Fuzzy Mixed-Sensitivity Control of Uncertain Nonlinear Induction Motor

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ABSTRACT
In this article we investigate on robust mixed-sensitivity H∞ control for speed and torque control of inductional motor (IM). In order to simplify the design procedure the Takagi–Sugeno (T–S) fuzzy approach is introduced to solve the nonlinear model Problem. Loop-shaping methodology and Mixed-sensitivity problem are developed to formulate frequency-domain specifications. Then a regional pole-placement output feedback H∞ controller is employed by using linear matrix inequalities(LMIs) teqnique for each linear subsystem of IM T-S fuzzy model. Parallel Distributed Compensation (PDC) is used to design the controller for the overall system . Simulation results are presented to validate the effectiveness of the proposed controller even in the presence of motor parameter variations and unknown load disturbance.

KEYWORDS: IM, LMIs, Mixed-Sensitivity Problem, Robust Control, T-S Fuzzy Model

1. INTRODUCTION

Inductional motors are extensively used in industry, due to their comparatively low cost and high reliability. Over the last decade, there have been numerous progresses for the development of miscellaneous controllers for induction motors. For example, M. Rodic et al. [1] proposed Speed-sensorless sliding-mode torque control of an induction motor. J. C. Basilio et al. [2] presented H_∞ design of rotor flux-oriented current-controlled induction motor drives: speed control, noise attenuation and stability robustness. R. Marino et al. [3] Studied a nonlinear tracking control for sensorless induction motors. H. A. Yousef et al. [4] has proposed an adaptive fuzzy MIMO control of induction motors. Recent researches show that a T-S fuzzy model can be utilized to approximate global behaviours of a highly complex nonlinear system .The published papers have used the T-S fuzzy model technique for different drive systems[5-12].

The main contribution of this research is speed and torque control of induction motor by using H_∞ mixed-sensitivity problem via T-S fuzzy model. In this paper the problem of robust mixed-sensitivity H_∞ control for an IM system which possesses not only parameter uncertainties but also external disturbances is considered. In the proposed method nonlinear plant is first represented by Takagi–Sugeno (T-S) fuzzy model. The fuzzy model is described by fuzzy IF-THEN rules which represent local input-output relations of a nonlinear system. So the overall fuzzy model of the system is achieved by fuzzy "blending" of the local linear subsystem models. Then loop-shaping methodology and mixed-sensitivity problems are proposed in order to obtain optimal weighting functions. Afterward, for each fuzzy linear subsystem a robust mixed-sensitivity H_∞ output feedback controller with regional pole-placement are designed based on LMI formulation. PDC technique is utilized to design the controller for the overall system. Finally simulation results show that the proposed method can effectively meet the performance requirements like robustness, good load disturbance rejection responses, good tracking responses and fast transient responses for the IM system. The paper is organized as follows: IM model and problem statement have been described in Section II. Section III describes the H_∞ loop-shaping and the mixed-sensitivity problem. The design of robust pole-placement controller is presented in section IV. Simulation result of the closed-loop system with the proposed controller are presented in Section V and finally the paper is concluded in Section VI.

2. IM MODEL AND PROBLEM STATEMENT

A. IM Dynamic Model

The nonlinear electrical and mechanical equations for the 3-phase induction motor in the d-q reference frame can be written as follows [13] :

$$\frac{d\Omega}{dt}=\frac{1}{J}(T_m-K\Omega-C_r)$$
$$\frac{d\varphi_{rd}}{dt}=\frac{R_r}{L_r}(Mi_{sd}-\varphi_{rd})-\rho\Omega\varphi_{rq}$$
$$\frac{d\varphi_{rq}}{dt}=\frac{R_r}{L_r}(Mi_{sq}-\varphi_{rq})+\rho\Omega\varphi_{rd}$$
$$\frac{di_{sd}}{dt}=M\rho\beta\Omega\varphi_{rq}+\frac{R_r}{L_r}M\beta\varphi_{rd}+\gamma i_{sd}+\beta L_r v_{sd}$$
$$\frac{di_{sq}}{dt}=-M\rho\beta\varphi_{rd}\varphi_{rq}+\frac{R_r}{L_r}M\beta\varphi_{rq}+\gamma i_{sq}+\beta L_r v_{sq}$$
$$\beta=\frac{1}{L_rL_s-M^2}\qquad \gamma=-\beta(R_sL_r+\frac{M^2R_r}{L_r}) \tag{1}$$

Where

$$\underline{x}=(\Omega,\varphi_{rd},\varphi_{rq},i_{sd},i_{sq})^T\qquad \underline{u}=(v_{sd},v_{sq})^T$$

In equation (1), Ω is the rotor angular speed, the (d, q) projections of the stator current and rotor flux are i_{sd}, i_{sq}, φ_{rd}, φ_{rq} respectively. The control inputs are v_{sd}, v_{sq}. R_s, L_s are the stator resistance and inductance, R_r, L_r are the rotor resistance and inductance, M is the mutual inductance between stator and rotor, ρ is the number of pole pairs, K is the damping coefficient, J is the moment of inertia. Motor torque of the motor can be described as

$$T_m=\frac{M\rho}{L_r}\left[\varphi_{rd}i_{sq}-\varphi_{rq}i_{rd}\right] \tag{2}$$

In this model the parameters Rs, Rr and K are supposed to differ from their nominal values.

