Static Voltage Stability Assessment Using Probabilistic Power Flow to Determine the Critical PQ Buses

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ABSTRACT
Nowadays, due to increased consumption and operation of electric power systems close to their stability boundaries, power systems may become unstable during severe disturbances. So, it is very important to determine the stability margin under different conditions. In this paper, the static voltage stability of an interconnected power system considering load and generation uncertainties is evaluated using probabilistic power flow. The Monte Carlo Simulation method is used to generate the probabilistic power flow scenarios. Then, the expected static voltage stability index and probability of stability for all of the PQ buses are obtained. Also, the standard deviations of the stability indexes are calculated. Finally, the critical PQ nodes are determined under given disturbance. The study has been carried out on 39-Bus New England and 118-bus IEEE test systems and results are presented.

KEYWORDS: Load Generation Uncertainty, Monte Carlo Simulation (MCS), Probabilistic Power Flow (PPF), Static Voltage Stability (SVS).

1. INTRODUCTION
According to the definition, power system stability is the ability of system to maintain an equilibrium state in the system under normal operating condition and after a disturbance. An interconnected power system may become unstable due to several reasons such as earthquakes, human operation errors, control system failures, protection system failures, etc. Nowadays, due to increased consumption and operation of electric power systems close to their stability boundaries, power systems may become unstable during severe disturbances. So, it is very important to determine the stability margin under different conditions.
In recent years, many methods have been proposed to assess the stability of a large-scale power system. The stability analysis in a power system is a complex issue and there are many approaches to solve this problem. These procedures can be divided into three general categories [1]:
a. Static voltage stability analysis
b. Small disturbance analysis
c. Dynamic voltage stability analysis
Static voltage stability analysis is based on the solution of conventional power flow equations and small disturbance methods are based on linearized system differential equations and dynamic voltage stability

approaches try to find why and how the voltage collapse has occurred. Because of nonlinear nature of differential equations, it is not possible to determine the exact distance to voltage collapse in dynamic methods. So, static methods are used to evaluate the voltage security margin.
Static voltage stability of a power system has been studied in [2] using continuation power flow considering the effect of contingencies on Mega Watt Margin (MWM) and loading point. Modal based analysis and its application in the evaluation of voltage stability of bulk power system is reported in [3]. This method makes use of the power system Jacobian matrix to determine the eigenvalues necessary for the evaluation of the voltage stability of the power system. This method was used to determine the components of the system that contribute to instability through the use of the participating factors. A criterion for finding the weakest bus in the system by using Artificial Immune System (AIS) clonal selection algorithm which is supported by evaluating eigenvalues and their participation factors is introduced in [4]. In [5], an Enhanced Radial Basis Function Neural Network (ERBFNN) and Winner-Take-All Neural Network (WTANN) have been proposed to examine whether the power system is secure under steady-state operating

conditions. The voltage stability assessment using mixed static and dynamic techniques is discussed in [6]. Also, several other methods are provided to assess the dynamic voltage stability and small disturbance stability. In recent years, many methods have been proposed to implement the probabilistic power flow. In [7], the use of Gaussian mixture models to represent non-Gaussian correlated input variables, such as wind power output or aggregated load demands in the probabilistic load flow is proposed. The main advantage of the Gaussian components method is that the probability density functions of any variable is directly obtained. In [8], a probabilistic load flow method is discussed that is based on the Nataf transformation and the Latin Hypercube Sampling. The main advantage of the proposed method is that high accurate solution can be obtained with less computation. A probabilistic load flow method based on polynomial normal transformation (PNT) and Latin hypercube sampling (LHS) is proposed in [9]. The correlation between input random variables has been taken into consideration. The proposed method uses the statistical moments and correlation matrix of input random variables instead of their marginal distribution functions and joint distribution functions, which are very difficult to be obtained, to establish their probability distribution models by PNT and LHS. The statistical moments and probability distribution functions of node voltage and line flow are calculated by Monte Carlo simulation method.

