Attitude Control of Satellite With Pulse-Width Pulse-Frequency (PWPF) Modulator Using Generalized Incremental Predictive Control

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ABSTRACT:
In this paper, we use generalized incremental predictive control (GIPC) to stabilize attitude of satellite. We compare Generalized Predictive Control (GPC) with GIPC algorithm and present that GIPC has better performance. The three-axis attitude control systems are activated in pulse mode. Consequently, a modulation of the torque command is compelling in order to avoid high non-linear control action. This work considers the Pulse-Width Pulse-Frequency modulator (PWPF) is composed of a Schmitt trigger, a first order filter, and a feedback loop. PWPF modulator has several advantages over classical bang-bang controllers such as close to linear operation, high accuracy, and reduced propellant consumption.

KEYWORDS: Predictive Control; Satellite; GIPC; PWPF.

1. INTRODUCTION

The GPC method was proposed by Clarke et al. [1] and has become one of the most popular predictive control methods both in academia and industry. It can handle many different control problems for a wide range of plants with a reasonable number of design variables and constraints.

For improving performance of generalized predictive control, Generalized Incremental Predictive Control (GIPC) method is proposed by Ghahramani et al. [2], whose remove instabilities caused by actuator saturation. Further, it has a higher robustness with respect to the modeling uncertainties in comparison with GPC method.

The primary task of the control system is to stabilize the attitude of the satellite against external torque disturbances. Such disturbances are produced by aerodynamic drag effects, solar radiation and solar wind torques, parasitic torques created by the propulsion thrusters, and so on. Unfortunately, reaction controllers do not possess the same linear relationship between the input to the controller and its output torque. In fact, they are activated in an on-off mode. Nonetheless, they can be used in a quasi-linear mode by modulating the width of the activated reaction pulse proportionally to the level of the torque command input to the controller. This is the often used Pulse Width modulation (PW) principle. A related design technique is based on the well-known Schmidt trigger, which implements a Pulse-Width Pulse-Frequency modulation (PWPF) in which the distance between the pulses is also modulated [4]. The basic goal is minimum time-fuel attitude control system to extend the life of a satellite.

This paper is organized as follows: In Section 2, Three degree of freedom satellite dynamics is modeled. In Section 3, MPC strategy and GIPC algorithm is presented. In Section 4,

PWPF modulator is explained. In Section 5, the results obtained from the simulation of GPC and GIPC methods for the satellite are given. Finally, Section 6 is devoted to conclusions.

2. THREE DEGREE OF FREEDOM SATELLITE DYNAMIC MODEL

Three degree of freedom (3-dof) rigid satellite model is presented in this section. Axes X_B,Y_B and Z_B define the satellite's body axis frame, and the axis system is considered centered at the center of gravity (Fig. 1). Thrusters are available to produce torques about each of the three principal axes. The physical interpretation

of the Euler angles for a satellite platform is illustrated in Fig. 1. The roll (ϕ),pitch (θ) and yaw (ψ) angles are defined by successive rotations around the coordinate axes X_B, Y_B and Z_B in the body fixed frame. For large angles and position control of spacecraft, quaternion rotation is used. This is more suitable as it avoids singularity functions and gimbal lock [8].

Euler angular moment ($\dot{H}$) which performs satellite attitude-rotational motion in space is expressed in the form of a matrix as follows:

$$M=\frac{dH}{dt}+[\omega\times H]$$

$$\frac{dH}{dt}=M-[\omega\times H]\ ,\ H=I.\omega$$

$$I.\frac{d\omega}{dt}=M-[\omega\times I.\omega]$$

$$\frac{d\omega}{dt}=\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix}=I^{-1}.M-\left[I^{-1}.\omega\right]\times[I.\omega] \quad (1)$$