B. T-S Fuzzy Model of IM

In this section, the T–S fuzzy dynamic model is described by fuzzy IF–THEN rules, which represent local linear input/output -relations of nonlinear systems [14].The fuzzy dynamic model is proposed by Takagi and Sugeno. The ith rule of T-S fuzzy dynamic model with parametric uncertainties can be described as follows:

$$IF\ \ v_1(t)\ \ is\ \ M_{i1}\ and\ldots and\ \ v_p(t)\ \ is\ \ M_{ip}\ \ THEN$$
$$\dot{x}(t)=\left[[A_i+\Delta A_i]x(t)+[B_{1_i}+\Delta B_{1_i}]w(t)+[B_{2_i}+\Delta B_{2_i}]u(t)\right],\ x(0)=0$$
$$z(t)=\left[[C_{1_i}+\Delta C_{1_i}]x(t)+[D_{11_i}+\Delta D_{11_i}]w(t)+[D_{12_i}+\Delta D_{12_i}]u(t)\right]$$
$$y(t)=\left[[C_{2_i}+\Delta C_{2_i}]x(t)+[D_{21_i}+\Delta D_{21_i}]w(t)+[D_{22_i}+\Delta D_{22_i}]u(t)\right]$$
$$i=1,2,\ldots,r \tag{3}$$

Where, M_{ip} is the fuzzy set; r is the number of IF THEN Rules and $v_1(t)\rightarrow v_p(t)$ are the premise variables; $x(t)\in R^n$ is the state vector; $u(t)\in R^m$ is the control input vector; $w(t)\in R^q$ is the disturbance input vector; $y(t)\in R^p$ is the output vector. The matrices: $\Delta A_i, \Delta B_{1i}, \Delta B_{2i}, \Delta C_{1i}, \Delta C_{2i}, \Delta D_{12i}, \Delta D_{21i}, \Delta D_{11i}, \Delta D_{22i}$ represent the uncertainties in the system (3). The quasi-linear system of the nonlinear state space model (1) can be expressed as

$$A=\begin{bmatrix}-\frac{K}{J} & 0 & 0 & -\frac{M\rho}{JL_r}x_3 & \frac{M\rho}{JL_r}x_2\\ -\rho x_3 & -\frac{R_r}{L_r} & 0 & \frac{MR_r}{L_r} & 0\\ \rho x_2 & 0 & -\frac{R_r}{L_r} & 0 & \frac{MR_r}{L_r}\\ M\rho\beta x_3 & \frac{\beta MR_r}{L_r} & 0 & \gamma & 0\\ 0 & -M\rho\beta x_3 & \frac{\beta MR_r}{L_r} & 0 & \gamma\end{bmatrix}$$

$$B_w=\begin{bmatrix}-\frac{1}{J} & 0 & 0 & 0 & 0\end{bmatrix}^T$$
$$B_u=\begin{bmatrix}0 & 0 & 0 & \beta L_r & 0\\ 0 & 0 & 0 & 0 & \beta L_r\end{bmatrix}^T \tag{4}$$

A, B_w and B_u are known as real matrices with appropriate dimensions in nonlinear model (1). According to local linearization approach, we can obtain the local linear models for the system (4) with mentioned uncertainties (Rs, Rr and K). The overall fuzzy model is shown as the following form

$$\dot{x}(t)=\sum_{i=1}^{r}\mu_i\left(v(t)\right)\left[[A_i+\Delta A_i]x(t)+[B_{1_i}+\Delta B_{1_i}]w(t)+[B_{2_i}+\Delta B_{2_i}]u(t)\right],x(0)=0$$
$$z(t)=\sum_{i=1}^{r}\mu_i\left(v(t)\right)\left[[C_{1_i}+\Delta C_{1_i}]x(t)+[D_{11_i}+\Delta D_{11_i}]w(t)+[D_{12_i}+\Delta D_{12_i}]u(t)\right]$$
$$y(t)=\sum_{i=1}^{r}\mu_i\left(v(t)\right)\left[[C_{2_i}+\Delta C_{2_i}]x(t)+[D_{21_i}+\Delta D_{21_i}]w(t)+[D_{22_i}+\Delta D_{22_i}]u(t)\right] \tag{5}$$

Where: $v(t)=[v_1(t)\ \ \ldots\ \ v_p(t)]$ and weighting function is

$$\mu_i(v(t)) = \frac{\varpi_i(v(t))}{\sum_{i=1}^{r}\varpi_i(v(t))}$$

$$\varpi_i(v(t)) = \prod_{k=1}^{p} M_{ik}(v_k(t)) \qquad (6)$$

And it should be noted that

$$\varpi_i(v(t)) \geq 0\,, i = 1,2,\dots,r \;;\; \sum_{i=1}^{r}\varpi_i(v(t)) > 0$$
$$\mu_i(v(t)) \geq 0\,, i = 1,2,\dots,r \;;\; \sum_{i=1}^{r}\mu_i(v(t)) = 1$$

3. H∞ MIXED-SENSITIVITY PROBLEM

Loop shaping is a design procedure to formulate frequency-domain specifications as H∞ constraints problems[15,16]. To get a feeling for the loop-shaping methodology, consider the general control structure in Fig. 1.