As mentioned, it is very important to evaluate the static voltage stability of an interconnected power system under different conditions. So, this paper introduces a new method based on modal analysis to determine the stability index and probability of static voltage stability for all of PQ buses in a large-scale power system considering load and generation uncertainties and probability of power system condition.

2. VOLTAGE STABILITY EVALUATION
2.1. Deterministic Newton-Raphson Power Flow
In Newton-Raphson approach, the linearized steady state system power flow equations can be written as follows [10]:

$$\begin{bmatrix} J_{P\delta} & J_{PV} \\ J_{Q\delta} & J_{QV} \end{bmatrix}\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix}=\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{1}$$

In this equation, ΔP is incremental change in bus real power injection and ΔQ is incremental change in bus reactive power injection, $\Delta\delta$ is incremental change in bus voltage angle, and ΔV is incremental change in bus voltage magnitude, $J_{P\delta}$ is sensitivity of incremental change in bus real power injection and incremental change in bus voltage angle, J_{PV} is

sensitivity of incremental change in bus real power injection and incremental change in bus voltage magnitude, $J_{Q\delta}$ is sensitivity of incremental change in bus reactive power injection and incremental change in bus voltage angle, J_{QV} is sensitivity of incremental change in bus reactive power injection and incremental change in bus voltage magnitude. Also, Jacobian matrix is defined as follows:

$$J =\begin{bmatrix} J_{P\delta} & J_{PV} \\ J_{Q\delta} & J_{QV} \end{bmatrix} \tag{2}$$

Jacobian matrix elements are obtained from the following relations. The diagonal elements of this matrix can be obtained as follows:

$$J_{P_i\delta_i} = \frac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}V_i^2 \tag{3}$$

$$J_{P_iV_i} = \frac{\partial P_i}{\partial V_i} = P_i + G_{ii}V_i^2 \tag{4}$$

$$J_{Q_i\delta_i} = \frac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii}V_i^2 \tag{5}$$

$$J_{Q_iV_i} = \frac{\partial Q_i}{\partial V_i} = Q_i - B_{ii}V_i^2 \tag{6}$$

The non-diagonal elements of Jacobian matrix can be calculated as follows:

$$J_{P_i\delta_k} = \frac{\partial P_i}{\partial \delta_k} = V_iV_k\left(G_{ik}\sin(\delta_i-\delta_k)-B_{ik}\cos(\delta_i-\delta_k)\right) \tag{7}$$

$$J_{P_iV_k} = \frac{\partial P_i}{\partial V_k} = V_iV_k\left(G_{ik}\cos(\delta_i-\delta_k)+B_{ik}\sin(\delta_i-\delta_k)\right) \tag{8}$$

$$J_{Q_i\delta_k} = \frac{\partial Q_i}{\partial \delta_k} = V_iV_k\left(-G_{ik}\cos(\delta_i-\delta_k)-B_{ik}\sin(\delta_i-\delta_k)\right) \tag{9}$$

$$J_{Q_iV_k} = \frac{\partial Q_i}{\partial V_k} = V_iV_k\left(G_{ik}\sin(\delta_i-\delta_k)-B_{ik}\cos(\delta_i-\delta_k)\right) \tag{10}$$

2.2. Load Generation Uncertainties
Power flow analysis is used on important problems in power system operation and planning, such as evaluation of system reliability and risk management in electricity market. So, it is very important to consider the power system uncertainties under different operating conditions. In a real power system, loads and generations are uncertain. So in this paper, normal probability distribution functions are used to model the load and generation uncertainties.

2.3. Static Voltage Stability Based on Modal Analysis

Static voltage stability is affected by both active and reactive power in an interconnected power system. In order to reduce the computational burden, at each operating point active power can be assumed constant. So, voltage stability will be evaluated by considering incremental relationship between Q and V [10]. Based on the above considerations, the relation between incremental change in bus reactive power injection and incremental change in bus voltage magnitude can be rewritten as follows:

$$\Delta Q = J_R \Delta V \tag{11}$$

$$J_R = J_{QV} - J_{Q\delta} J_{P\delta}^{-1} J_{PV} \tag{12}$$

Recent matrix is called the reduced Jacobian matrix which directly relates the bus voltage magnitude and bus reactive power injection as follow:

$$\Delta V = J_R^{-1} \Delta Q \tag{13}$$

In (13), i^{th} diagonal element of reduced Jacobian matrix is Q-V sensitivity at bus i. Because of nonlinear nature of Q-V relationships, the magnitudes of the sensitivities do not provide a direct measure of the relative degree of stability. Therefore, the eigenvalues of the reduced Jacobian matrix are used to determine the voltage stability index. So, we have:

$$\boldsymbol{J_R} = \xi \Lambda \eta \tag{14}$$

In (14), ξ is right eigenvector matrix of matrix J_R, Λ is diagonal eigenvalues of matrix J_R, η is left eigenvector matrix of J_R.

$$J_R^{-1} = \xi \Lambda^{-1} \eta \tag{15}$$

$$\Delta V = \xi \Lambda^{-1} \eta \Delta Q \tag{16}$$

$$\Delta V = \sum_i \frac{\xi \eta}{\lambda_i} \Delta Q \tag{17}$$

$$\eta = \xi^{-1} \tag{18}$$

$$\eta \Delta V = \Lambda^{-1} \eta \Delta Q \tag{19}$$

$$v = \eta \Delta V \tag{20}$$

$$q = \eta \Delta Q \tag{21}$$

$$v_i = \frac{1}{\lambda_i} q_i \tag{22}$$

In (20) and (21), v and q are the vector of modal voltage variation and the vector of modal reactive power variation, respectively. In (22), λ_i is the ith eigenvalue. If $\lambda_i > 0$, the ith modal voltage and the ith modal reactive power variation are along the same direction and the system is stable. If $\lambda_i < 0$, the ith modal voltage and the ith modal reactive power variation are along opposite direction and the system is unstable. If $\lambda_i = 0$, the ith modal voltage collapses.

3. PROPOSED PROBABILISTIC METHOD

3.1. Probability of Power System Condition

In an interconnected power system, failure of equipments such as power transformers, transmission lines and generation units leads to protection system operation and will affect the power system stability. So, it is considered the probability of equipment outage as another factor to determine the probabilistic nature of the static voltage stability index. Therefore, the probability of power system condition is obtained as follows:

$$P_C = p\left(\prod_{i=1}^{s}(outage_i)\right) = \prod_{i=1}^{s} p(outage_i) \tag{23}$$

Where, P_C is the probability of power system condition and $p(outage_i)$ is the probability of ith outage and s shows the number of simultaneous outages. According to the recent relation, the probability of power system equipment simultaneous outages is smaller than the probability of each element outage. In other words, we have:

$$P_C = \prod_{i=1}^{s} p(outage_i) \le p(outage_j) \quad j = 1, 2, ..., s \tag{24}$$

3.2. Static Voltage Stability Assessment

The static voltage stability of an interconnected power system is evaluated considering the load and generation uncertainties and the probability of power system condition according to the following steps:

1. N = Enter the number of Monte Carlo Simulations.
2. Determine power system condition and calculate P_C.
3. k=1
4. Generate $U_{L_i}(k) = \text{rand}(i,k)$ for $i = 1, 2, ..., N_L$, N_L refers to the number of load buses.
5. Generate $U_{G_j}(k) = \text{rand}(j,k)$ for $j = 1, 2, ..., N_G$, N_G refers to the number of generation buses.
6. Calculate:

$$P_{L_i}(k) = \text{InverseCDF}('Normal', U_{L_i}(k), \mu_{P_{L_i}}, \sigma_{P_{L_i}})$$

$$Q_{L_i}(k) = \text{InverseCDF}('Normal', U_{L_i}(k), \mu_{Q_{L_i}}, \sigma_{Q_{L_i}})$$

$$P_{G_j}(k) = \text{InverseCDF}('Normal', U_{G_j}(k), \mu_{P_{G_j}}, \sigma_{P_{G_j}})$$

$$Q_{G_j}(k) = \text{InverseCDF}('Normal', U_{G_j}(k), \mu_{Q_{G_j}}, \sigma_{Q_{G_j}})$$