Where vector M, the applied moment (thrusters), is the input u, vector ω is the angular rate and matrix I is the inertia around the body principal axes X_B,Y_B and Z_B. These are given as follow:

$$u=[M_x \quad M_y \quad M_z]^T$$

$$\omega=[p \quad q \quad r]^T$$

$$I=\begin{bmatrix}I_{xx} & 0 & 0\\ 0 & I_{yy} & 0\\ 0 & 0 & I_{zz}\end{bmatrix} \quad (2)$$

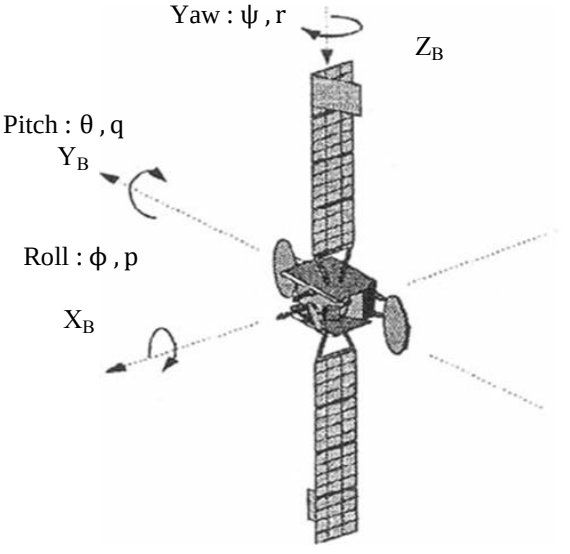


Fig. 1. Satellite reference and body coordinates.

Solving (1) and (2)

$$\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix}=\begin{bmatrix}\frac{M_x-q.I_{zz}.r+r.I_{yy}.q}{I_{xx}}\\ \frac{M_y-r.I_{xx}.p+p.I_{zz}.r}{I_{yy}}\\ \frac{M_z-p.I_{yy}q+q.I_{xx}.p}{I_{zz}}\end{bmatrix} \quad (3)$$

The relationship between body rate $\omega_b=[p_b \quad q_b \quad r_b]^T$ of the satellite and angular rates $\omega=[\dot{\phi} \quad \dot{\theta} \quad \dot{\psi}]^T$ is defined by the following transformation:

$$\begin{bmatrix}\dot{\varphi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}1 & \frac{\sin(\varphi)\sin(\theta)}{\cos(\theta)} & \frac{\sin(\varphi)\sin(\theta)}{\cos(\theta)}\\ 0 & \cos(\varphi) & -\sin(\varphi)\\ 0 & \frac{\sin(\varphi)}{\cos(\theta)} & \frac{\cos(\varphi)}{\cos(\theta)}\end{bmatrix}.\begin{bmatrix}p\\ q_b\\ r_b\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}\dot{\varphi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}p+\frac{(q\sin(\varphi)+r\cos(\varphi))\sin(\theta)}{\cos(\theta)}\\ q\cos(\varphi)-r\sin(\varphi)\\ \frac{q\sin(\varphi)+r\cos(\varphi)}{\cos(\theta)}\end{bmatrix} \quad (4)$$

The non-linear state model of the satellite can be derived by partial derivatives of the model states $x=[p_b, q_b, r_b, \phi_I, \theta_I, \psi_I]^T$

$$\begin{bmatrix}\dot{p}_b\\ \dot{q}_b\\ \dot{r}_b\\ \dot{\varphi}_I\\ \dot{\theta}_I\\ \dot{\psi}_I\end{bmatrix}==\begin{bmatrix}\frac{M_x-q.I_{zz}.r+r.I_{yy}.q}{I_{xx}}\\ \frac{M_y-r.I_{xx}.p+p.I_{zz}.r}{I_{yy}}\\ \frac{M_z-p.I_{yy}q+q.I_{xx}.p}{I_{zz}}\\ p+\frac{(q\sin(\varphi)+r\cos(\varphi))\sin(\theta)}{\cos(\theta)}\\ q\cos(\varphi)-r\sin(\varphi)\\ \frac{q\sin(\varphi)+r\cos(\varphi)}{\cos(\theta)}\end{bmatrix} \quad (5)$$