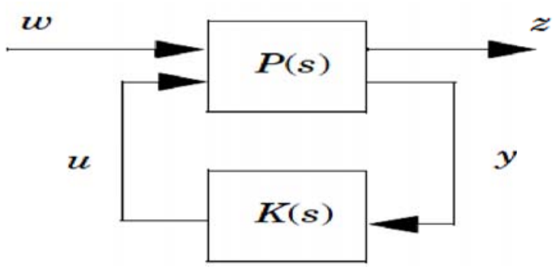


Fig. 1.The Control structure

In this figure, P(s) is the generalized plant, K(s) is the controller, u is the control signals, y is the measured variables, w is the exogenous signals and z is the cotrolled output. In this frequency domain method, the design specifications are reflected as gain constraints on the various closed- loop transferring functions. Where the main closed-loop transfer functions are sensitivity function and complementary sensitivity function and so gain constraints are shaping filters . The optimal H∞ control problem can be interpreted as minimizing the effect of the worst-case disturbance w on the output z. Hence the optimal H∞ control seeks to minimize $\|F(P,K)\|_\infty$ over all stabilizing controllers K(s). Where $\|F(P,K)\|_\infty$ is the closed-loop transfer function from w to z .Alternatively, we can specify some maximum value γ for the closed-loop RMS gain as $\|F(P,K)\|_\infty < \gamma$. Where γ is guaranteed H∞ norm constraint, ratio between z and w. In this case, the closed-loop transfer function $T_{zw}(s)$ is as follows:

$$T_{zw}(s) = F(P,K) = \begin{bmatrix} W_T(s)T(s) \\ W_s(s)S(s) \end{bmatrix} \qquad (7)$$

Where S(s) is the sensitivity transfer matrix (transfer function from r to e) and T(s) is the complementary sensitivity transfer matrix (transfer function from r to y):

$$S(s) = (I + G(s)K(s))^{-1}$$
$$T(s) = G(s)K(s)(I + G(s)K(s))^{-1} \qquad (8)$$

The $W_S(s)$ and $W_T(s)$ are two frequency dependent weighting functions (shaping filters), sensitivity weighting function and the complementary sensitivity weighting function respectively. The design procedure is to find out a controller, K which can

$$\sigma_{max}(S(jw)) < \gamma\, \sigma_{min}(W_s^{-1}(jw))$$
$$\sigma_{max}(T(jw)) < \gamma\, \sigma_{min}(W_T^{-1}(jw)) \qquad (9)$$

In this paper, because the number of control objectives are equal to 2, the size of weighting functions $W_S(s)$ and $W_T(s)$ are 2×2 matrices and in this case S(s) and T(s) are:

$$T(s) = T_{yr} = \begin{bmatrix} T_1 = T_{y_1r_1} \\ T_2 = T_{y_2r_2} \end{bmatrix} \qquad S(s) = T_{er} = \begin{bmatrix} S_1 = T_{e_1r_1} \\ S_2 = T_{e_2r_2} \end{bmatrix} \qquad (10)$$

Where y_1 and y_2 are rotor angular speed and motor torque, r_1 and r_2 are speed command and torque reference inputs and e_1 and e_2 are the tracking errors. Thereby $W_T(s)$ consists of a 2×2 square diagonal matrix with all its diagonal elements with the same transfer function $W_{T_{ii}}(s)$ and so $W_S(s)$ is proposed to be a square diagonal matrix with the same diagonal elements $W_{S_{ii}}(s)$:

$$W_T(s) = W_{T_{ii}}(s)\, I_{2\times2} \qquad W_s(s) = W_{S_{ii}}(s)\, I_{2\times2} \qquad (11)$$

The transfer functions $W_{S_{ii}}(s)$ and $W_{T_{ii}}(s)$ must be stable, minimum phase and additionally, they should be proper, As well $W_{S_{ii}}(s)$ and $W_{T_{ii}}(s)$ must be low-pass and high-pass filters respectively. A practical formula to determine the performance and robustness weights are as follows

$$W_{T_{ii}}(s) = \frac{ds+1}{ds+2} \qquad W_{S_{ii}}(s) = \frac{as+w_c}{s+w_c b} \qquad (12)$$

Where a is the gain for high frequency disturbances, b is the gain for low frequency control signal, d is a constant and w_c is the crossover frequency. In order to select optimal weighting functions to formulate performance and robustness specifications of close-loop system, a, b, c, w_c values should be decremented or incremented until the inequalities (9) are realized and $\gamma < 1$.

4. DESIGN OF ROBUT POLE-PLACEMENT CONTROLLER

In this section we focus on design of a local pole-placement output feedback controller for each linear subsystem(3):

IF $v_1(t)$ is M_{i1} and ... and $v_p(t)$ is M_{ip} THEN

$$u(t) = K_i y(t) \quad , \qquad i = 1,2,\dots,r \tag{13}$$

Where Ki (i = 1,2, … , r) are the local controller gains to be determined. For the system (3), the concept of parallel distributed compensation (PDC) is employed. According to PDC approach, the control law of the whole system is the weighted sum of the local feedback control of each subsystem. That is:

$$u(t) = \sum_{j=1}^{r} \mu_j K_j y(t) \tag{14}$$

Where, the local pole-placement output feedback gains K_j are determined by LMI-based design techniques in order to achieve the design requirements[16].The LMI formulation is applicable to design local controller that are introduced in Theorem1[15].