7. Update system topology and run probabilistic power flow for k^{th} scenario.
8. Modal analysis to obtain $\lambda_n(k)$ $for\ n = 1, 2, ..., m$, m shows the number of the PQ buses.
9. If $real(\lambda_n(k)) > 0$, calculate the Stability Index for n^{th} PQ bus in k^{th} scenario.

$$SI(n,k) = \frac{P_C}{|\lambda_n(k)|} \qquad for\ n = 1, 2, ..., m \qquad (25)$$

10. If $k = N$, go step 11. Else, $k = k + 1$ and go step 4.
11. Calculate the expected values of stability indexes in N scenarios:

$$ESI(n) = \frac{1}{N}\sum_{k=1}^{N} SI(n,k) \qquad for\ n = 1, 2, ..., m \qquad (26)$$

12. Obtain the standard deviations of stability indexes as follows:

$$SD(n) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(SI(n,k) - ESI(n)\right)^2} \qquad for\ n = 1, 2, ..., m \qquad (27)$$

13. Determine Binary Variables for all of the PQ buses in each scenario:

14. $BV(n,k) = \begin{cases} 0 & if\ real(n,k) \le 0 \\ 1 & if\ real(n,k) > 0 \end{cases}$ $for\ n = 1, 2, ..., m$ (28)

15. Calculate probability of stability as follow:

$$SP(n) = \frac{1}{N}\sum_{k=1}^{N} BV(n,k) \qquad for\ n = 1, 2, ..., m \qquad (29)$$

3.3 Determination of Critical PQ Buses

After evaluation of power system stability under normal operating condition and after occurring contingencies, the critical PQ buses can be determined according to the following index:

$$CBI(i) = \frac{ESI(i)|_{Under\ Contingencies}}{ESI(i)|_{Under\ Normal\ Operation}} \qquad for\ i = 1, 2, ..., m \qquad (30)$$

$CBI(i)$ refers to the ith Critical Bus Index. The Critical Bus Index is between 0 and 1. Whatever $CBI(i)$ be larger, i^{th} PQ bus will be non-critical and whatever $CBI(i)$ be close to zero, i^{th} PQ bus will be critical. If $CBI(i)$ is zero, i^{th} PQ bus will collapse.

4. SIMULATION RESULTS

4.1 Simulation on 39-bus New England test system

The study has been carried out on the New England 39-bus and 46- line power system [12]. At first, it is assumed that the power system is under normal operating condition and the number of Monte Carlo Simulations is 1000 and loads and generations are uncertain. The probability of normal condition is considered 0.9. The standard deviations of load and generation under normal operation are assumed 0.99, the amounts of the loads and the generations are considered as the expected values of loads and generations. The test system is shown in Fig. 1.
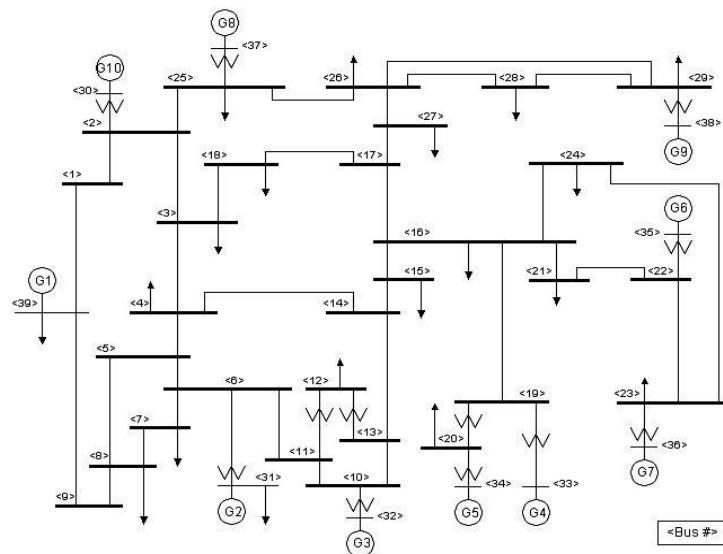