Table 1. Satellite Parameters

Parameters	Description	Value
I_{xx}	Moment of inertia (x-axis)	1.928 kgm^2
I_{yy}	Moment of inertia (y-axis)	1.928 kgm^2
I_{zz}	Moment of inertia (z-axis)	4.953 kgm^2
Thruster	Input to satellite (Mx , My , Mz)	1 kgm^2
ϕ_0	Initial roll Euler angle	0.362 rad
θ_0	Initial pitch Euler angle	0.524 rad
ψ_0	Initial yaw Euler angle	-0.262 rad
p	Body pitch roll rate	0 rad/s
q	Body yaw rate	0 rad/s
r	Body roll rate	0 rad/s

A=Jacobian(f(x),x), B=Jacobian(f(x),u)

A =

$$\begin{bmatrix} 0 & \frac{-rI_{zz}+rI_{yy}}{I_{xx}} & \frac{-qI_{zz}+qI_{yy}}{I_{xx}} & 0 \\ \frac{-rI_{xx}+rI_{zz}}{I_{yy}} & 0 & \frac{-pI_{xx}+pI_{zz}}{I_{yy}} & 0 \\ \frac{-qI_{yy}+qI_{xx}}{I_{zz}} & \frac{-pI_{yy}+pI_{xx}}{I_{zz}} & 0 & 0 \\ 1 & \frac{\sin\varphi(\sin\theta)}{\cos\theta} & \frac{\cos\varphi(\sin\theta)}{\cos\theta} & \frac{q\cos\varphi-r\sin\varphi(\sin\theta)}{\cos^2\theta} \\ 0 & \cos\varphi & -\sin\varphi & -q\sin\varphi-r\cos\varphi \\ 0 & \frac{\sin\varphi}{\cos\theta} & \frac{\cos\varphi}{\cos\theta} & \frac{-q\cos\varphi-r\sin\varphi}{\cos\theta} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ q\sin\varphi + r\cos\varphi + \frac{(q\sin\varphi + r\cos\varphi)(\sin^2\theta)}{\cos^2\theta} & 0 \\ 0 & 0 \\ q\sin\varphi + \frac{r\cos\varphi(\sin\theta)}{\cos^2\theta} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{Ixx} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{Iyy} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{Izz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

Where matrix A(x) and B(u) are Jacobian matrix of non-linear function f(x) with respect to state vector $x = [p, q, r, \phi, \theta, \psi]^T$and input vector $u = [M_x, M_y, M_z, 0,0,0]^T$, respectively. The satellite parameters in Table 1 are given in [3].

3. MODEL PREDICTIVE CONTROL AND GIPC ALGORITHM

The success of the methods based on model based predictive control is their ability to explicitly handle constraints, processes with delays, and nonlinear systems [5]. As shown in Fig. 2, model uses past inputs and outputs of system to predict N_y step future outputs. N_y is called prediction horizon. Vector of future optimal control signals obtain by measuring prediction error and minimizing cost function.

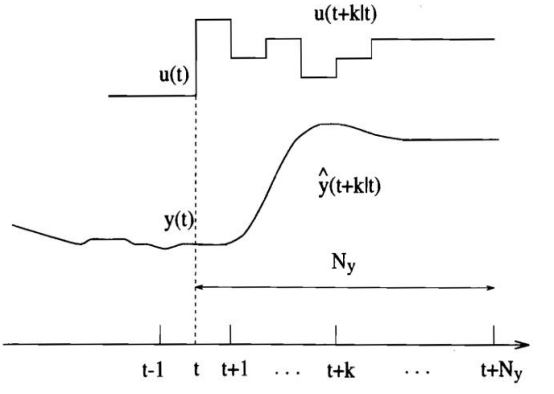


Fig. 2. MPC strategy

$$\Delta U = \left[\Delta u(t|t) \quad \Delta u(t+1|t) \quad \cdots \quad \Delta u(t+N_u-1|t)\right]^T \quad (7)$$

Where Nu is called control horizon and equals to length of obtained vector from optimization problem. Therefore, first control signal $\Delta u(t|t)$ is applied to system and next signals are rejected. At the next sampling instant, prior step is repeated with this new value and all the sequences are brought up to date. Thus the $\Delta u(t+1|t+1)$ is calculated (which in principle will be different to the $\Delta u(t+1|t)$ because of the new information available) using the receding horizon concept [1].