Theorem1. Main objective is to design an output-feedback controller u = K y as:

- maintain the H_∞ norm of $T\infty(s)$ (RMS gain) below some prescribed value $\gamma_0 > 0$
- maintain the H_2 norm of $T_2(s)$ (LQG cost) below some prescribed value $v_0 > 0$
- place the closed-loop poles in some prescribed LMI region D
- Minimize a trade-off criterion of the form $\alpha \|T_\infty\|_\infty{}^2 + \beta \|T_2\|_2{}^2$.

$T_\infty(s)$ and $T_2(s)$ are the closed-loop transfer functions from w to z_∞ and z_2, respectively. For the control structure shown in Fig. 1, the linear fuzzy sub plant P(s) is given in state-space form by

$$\begin{cases} \dot{x} = A_i x + B_{1_i} w + B_{2_i} u \\ z_\infty = C_{\infty_i} x + D_{\infty 1_i} w + D_{\infty 2_i} u \\ z_2 = C_{2_i} x + D_{21_i} w + D_{22_i} u \\ y = C_{y_i} x + D_{y1_i} w \end{cases} \tag{15}$$

And related controller K(s) is introduced by

$$\begin{cases} \dot{\xi} = A_k \xi + B_k y \\ \dot{u} = C_k \xi + D_k y \end{cases} \tag{16}$$

With regard to P(s), K(s) and u = K y the closed-loop system is

$$\begin{cases} \dot{x}_{cl} = A_{cl_i} x_{cl} + B_{cl_i} w \\ z_\infty = C_{cl1_i} x_{cl} + D_{cl1_i} w \\ z_2 = C_{cl2_i} x_{cl} + D_{cl2_i} w \end{cases} \tag{17}$$

Our three design objectives can be expressed as follows[9]:

•**H_∞ performance:** The closed-loop RMS gain from w to z_∞ does not exceed γ if and only if there exists a symmetric matrix X_∞ such that

$$\begin{bmatrix} A_{cl} X_\infty + X_\infty A_{cl}{}^T & B_{cl} & X_\infty C_{cl1}{}^T \\ B_{cl}{}^T & -I & D_{cl1}{}^T \\ C_{cl1} X_\infty & D_{cl1} & -\gamma^2 I \end{bmatrix} < 0, X_\infty > 0 \tag{18}$$

•**H_2 performance:** The H_2 norm of the closed-loop transfer function from w to z_2 does not exceed ν if and only if $D_{cl2} = 0$ and there exist two symmetric matrices X_2 and Q such that

$$\begin{bmatrix} Q & C_{cl2} X_2 \\ X_2 C_{cl2}{}^T & X_2 \end{bmatrix} > 0 \qquad \begin{bmatrix} A_{cl} X_2 + X_2 A_{cl}{}^T & B_{cl} \\ B_{cl}{}^T & -I \end{bmatrix} < 0$$

$$\mathrm{Trace}(Q) < \nu^2 \tag{19}$$

•**Pole placement:** The closed-loop poles lie in the LMI region

$$\mathcal{D} = \{z \in C : L + Mz + M^T \bar{z} < 0\}$$

$$L = L^T = \{\lambda_{ij}\}_{1 \le i,j \le m} \qquad M = \{\mu_{ij}\}_{1 \le i,j \le m} \tag{20}$$

If and only if there exists a symmetric matrix Xpol, it will be satisfied as follows:

$$\left[\lambda_{ij} X_{pol} + \mu_{ij}(A + B_2 K) X_{pol} + \mu_{ij} X_{pol} + \mu_{ji} X_{pol} (A + B_2 K)^T\right]_{1 \le i,j \le m} < 0 \text{ , } X_{pol} > 0 \tag{21}$$

For tractability in the LMI framework, we seek a single Lyapunov matrix: $X := X_\infty = X_2 = X_{pol}$ that enforces all three sets of constraints. Factorizing X as follows:

$$X = X_1 X_2{}^T \qquad X_1 := \begin{bmatrix} R & I \\ M^T & 0 \end{bmatrix} \qquad X_2 := \begin{bmatrix} 0 & S \\ I & N^T \end{bmatrix}$$

And introducing the change of controller variables

$$\begin{aligned} A_k &:= N A_k M^T + N B_k C_y R + S B_2 C_k M^T + S(A + B_2 D_k C_y) R \\ B_k &:= N B_k + S B_2 D_k \\ C_k &:= C_k M^T + D_k C_y R \end{aligned} \tag{22}$$