Fig. 1. New England 39-bus test power system

Voltage profile for 1000 times Monte Carlo Simulations is shown in Fig. 2. The stability index and the standard deviations of the stability indexes under normal operation are shown in Figs. 3 and 4, respectively. The simulation results shows that in an interconnected power system under normal operating condition, all of the eigenvalues are positive and it is satisfied the static voltage stability constraint considering load and generation uncertainties. So, the probability of stability for all of the PQ buses is equal to 1.
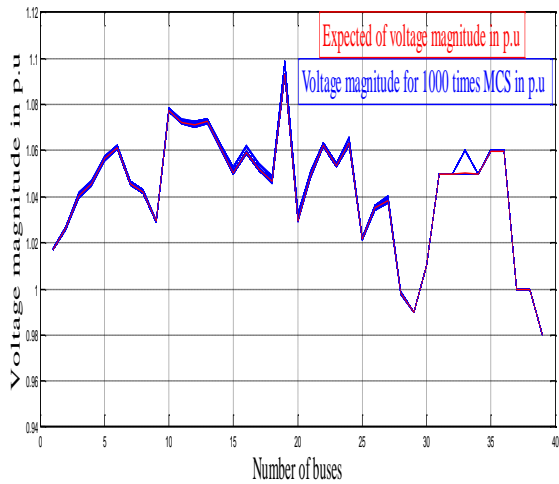
Fig. 2. Voltage profile under normal condition
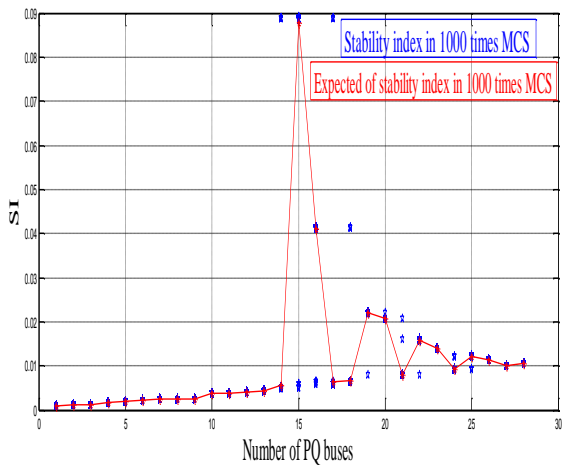
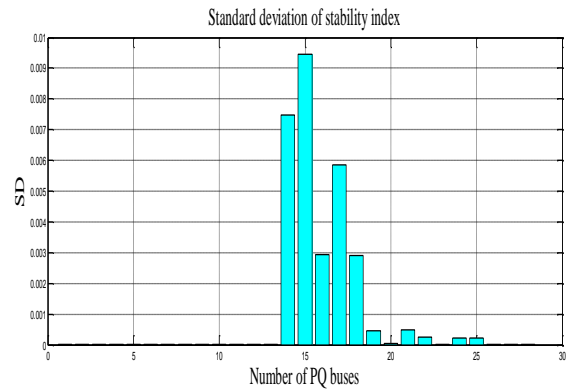**Fig. 3**. Stability indexes for 1000 scenarios under normal operation

Fig. 4. Standard deviation of stability index under normal operation

In order to assess the stability of system under contingency, branch 28 to 29 and generator 34 have been out of the system. The number of Monte Carlo Simulations is 1000. Also, the probabilities of branch and generator outages are assumed 0.1 and 0.05, respectively. The standard deviations of load and generation under given contingencies are assumed 0.99. The amounts of loads and generations are considered as the expected values of loads and generations. The simulation results are shown in Figs. 5 to 8.
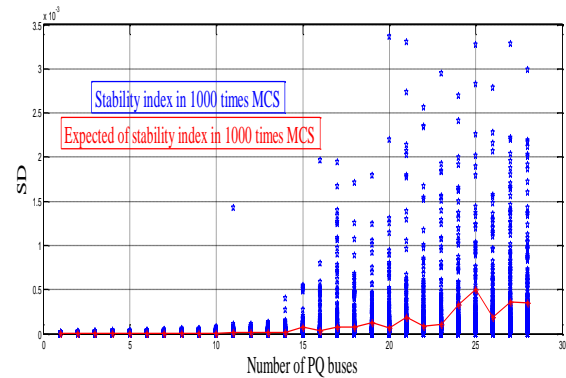

