Model Order Reduction by Using Legendre Expansion and Harmony Search Algorithm

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ABSTRACT
In this paper, a new approach is investigated for order reduction based on Legendre expansion. Harmony Search is used in this approach, to determine the reduced order model parameters. The Routh criterion is applied to specify the stability conditions. Then, the stability conditions are constraints in optimization problem. To present the efficiency of the proposed method, three test systems are reduced. The obtained results were compared to other existing techniques. The comparison showed that the proposed approach performs well.

KEYWORDS: Harmony Search Algorithm, Legendre Expansion, Order Reduction, Routh Array, Stability Constraints.

1. INTRODUCTION

Many of Various methods are reported in the literature for order reduction in time domain and frequency domain. Model reduction started by Davison in 1966 [1] and followed by Chidambara by suggesting several modifications to Davison's approach [2-4]. After that different approaches were proposed such as: dominant pole retention [5], Routh approximation [6] , Hurwitz polynomial approximation [7-8], stability equation method [9-10], moments matching [11-l4], continued fraction method [15-17], Pade approximation [18] and etc.

The issue of optimality in model reduction was considered by Wilson [19-20] who suggested an optimization approach based on minimization of the order model's parameters are achieved by minimizing a fitness function which is often

Integral Square Error (ISE), Integral Absolute Error (IAE), H_2 norm or H_∞ norm [34-36].

This paper introduces a new alternative method for order reduction using orthogonal polynomials through shifted Legendre functions. In this method, the full order system is expanded by shifted Legendre functions and then the l first coefficients of shifted Legendre functions are obtained. A desire fixed structure for reduced order model is considered and a set of parameters are defined, whose values determine the reduced order system. These unknown parameters are determined using harmony search (HS) algorithm by minimizing the errors between the l first coefficients of integral squared impulse response error between the full and reduced-order models. This attempt was continued by other researches through other approaches [21-24].

In 1981 [25], the controllability and observability of the states was considered in model reduction by Moore. The suggested approach suffered from steady state errors but the stability of the reduced model was assured if the original system was also stable [26]. Furthermore, the concept of H_∞, H_2, L_2 and L_∞ were used in model reduction [27-30].

In recent decades, the evolutionary techniques such as Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) are used for order reduction of systems [31-33]. In these approaches, the reduced shifted Legendre functions expansion of full and reduced systems. To satisfy the stability, Routh criterion is applied as it is used in [37] where, it states in optimization problem as constraints and subsequently, optimization problem converted to a constrained optimization problem. To show the accuracy of the proposed method, three systems are reduced by the proposed method and were compared to those available in the literature.

To make a proper background, Shifted Legendre functions and harmony search algorithm are briefly explained in Sections 2 and 3, respectively. The proposed method is explained in Section 4. The ability of the proposed approach is shown in Section 5. The paper is concluded in Section 6.

2. THE SHIFTED LEGENDRE FUNCTIONS

The shifted Legendre polynomials which are orthogonal and represented by $L_i(z),\ i=0,1,2,\ldots$ can be defined on interval $z\in[-1,1]$ by recursive formula as follows [38]:

$$(i+1)L_{i+1}(z)=(2i+1)zL_i(z)-iL_{i-1}(z),\ i=1,2,\ldots \quad (1)$$

where $L_0(z)$ and $L_1(z)$ are equal to 0 and *z*, respectively.

In order to use these polynomials on the interval $[0,t_f]$, shifted Legendre polynomials $P_i(t)$ is used by changing the variables as follows:

$$z=2\left(\frac{t}{t_f}-1\right),\ 0\le t\le t_f \quad (2)$$

Based on (1), the shifted Legendre polynomials can be defined by using recursive formula as below:

$$(i+1)P_{i+1}(z)=$$
$$(2i+1)\left(\frac{2t}{t_f}-1\right)P_i(z)-iP_{i-1}(z),\ i=1,2,\ldots \quad (3)$$

Where $P_0(z)$ and $P_1(z)$ are equal to 0 and $\left(\frac{2t}{t_f}-1\right)$, respectively.