The inequality constraints on χ are readily turned into LMI constraints in the variables R, S, Q, A_K, B_K, C_K, and D_K. This leads to the following suboptimal LMI formulation of our multi-objective synthesis problem [10,11]:
Minimize $\alpha\gamma^2 + \beta$ Trace(Q) over R, S, Q, A_K, B_K, C_K, D_K and γ^2 satisfying:

$$\begin{bmatrix} AR + RA^T + B_2C_K + C_K{}^TB_2{}^T & A_K{}^T + A + B_2D_kC_y & B_1 + B_2D_kD_{y1} & H \\ H & A^TS + SA + B_KC_y + C_y{}^TB_K{}^T & SB_1 + B_kD_{y1} & H \\ H & H & -I & H \\ C_\infty R + D_{\infty 2}C_K & C_\infty + D_{\infty 2}D_KC_y & D_{\infty 1} + D_{\infty 2}D_KD_{y1} & -\gamma^2 I \end{bmatrix} < 0$$

$$\begin{bmatrix} Q & C_2R + D_{22}C_K & C_2 + D_{22}D_KC_y \\ H & R & I \\ H & I & S \end{bmatrix} > 0$$

$$\left[\lambda_{ij}\begin{pmatrix} R & I \\ I & S \end{pmatrix} + \mu_{ij}\begin{pmatrix} AR + B_2C_K & A + B_2D_kC_y \\ A_K & SA + B_KC_y \end{pmatrix} + \mu_{ji}\begin{pmatrix} RA^T + C_K{}^TB_2{}^T & A_K{}^T \\ (A + B_2D_kC_y)^T & A^TS + C_y{}^TB_K{}^T \end{pmatrix}\right]_{1\le i,j\le m} < 0$$

$$\text{Trace}(Q) < \nu_0{}^2 \qquad \gamma^2 < \gamma_0{}^2 \qquad D_{21} + D_{22}D_KD_{y1} = 0 \tag{23}$$

Given optimal solutions γ^*, Q^* of this LMI problem, the closed-loop H_∞ and H_2 performances are bounded by

$$\|T_\infty\|_\infty < \gamma^* \qquad \|T_2\|_2 < \sqrt{\text{Trace}(Q^*)} \tag{24}$$

The purpose of this section is to design a suitable control which guarantees robust performance in the presence of parameters-variation and load torque-disturbance. In this case, there are two control objectives. First rotor angular speed and second the motor torque that both of them must track reference trajectories r_1 and r_2, respectively. The cause of second objective is that load torque disturbance is unknown and it may obtain various values, consequently in order to drive motor we have got to select a motor torque specific value greater than load torque value. Therefore, to achieve these objectives just both tracking errors are minimized. Mentioned goals are realized through constructing the objectives z in an appropriate control loop. Under the above considerations, the structure of the fuzzy robust control loop is proposed that shown schematically in Fig. 2.

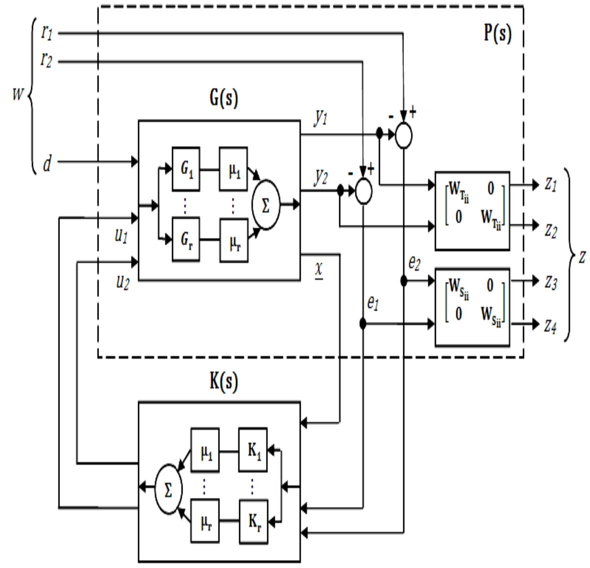


Fig. 2. The fuzzy robust control loop structure

In above structure first nonlinear dynamic model is approximated with some local linear models that each rule is represented by T–S fuzzy approach. Then, two shaping filters $W_S(s)$ and $W_T(s)$ are designed and Built up the augmented plant P and the controllers are designed for each linear sub plant based on LMI approach. After that, the total linear system is obtained by using the weighted sum of the local linear system and it is utilized rather than original nonlinear system. So, according to the PDC approach, the control law of the whole system is the weighted sum of the local feedback control of all subsystems. Finally, by using whole system and so that, global controller is applied as a tracking loop in order to achieve the desirable specifications such as tracking performance, bandwidth, disturbance rejection, and robustness for close-loop system.

5. SIMULATION RESULT

In this section, we show effectiveness of the proposed method by doing simulation for a three-phase-two pole pairs of induction motor. The parameters of the IM are shown in Table I. In this case, stator and rotor-resistances and damping coefficient are varied between $\pm 50\%$ and load torque disturbance is unknown [13]. In the quesi-linear system of IM (4), the number of nonlinearity terms are 2 (x_2, x_3). According to IM characteristic and system operating points we can assume that: $\varphi_{rd} = x_2 \in [-2 \;\; 4]$, $\varphi_{rq} = x_3 \in [-1 \;\; 2]$

Table.1. The paremeters of IM

Ls	*0.38 H*
Lr	*0.3 H*
M	*0.3 H*
Rs	*10 Ω*
Rr	*3.5 Ω*
K	*0.04 N m s/rad*
J	*0.02 kg.m²*
p	*2*

With regard to the above limitations, the membership functions can be demonstrated as Fig. 3.