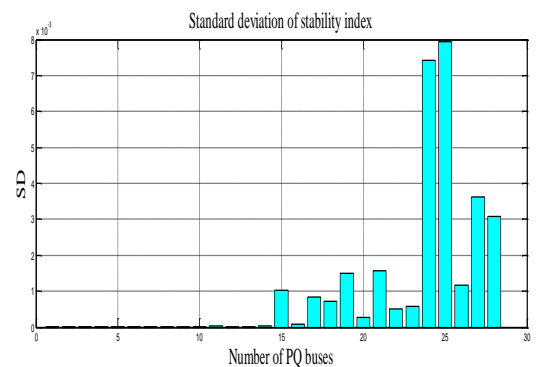**Fig. 5.** Stability indexes for 1000 scenarios under given contingencies

Fig. 6. Standard deviation of stability indexes under given contingencies
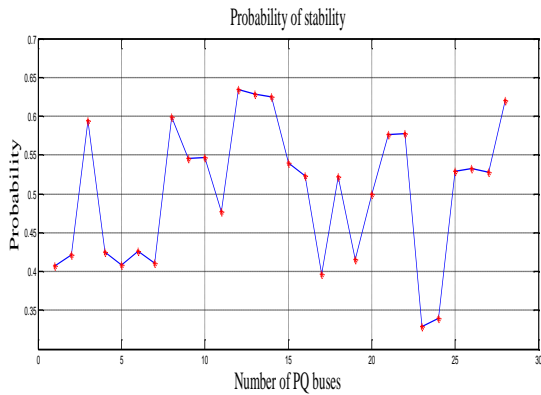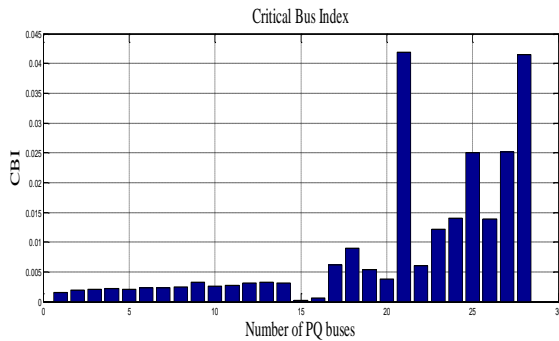
Fig. 7. Probability of stability under given contingencies

Fig. 8. Critical Bus Indexes under given contingencies

As shown in Figs. 5 and 6, 25th, 24th, 27th, 28th PQ buses have the maximum standard deviations, respectively. According to the Figs. 7 and 8, 15th and 16th PQ buses will be the critical PQ nodes with probability of stability 0.54 and 0.52, respectively.

4.2 Simulation on 118 bus IEEE test system
Also, the study has been carried out on 118 buses IEEE [13]. At first, it is assumed that the power system is under normal operating condition and the number of Monte Carlo Simulations is 1000 and loads and generations are uncertain. The probability of normal condition is considered 0.87. The standard deviations of load and generation under normal operation are considered 0.99. The amounts of loads and generations are assumed as the expected values of loads and generations. The test system is shown in Fig. 9.

Fig. 9. 118-bus IEEE test power system

The simulation results under normal operation are shown in Figs. 10 to 12.
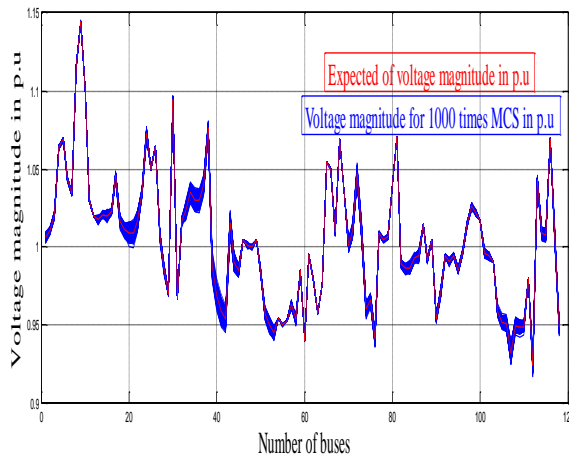