The orthogonal property is given by

$$\int_0^1 P_i(t)P_j(t)\,dt=\begin{cases}\frac{2t_f}{2i+1} & i=j\\ 0 & i\ne j\end{cases} \quad (4)$$

A function $f(t)$ which is absolutely integrable on interval $[0,t_f]$ may be expressed in terms of a shifted Legendre series as

$$f(t)=\sum_{i=0}^{\infty}\alpha_i P_i(t) \quad (5)$$

Where

$$\alpha_i=\frac{2i+1}{t_f}\int_0^1 f(t)P_i(t)\,dt \quad (6)$$

The above equation indicates that the expansion coefficients, α_i, can be achieved by integrating of $f(t)P_i(t)$.

3. HARMONY SEARCH ALGORITHM

The HS is based on natural musical performance of a process that searches for a perfect state of harmony. In general, the HS algorithm works as follows [39]-[40]:

Step1. Initialization: Define the objective function and decision variables. Input the system parameters and the boundaries of the decision variables. The optimization problem can be defined as:

Minimize $f(x)$ subject to $x_{iL}<x_i<x_{iU}\ \ (i=1,2,\ldots,N)$ where x_{iL} and x_{iU} are the lower and upper bounds for decision variables.

The HS algorithm parameters are also specified in this step. They are the harmony memory size (*HMS*) or the number of solution vectors in harmony memory, harmony memory considering rate (*HMCR*), distance bandwidth (bw), pitch adjusting rate (*PAR*), and the number of improvisations (*K*), or stopping criterion. *K* is the same as the total number of function evaluations.

Step2. Initialize the harmony memory (*HM*). The harmony memory is a memory location where all the solution vectors (sets of decision variables) are stored. The initial harmony memory is randomly generated in the region $[x_{iL},x_{iU}]\ (i=1,2,\ldots,N)$. This is done based on the following equation:

$$x_i^j=x_{iL}+rand(\ )\times(x_{iU}-x_{iL})\ \ j=1,2,\ldots,HMS \quad (7)$$

Where $rand(\ )$ is a random from a uniform distribution on $[0,1]$.

Step3. Improvise a new harmony from the harmony memory. Generating a new harmony, x_i^{new}, is called improvisation where it is based on 3 rules: memory consideration, pitch adjustment and random selection. First of all, a uniform random number *r* is generated in the range $[0,1]$. If *r* is less than *HMCR*, the decision variable x_i^{new} is generated by the memory consideration; otherwise, x_i^{new} is obtained by a random selection. Then, each decision variable x_i^{new} will undergo a pitch adjustment with a probability of *PAR* if it is produced by the memory consideration. The pitch adjustment rule is given as follows:

$$x_i^{new}=x_i^{new}\pm r\times bw \quad (8)$$

Step4. Update harmony memory. After generating a new harmony vector, x^{new}, the harmony memory will be updated. If the fitness of the improvised harmony vector $x^{new} = \left(x_1^{new}, x_2^{new}, \ldots, x_N^{new}\right)$ is better than that of the worst harmony, the worst harmony in the *HM* will be replaced with x^{new} and become a new member of the *HM*.
Step5. Repeat steps 3-4 until the stopping criterion (maximum number of improvisations *K*) is met.