3.1. GIPC algorithm

In state-space modeling of the standard MPC, the present states are used to predict the states. But, in the new receding-horizon algorithm named Generalized Incremental Predictive Control, both present and previous states rather than present states are considered in the j step ahead prediction of the states and outputs.

Therefore, the optimal solution includes the weighted difference of the current and the previous states and the summation of the control action increments. Using the weighted difference of the states in control action provides a faster response and improves the robustness of the system [2]. Consider the following state-space model:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k+1) = Cx(k+1) \end{cases} \quad (8)$$

$x \in R^n$, $u \in R$ and $y \in R$ denote the state vector, system input, and system output, respectively. Also matrices $A \in R^{n\times n}$, $B \in R^n$, and $C \in R^n$ are system matrices. The incremental form of the state predictions is given by:

$$\Delta x(k+1) = A\Delta x(k) + B\Delta u(k) \quad (9)$$

Where

$$\Delta u(k) = u(k) - u(k-1), \Delta x(k) = x(k) - x(k-1)$$
$$\Delta x(k+1) = x(k+1) - x(k) \quad (10)$$

By combining (9), (10) gives:

$$x(k+1) = (A+I)x(k) - Ax(k-1) + B\Delta u(k) \quad (11)$$

Where **I** is the identity matrix. This equation describes the one-ahead prediction of states in terms of the current states $x(k)$ and the previous states $x(k-1)$. Finally, the general form of j step-ahead predictions of the states is given by:

$$x(k+j) = \left(\sum_{m=0}^{j} A^m\right)x(k) - \left(\sum_{m=1}^{j} A^m\right)x(k-1) + \sum_{n=1}^{j}\left(\sum_{m=0}^{n-1} A^m\right)B\Delta u(k+j-n) \quad (12)$$

From Eq. (12), outputs prediction over the prediction horizon N_y and the control horizon N_u can be written as the following compact matrix/vector form:

$$Y = P_{y0}x(k) - P_{y1}x(k-1) + H_y\Delta U \quad (13)$$

Where

$$Y = \begin{bmatrix} y(k+1) \\ y(k+2) \\ \vdots \\ y(k+N_y) \end{bmatrix}, \Delta U = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N_u-1) \end{bmatrix},$$

$$P_{y0} = \begin{bmatrix} C(A+I) \\ C(A^2+A+I) \\ \vdots \\ C\left(\sum_{i=0}^{N_y} A^i\right) \end{bmatrix}$$

$$P_{y1} = \begin{bmatrix} CA \\ C(A2+A) \\ \vdots \\ C\left(\sum_{i=1}^{N_y} A^i\right) \end{bmatrix},$$

$$H_y = \begin{bmatrix} CB & 0 & \cdots & 0 \\ C(A+I)B & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C\left(\sum_{i=0}^{N_y-1} A^i\right)B & C\left(\sum_{i=0}^{N_y-2} A^i\right)B & \cdots & CB \end{bmatrix} \quad (14)$$

Now, consider following cost function

$$J = (W-Y)^T Q(W-Y) + \Delta U^T \Delta U \quad (15)$$

Where

$$W = \begin{bmatrix} w(k+1) & w(k+2) & \cdots & w(k+N_y) \end{bmatrix}^T$$

is the reference trajectory. **Q** and **R** are positive definite and the weight of error and control signals respectively.

