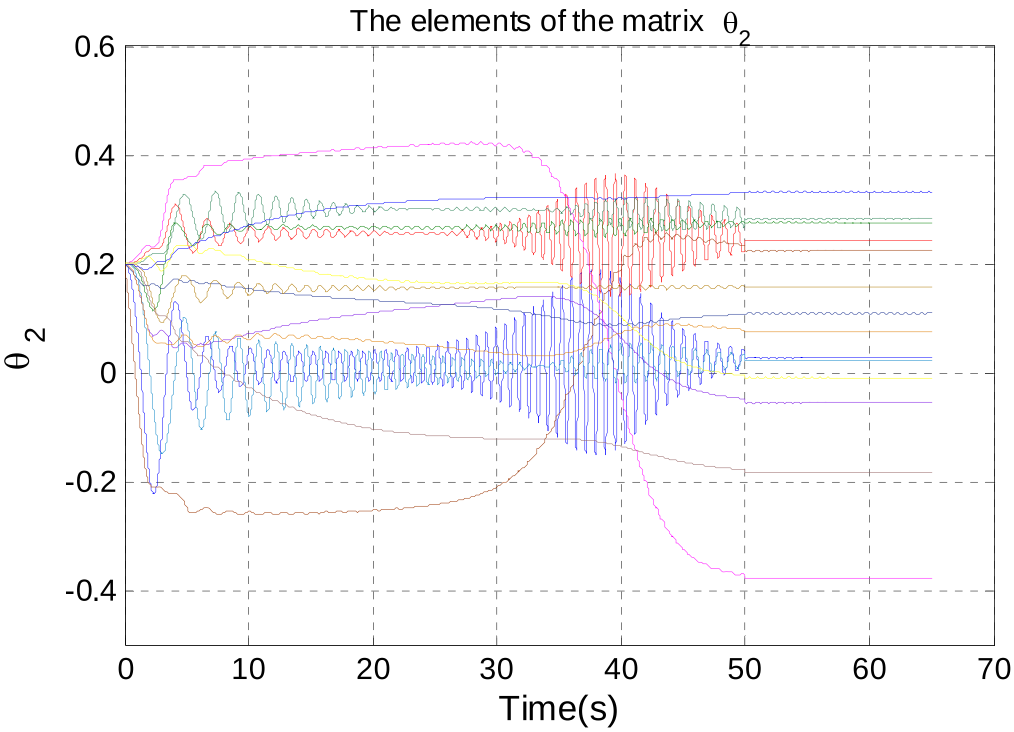

Fig. 9.Convergence of adaptive gains, Matrix θ_2