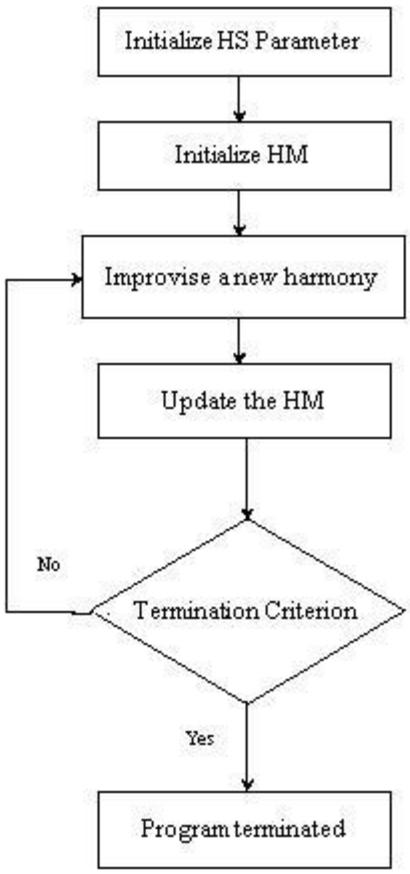

Fig. 1. Basic flowchart diagram for HS algorithm

4. THE PROPOSED MODEL REDUCTION METHOD

Consider a stable single-input single-output (SISO) system described by the transfer function of order *n* as follows:

$$G(s) = \frac{a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n}{s^n + b_1 s^{n-1} + b_2 s^{n-2} + \cdots + b_n} \tag{9}$$

Where a_i and b_i are constants.

The objective is to obtain a reduced model of order *r* so that *r* is smaller than *n* such that the principal and important specification of the full order system are retained in the reduced order model. This reduced order system is presented by

$$G_r(s) = \frac{c_1 s^{r-1} + c_2 s^{r-2} + \cdots + c_r}{s^r + d_1 s^{r-1} + d_2 s^{r-2} + \cdots + d_r} \tag{10}$$

Where c_i and d_i are unknown constants.

To obtain the reduced model by the proposed method, firstly, the full order system is expanded. Then the *l* first coefficients of Legendre expansion of original system are obtained by equation (6). By considering the fixed structure for reduced order model, the harmony search algorithm is applied for determining the unknown coefficients of reduced order model. If g_i and g_i' $i = 1, 2, \ldots$ are defined as Legendre expansion of full order and reduced order model, respectively, the reduced order model's parameters are determined by minimizing the following fitness function:

$$J = \sum_{i=0}^{l} \left| g_i - g_i' \right| \tag{11}$$

Since, the proposed approach must guarantee the stability of the reduced system, the Routh criterion is applied for specifying the stability conditions as follows:
The denominator of reduced order model which is presented by (10) can be shown as below [41]:

$$\begin{aligned} & s^r + h_1 s^{r-1} + \left(h_2 + h_3 + \cdots + h_r\right) s^{r-2} + \\ & + h_2\left(h_3 + h_4 + \cdots + h_r\right) s^{r-3} + \\ & + \begin{bmatrix} h_2\left(h_4 + h_5 + \cdots + h_r\right) + \\ + h_3\left(h_5 + h_6 + \cdots + h_r\right) + \\ + h_4\left(h_6 + h_7 + \cdots + h_r\right) + \cdots h_{r-2} h_r \end{bmatrix} s^{r-4} + \\ & + \cdots + h_{1+k} h_{3+k} \cdots h_{r-2} h_r \end{aligned} \tag{12}$$

Which is constructed by taking the coefficients of the first two rows of the Routh array with the elements of its first column given by

$$1, h_1, h_2, h_1 h_3, h_2 h_4, h_1 h_3 h_5, \ldots h_{1+k} h_{3+k} \ldots h_{r-2} h_r \tag{13}$$

Where, *k* is equal to 1 for even *r* and *k* is equal to 0 for odd *r*.
Comparing the entries of the first row with $1, d_2, d_4, \ldots$ and those of the second row with $d_1, d_3, d_5, \ldots$ the relations defined in (14) is obtained:

$$\begin{cases} d_1 = h_1 \\ d_2 = (h_2 + h_3 + \cdots + h_r) \\ d_3 = h_1 (h_3 + h_4 + \cdots + h_r) \\ \quad \vdots \\ d_r = (h_{1+k} h_{3+k} \cdots h_{r-2} h_r) \end{cases} \tag{14}$$