By posing eq. (13) in cost function eq. (15) and derivative toward ΔU and equals it to zero, the optimal control vector is obtained as follows: [2]

$$\Delta U = \left(H_y^T Q H_y + R\right)^{-1} H_y^T Q \left(W - P_{y0}x(k) + p_{y1}x(k-1)\right) \quad (16)$$

4. PWPF MODULATOR

Pulse thruster devices can provide only on-off signals generating nonlinear control action. Nonetheless, those can be used in a quasi-linear mode by modulating the width of the activate reaction pulse proportionally to the level of the torque command input. This is known as pulse-width modulation (PW). In the pulse-width pulse-frequency (PWPF) modulation the distance between the pulses is also modulated. Its basic structure is shown in Fig. 3.

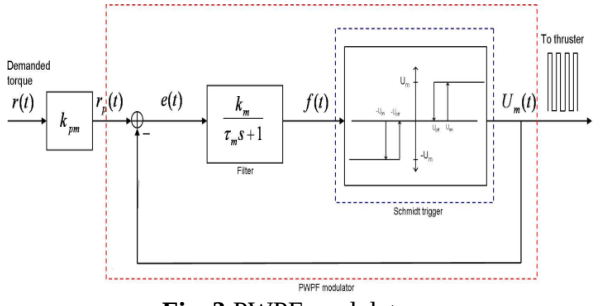


Fig. 3.PWPF modulator

The modulator includes a Schmitt trigger which is a relay with dead zone and hysteresis, it includes also a first-order-filter, lag network type, and a negative feedback loop. When a positive input to the Schmitt trigger is greater than U_{on} , the trigger input is U_m. If the input falls below U_{off} the trigger output is 0. This response is also reflected for negative inputs in case of two side-thrusters or those thruster that produce negative torques (clockwise direction). The error signal e(t) is the difference between the Schmitt trigger output Uon and the system input r(t). The error is fed into the filter which output signal f(t) and it feeds the Schmitt trigger. For designing PWPF, consider these parameters:

- The filter coefficients K_m and τ_m .
- The Schmitt trigger parameters U_{on}, U_{off}, it defines the hysteresis as $h=U_{on}-U_{off}$.
- The maximal/minimal $\pm U_m$.

The PWPF modulator can incorporate an additional gain K_{pm} which will be considered separately from the control gain. In the case of a constant input, the PWPF modulator drives the thruster valve with on-off pulse sequence having a nearly linear duty cycle with input amplitude [6, 7].

Table 2. Recommended value for the PWPF parameters

K_m	τ_m	U_{on}	H	K_{pm}
1	0.1	0.45	0.3	50

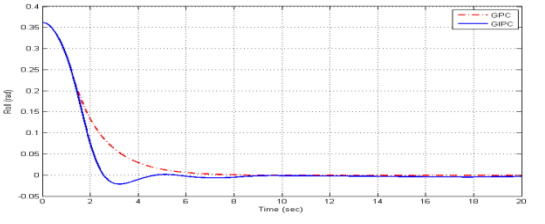
5. NUMERICAL SIMULATION

In this section, GPC and GIPC algorithm are applied to three degree of freedom satellite and are being compared both algorithms. These simulations are implemented with table 3 values.

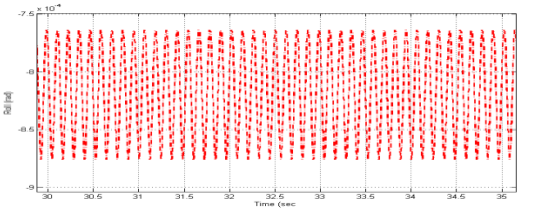
Table 3. Recommended value for the PWPF parameters

Q	R	N_u	N_y
7.I	1.I	3	200

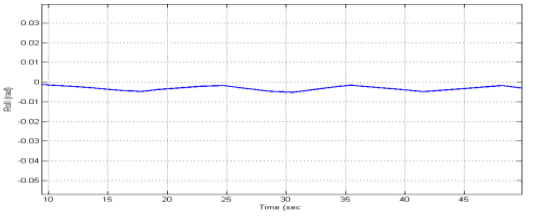
Fig. 4, Fig. 5 and Fig. 6 are presented Euler angles ϕ , ψ , θ under GPC and GIPC control. As seen in Fig. 4-a, Fig. 5-a and Fig. 6-a, it seems that GPC has faster response than GIPC, but according to more accurate Fig. 4-b,c , Fig. 5-b,c , Fig. 6-b,c, we find that the angles under GPC control with PWPF modulator have higher frequency fluctuations and more severe nonlinear operation toward GIPC. In fact, GIPC has pseudo-linear (nearly linear) operation.