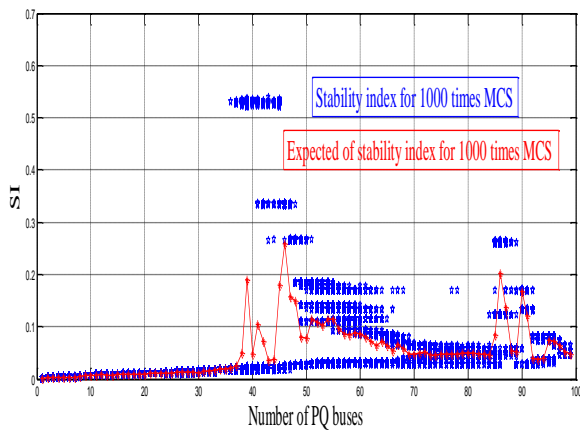**Fig. 10**. Voltage profile under normal condition

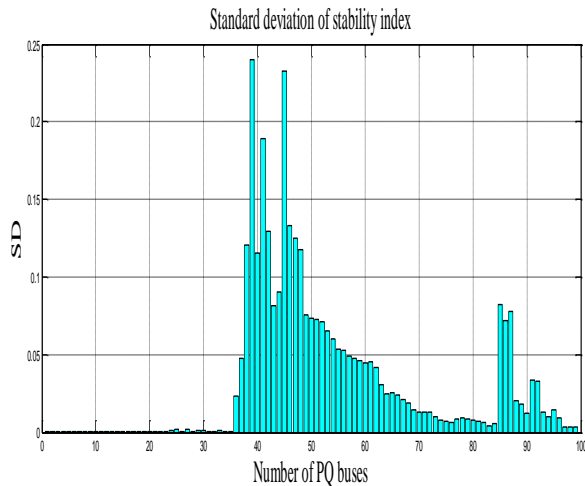**Fig. 11**. Stability indexes under normal condition

Fig. 12. Standard deviation of stability index under normal condition

In order to evaluate the stability of system under contingency, branch 35 to 37 and generator 80 have been out of the system. The number of Monte Carlo Simulations is 1000. The probabilities of branch and generator outages are assumed 0.2 and 0.05, respectively. The standard deviations of load and generation under given contingencies are assumed 0.99. The amounts of loads and generations are considered as the expected values of loads and generations. The simulation results are shown in Figs. 13 to 16.
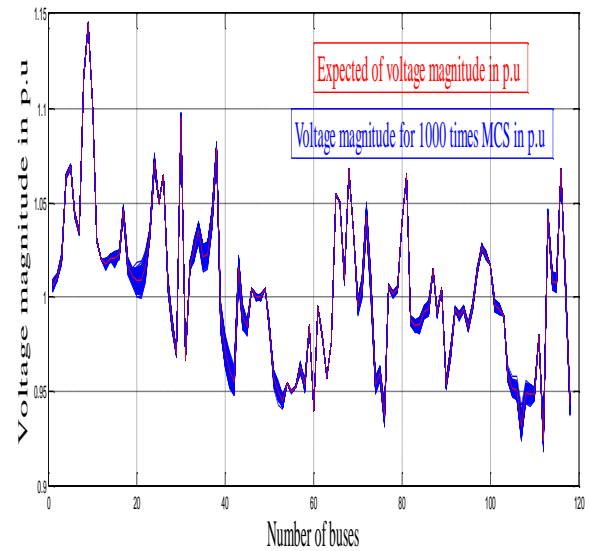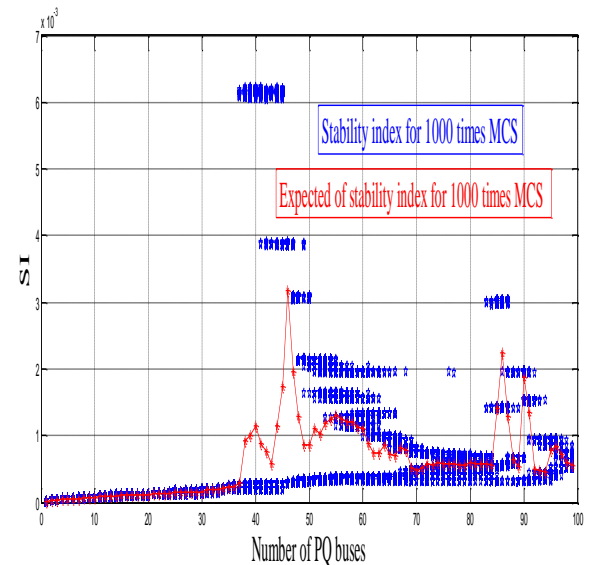