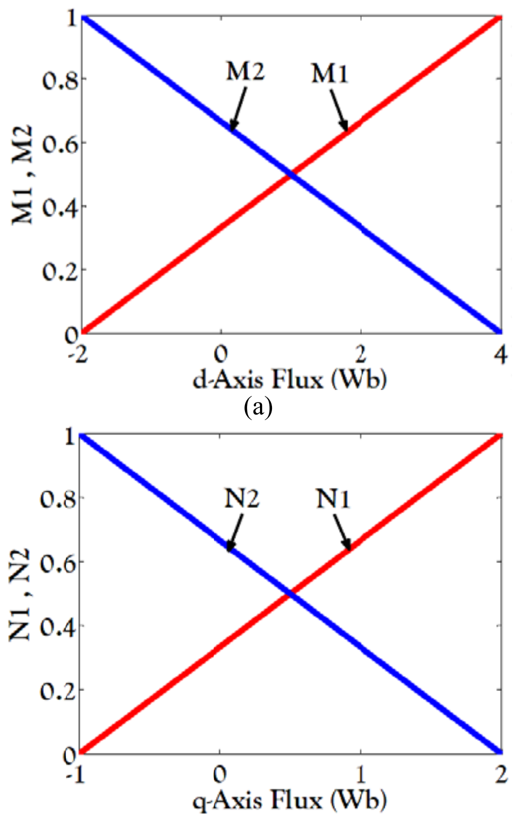


Fig. 3. (a) The membership functions for $M_1(x_2)$ and $M_2(x_2)$, (b) the membership functions for $N_1(x_3)$ and $N_2(x_3)$

In the first step, the system (4) is represented by T-S fuzzy model within the using of the fuzzy rules. For this design problem, the rules r_1- r_4 are constructed for T-S fuzzy dynamic model. Referring to (3)-(4) the four linear sub system model is given by:

$$A_{0_1}=\begin{bmatrix} -2 & 0 & 0 & -200 & 400 \\ -4 & -11.6 & 0 & 3.5 & 0 \\ 8 & 0 & -11.6 & 0 & 3.5 \\ 50 & 145.8 & 0 & -168.7 & 0 \\ 0 & -50 & 145.8 & 0 & -168.7 \end{bmatrix}$$

$$A_{0_2}=\begin{bmatrix} -2 & 0 & 0 & 100 & 400 \\ 2 & -11.6 & 0 & 3.5 & 0 \\ 8 & 0 & -11.6 & 0 & 3.5 \\ -25 & 145.8 & 0 & -168.7 & 0 \\ 0 & 25 & 145.8 & 0 & -168.7 \end{bmatrix}$$

$$A_{0_3}=\begin{bmatrix} -2 & 0 & 0 & -200 & -200 \\ -4 & -11.6 & 0 & 3.5 & 0 \\ -4 & 0 & -11.6 & 0 & 3.5 \\ 50 & 145.8 & 0 & -168.7 & 0 \\ 0 & -50 & 145.8 & 0 & -168.7 \end{bmatrix}$$

$$A_{0_4}=\begin{bmatrix} -2 & 0 & 0 & 100 & -200 \\ 2 & -11.6 & 0 & 3.5 & 0 \\ -4 & 0 & -11.6 & 0 & 3.5 \\ -25 & 145.8 & 0 & -168.7 & 0 \\ 0 & 25 & 145.8 & 0 & -168.7 \end{bmatrix}$$

And for $i=1,\dots,4$

$$B_{wi}=\begin{bmatrix} -50 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$B_{ui}=\begin{bmatrix} 0 & 0 & 0 & 12.5 & 0 \\ 0 & 0 & 0 & 0 & 12.5 \end{bmatrix}^T$$

Referring to the section IV, the weighting matrices $W_T(s)$ and $W_S(s)$ have been designed as follows:

$$W_T(s)=W_{T_{ii}}(s)\,I_{2\times2}=\frac{0.001s+1}{0.001s+2}I_{2\times2}$$

$$W_s(s)=W_{S_{ii}}(s)\,I_{2\times2}=\frac{0.5s+50}{s+0.05}I_{2\times2}$$

Then by using purposed control loop (Fig. 2) and mentioned weighting matrices and the theorem 1 we can calculate the local controller for each linear subsystem.

In order to design output feedback gains (K_i) for each subsystem, below steps are done:

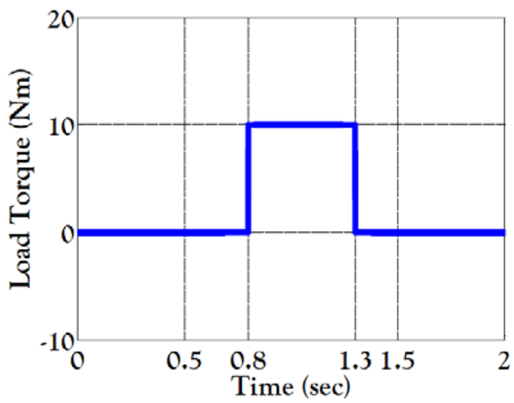
• Specify the LMI region (20), in order to place the closed-loop poles in this region (pole placement) and also to guarantee some minimum decay rate and closed-loop damping. The characteristic of appointed region is: the intersection of the half-plane is x<-2 and it's of the sector centered at the origin and with inner angle 5π/6.

• Choose a four-entry vector specifying the H_2/H_∞ trade-off criterion in theorem1: [γ0 ν0 α β]=[0 0 1 0]. As a matter of fact, in this case, constraint and criterion of H2 are not used(β=0) and just two design objectives, H∞ performance and pole placement are employed.