a. Roll angle

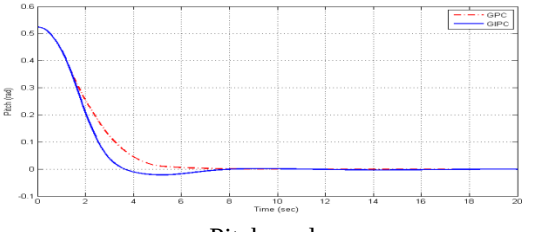


b. More accurate under GPC method

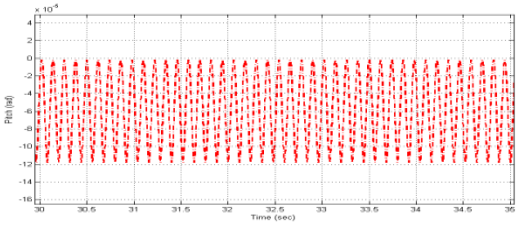


c. More accurate under GIPC method

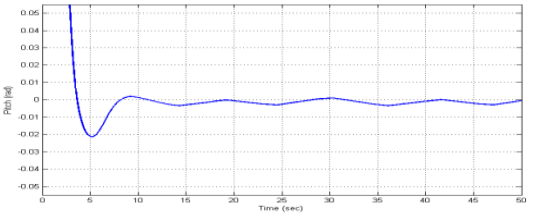
Fig .4.



a. Pitch angle

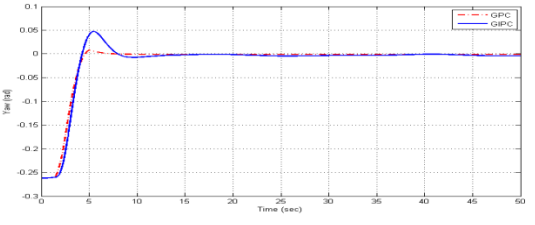


b. More accurate under GPC method

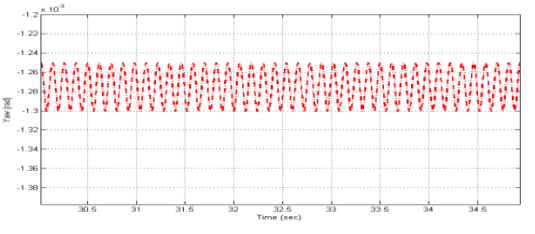


c. More accurate under GIPC method

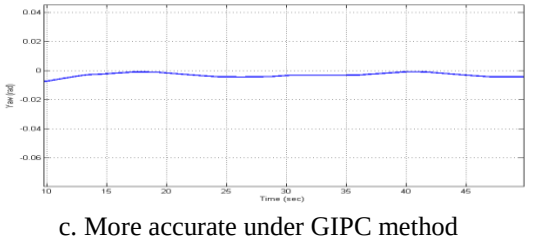
Fig. 5.



a. Yaw angle



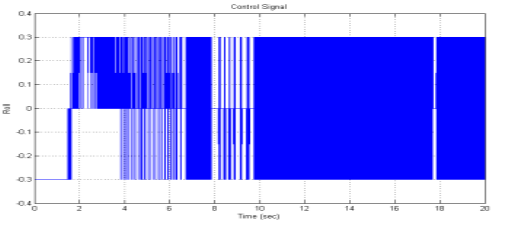
b. More accurate under GPC method



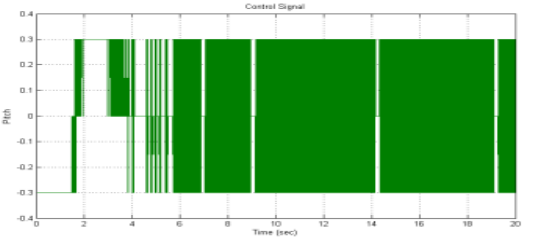
c. More accurate under GIPC method

Fig.6.