Substituting the above relations in reduced order model's denominator, the equation (12) is achieved.
Therefore, the necessary and sufficient condition for all the poles of the reduced system to be strictly in the left-half plane is

$$\begin{cases} h_1 > 0 \\ h_2 > 0 \\ \quad \vdots \\ h_r > 0 \end{cases} \tag{15}$$

and subsequently

$$\begin{cases} d_1 > 0 \\ d_2 > 0 \\ \quad \vdots \\ d_r > 0 \end{cases} \tag{16}$$

Therefore, to have a stable reduced system, the reduced order model's parameters are determined by minimizing (11) subject to (16). In other words, the reduced order model is obtained by minimizing the following fitness function:

$$J = \sum_{i=0}^{l} |g_i - g_i'| \quad i = 1, 2, \ldots, l$$
$$\text{subject to } d_j > 0 \quad j = 1, 2, \ldots, r \tag{17}$$

Thus, the reduced order model is achieved such that the l first coefficients of Legendre expansion of the full order system are equal (or very close) to the l first coefficients of Legendre expansion of reduced order model. The reduced order model that is achieved by this method, tries retaining the important characteristic of the original system.
The proposed method can be summarized in the following steps:
Step 1: The Legendre expansion of the full order system in (9) is obtained.
Step 2: A desire fixed structure is considered for reduced order model as defined in (10) where c_i and d_i are unknown parameters of reduced order model that are obtained in the next step.
Step 3: To obtain the unknown parameters, HS is applied. The goal of the optimization is to find the best parameters for $G_r(s)$. Therefore, each harmony is a d-dimensional vector in which d is $c_i + d_i$. Each harmony is a solution to G_r and for each solution (harmony), the Legendre expansion are obtained. Each harmony in the population is evaluated using the objective function defined by (17) searching for the harmony associated with J_{best} (the best J) until the termination criteria are met. At this stage the best parameters are given as parameters of reduced order model.

5. SIMULATION AND RESULTS

To assess the efficiency of the proposed approach, it has been applied on three test systems. To obtain a reduced-order model, a step-by-step procedure is given for the first test system.
Test system 1: The first system to be reduced is a system given in [31] by Mukherjee, where a procedure is presented to obtain the reduced system. The system is as follows:

$$G(s) = \frac{18s^7 + 514s^6 + 5982s^5 + 3638s^4 + 122664s^3 + 222088s^2 + 185760s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109584s + 40320} \tag{18}$$

The reduced-order model can be achieved by the following steps, using Legendre expansion and HS:
Step 1: Based on section 2, by considering t_f =20, the Legendre expansion of the full order system in (18) is obtained as:

$$\begin{aligned} G_{Org} &= (1.097\times10^3)\times(1) + (-559.214)\times\left(\frac{s}{10} - 1\right) + \\ &+ (88.760)\left(\frac{3s^2}{200} - \frac{3s}{10} + 1\right) + \\ &+ (-337.428)\times\left(\frac{s^3}{400} - \frac{3s^2}{40} + \frac{19s}{30} - \frac{4}{3}\right) + \\ &+ (1.920\times10^3)\times\left(\frac{7s^4}{16000} - \frac{7s^3}{400} + \frac{143s^2}{600} - \frac{19s}{15} + \frac{25}{12}\right) \end{aligned} \tag{19}$$