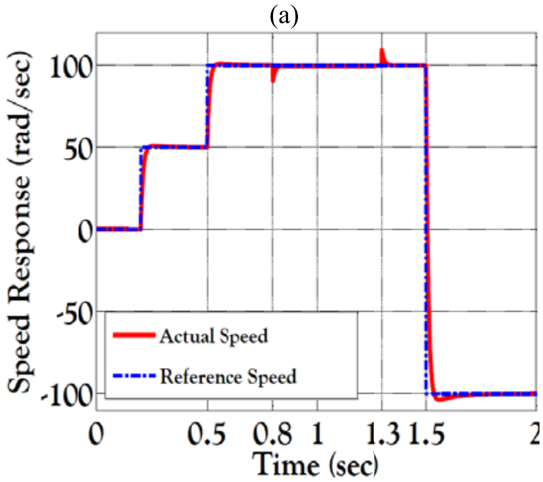
• Minimize H_2/H_∞ cost function based on theorem 1 subject to the mentioned pole placement constraint by using (18)-(19)-(22)-(23)-(24).

Finally, by using a weighted average defuzzifer, the overall fuzzy system and the control law of the whole system are obtained. Global proposed T-S fuzzy model can exactly represents the nonlinear system in the region [-2 , 4] Wb× [-1 , 2] Wb on the x_2-x_3 space for various operating point.

In actuality, the motor is used to convert the electrical energy into mechanical energy. Accordingly, an external load is added to the drive system. The first test concerns a no-load starting of the motor with a reference speed. A load torque (T_L = 10 Nm) is then applied between t = 0.8 sec and t = 1.3 sec, which is followed at t = 1.5 sec by a reverse of a speed from 100 rad/sec to –100 rad/sec. Fig. 4(a) and 4(b) demonstrate load torque and angular speed tracking against this disturbance respectively.



(a)



(b)

Fig. 4. (a) load torque (b) motor speed

Fig. 5 illustrates the motor torque tracking responses at different torque commands by using the proposed controller. According to this figure, the proposed system has satisfactory performance for various torque commands in order to conquest over various load torques between t = 0.8 sec and t = 1.3 sec.

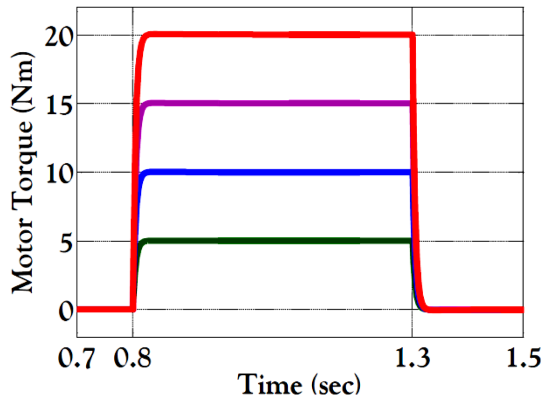
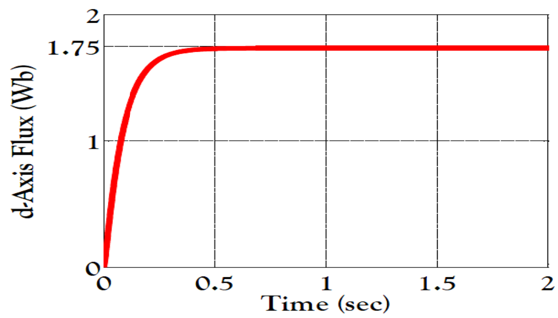
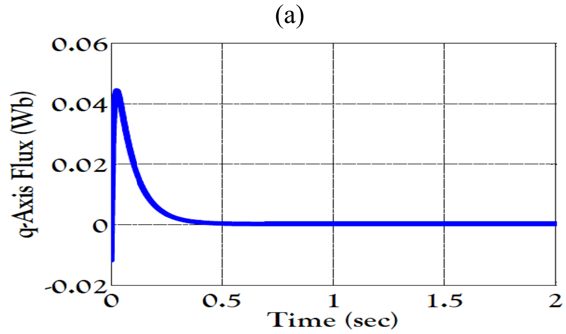


Fig. 5. Motor torque tracking responses at different torque references

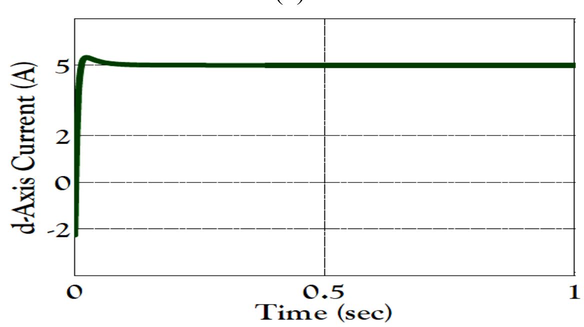
Fig. 6 shows the d-q components of stator current and rotor flux.