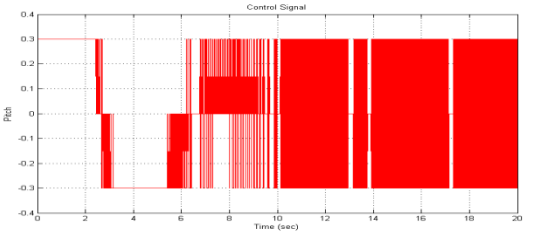
Modulated control signals are shown in Fig 7. Modulator present small pulse-width modulation which leads small impulses and hence less fuel consumption.



a. Roll control signal and PWPF modulation



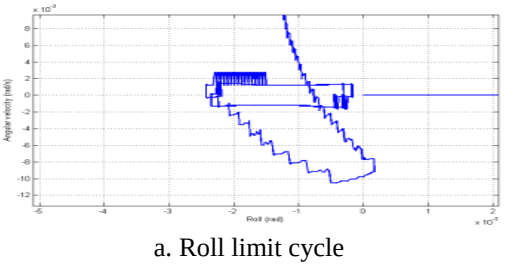
b. Pitch control signal and PWPF modulation



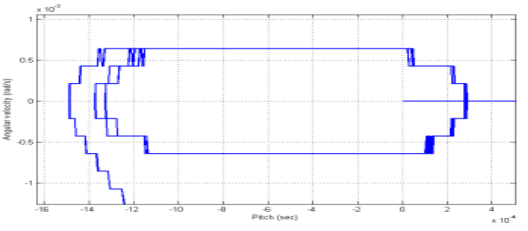
c. Yaw control signal and PWPF modulation

Fig .7. Control signals and PWPF modulation

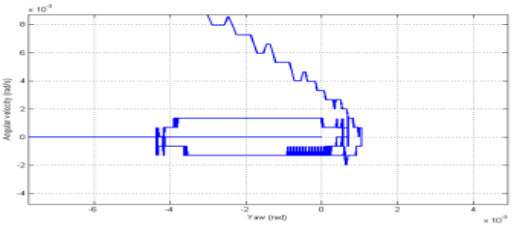
Another aspect of PWPF modulated systems is that they will be subject to self sustained oscillations because of the nonlinearities in the system. These oscillations are called limit cycles. In this satellite system, the limit cycles occur because of the minimum thruster on-time. In the simulations, the limit cycling will appear as a oscillation around the steady state. The duty cycle generated by the PWPF modulation is shown in Fig. 8. The specification of pointing accuracy is achieved, less than 0.01 degrees. It shows a high accurate performance of the reaction thruster which is possible by modulating the control signal using the PWPF modulator.



a. Roll limit cycle



b.Pitch limit cycle behavior about the steady state point in phase plane



c. Yaw limit cycle behavior about the steady state point in phase plane

Fig .8. Limit cycles behavior about the steady state point in phase plane

6. CONCLUSIONS

As it was observed, satellite system under GIPC method has better performance and more stability operation than GPC. The obtained results demonstrate the feasibility of combining GIPC/PWPF modulator in a unique controller for on-off thruster reaction attitude control system. Stability remains by adding the PWPF modulator and reasonable accuracy in attitude is achieved. GIPC method considers the present states as well as the previous states. Consequently, the resulting controller GIPC includes the weighted difference of the current and the previous states and the summation of the control action increments. Meanwhile, magnitude of the limit cycle has reduced i.e. response of system under GIPC/PWPF has extremely Low steady-state error.

It should be noted that GIPC controller can be made to eliminate disturbance because of the integrator and that is why the limit cycle oscillation frequency is less.

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