Fig. 13. Voltage profile under given outages

Fig. 14. Stability indexes under given outages

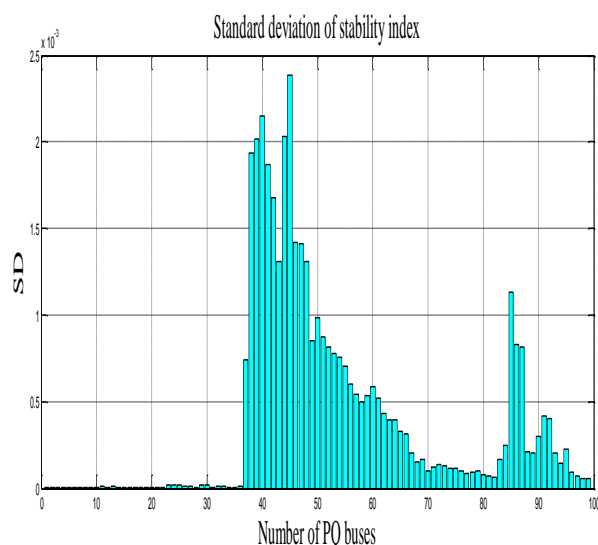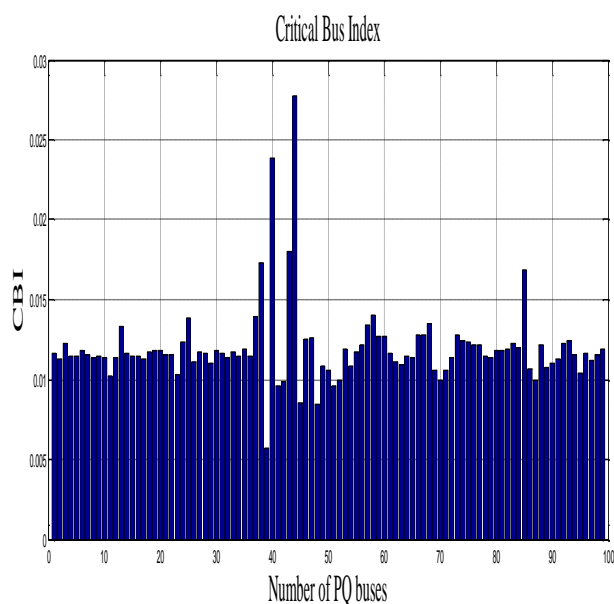**Fig. 15**. Standard deviation of stability indexes under given outages

Fig. 16. Critical Bus Indexes under given outages

As shown in Figs. 14 and 15, 38th to 48th and 85th PQ buses have the maximum standard deviations, respectively. According to the Fig. 16, 39th and 45th and 48th PQ buses will be the critical PQ nodes with probability of stability 1.

5. CONCLUSION
In this paper, the static voltage stability of an interconnected power system considering load and generation uncertainties is evaluated using probabilistic power flow. The Monte Carlo Simulation method is used to generate the probabilistic power flow scenarios. Then, the expected static voltage stability index and probability of stability for all of the PQ buses are obtained. Also, the standard deviations of the stability indexes are calculated. Finally, the critical PQ nodes are determined under given disturbance.

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