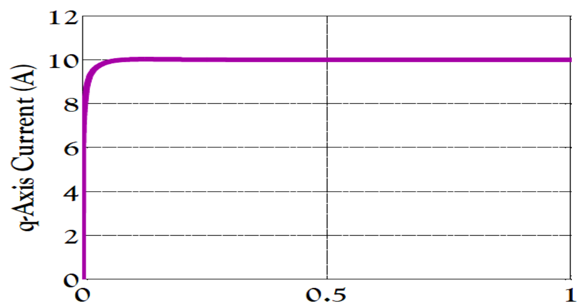
(a)



(b)



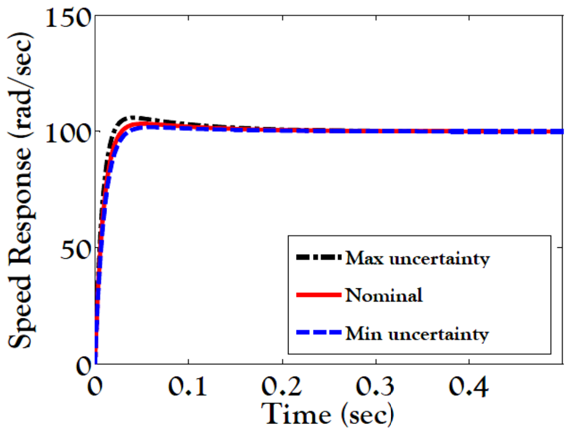
(c)



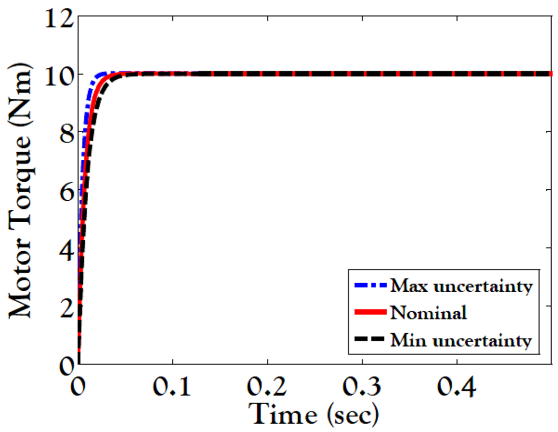
(d)

Fig. 6. (a), (b) d-q projections of the rotor flux (c), (d) d-q projections of the stator current

Fig. 7(a) and 7(b) illustrate the rotor speed and torque responses respectively, when the parameters of the stator and rotor resistances R_s, R_r and the damping coefficient K are varied between ±50%. As you can see, the system has good robustness when the parameters in the systems dynamic are varied in a wide range.



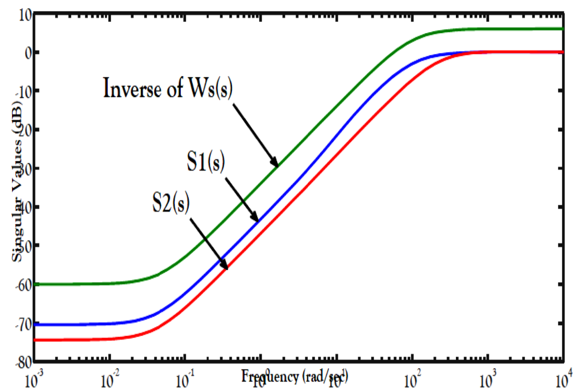
(a)



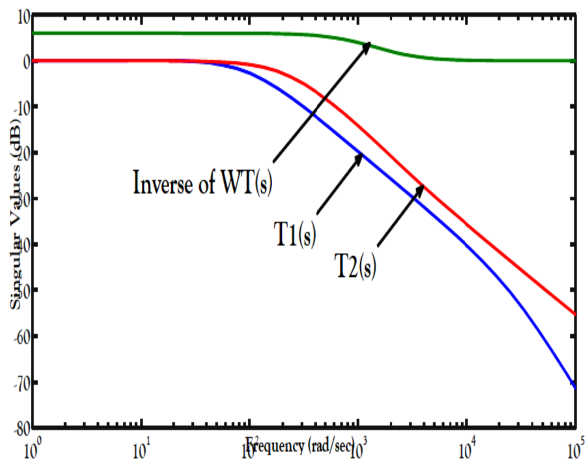
(b)

Fig. 7. (a) Angular speed responses with varying Rs, Rr, K (b) Motor torque responses within varying Rs, Rr, K

Fig. 8(a) and 8(b) demonstrate W_s^{-1} and W_T^{-1}, that they are greater than S_1, S_2 and T_1, T_2 on frequency domain respectively .



(a)



(b)

Fig. 8. (a) W_s^{-1} matrix as an upper bound of the $S_1$and S_2 (b) W_T^{-1} matrix as an upper bound of the $T_1$and T_2

6. CONCLUSION

In this paper, a robust mixed-sensitivity H_∞ controller has been designed in terms of tracking and disturbance attenuation of speed and torque, for a MIMO nonlinear uncertain IM system. First to approximate uncertain nonlinear system, the T-S fuzzy technique is employed. Next, Both loop-shaping methodology and Mixed-sensitivity problem are presented to improve frequency-domain specifications. After that, based on each linear model, a robust pole-placement output feedback H∞ controller is determined by LMI-based design techniques in order to achieve the robustness design of nonlinear uncertain systems. Final PDC is used to design the controller for the overall system and the total linear system is obtained by using of the weighted sum of the local linear system. The simulation results on IM show that the robust control system has suitable speed and torque tracking error and it has also desired robustness against load torque disturbance and parameter variations. Proposed speed and torque control-system have good transient responses and load disturbance rejection and tracking responses.

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