Step 2: The full order system in (18) is going to be reduced to a third-order system with the following transfer function:

$$G_r(s) = \frac{c_1 s + c_2}{s^2 + d_1 s + d_2} \tag{20}$$

Where c_i and d_i are the unknown parameters of reduced order model.
Step 3: HS is applied to obtain the unknown parameters. Since, the goal of the optimization is to find the best parameters for $G_r(s)$, therefore, each

harmony is a d-dimensional vector in which $d = 6$. The *HMS* is selected to be 6, *HMCR* and evaluation number is set to be 0.9 and 1000, respectively. Each harmony is a solution to G_r and for each solution (harmony), the Legendre expansion are obtained. Each harmony in the population is evaluated using the objective function defined by (17) searching for the best J until the termination criteria are met. At this stage the best parameters are given for reduced order model where, the following reduced order model is obtained:

$$G_{Legendre} = \frac{17.8387s + 5.4503}{s^2 + 7.4171s + 5.4289} \tag{21}$$

The Legendre expansion of obtained reduced order model is as:

$$\begin{aligned} G_{Legendre} &= \left(1.096\times10^3\right)\times(1) + (-556.535)\times\left(\frac{s}{10}-1\right) + \\ &(84.846)\left(\frac{3s^2}{200}-\frac{3s}{10}+1\right) + \\ &(-334.383)\times\left(\frac{s^3}{400}-\frac{3s^2}{40}+\frac{19s}{30}-\frac{4}{3}\right) + \\ &\left(1.915\times10^3\right)\times\left(\frac{7s^4}{16000}-\frac{7s^3}{400}+\frac{143s^2}{600}-\frac{19s}{15}+\frac{25}{12}\right) \end{aligned} \tag{22}$$

Comparing (19) and (22) shows that a good approximant is achieved for $G(s)$.

The step response of the full order system and that of the system with second-order reduced models are shown in Fig. 2. This figure shows that, the obtained reduced order model is an adequate low-order model that retains the characteristics of full order model. Also, to show the efficiency of the proposed method, the step and frequency responses of the obtained reduced model are compared with those available in the literature. Figs. 3-4 , show the comparison of the results obtained to the one proposed by Mukherjee [31], the one proposed by Mittal [31], Optimal Hankel norm approximation (HSV) [42] and Balanced Truncation (BT)[42]

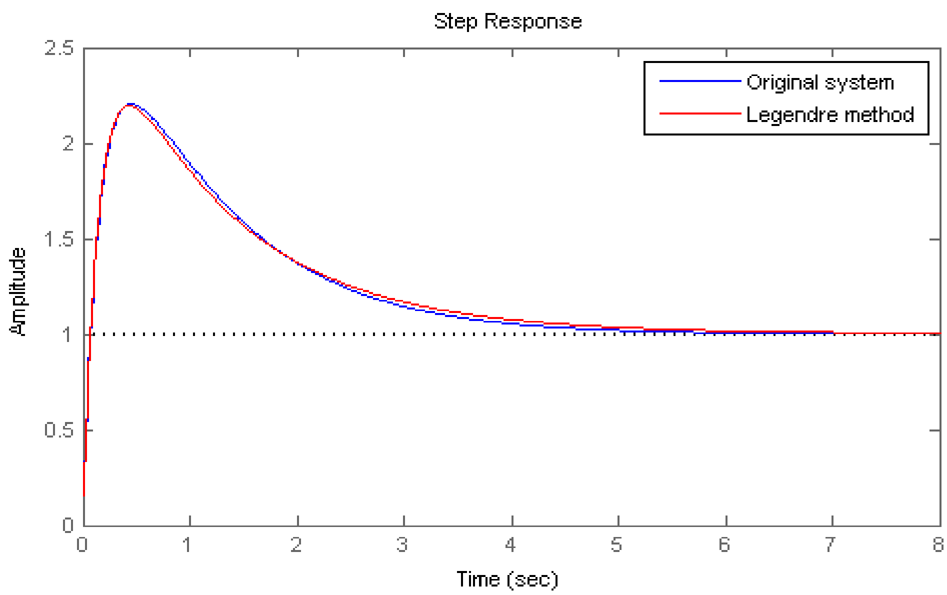

Fig. 2. Step response of full order and obtained reduced order model for test system 1

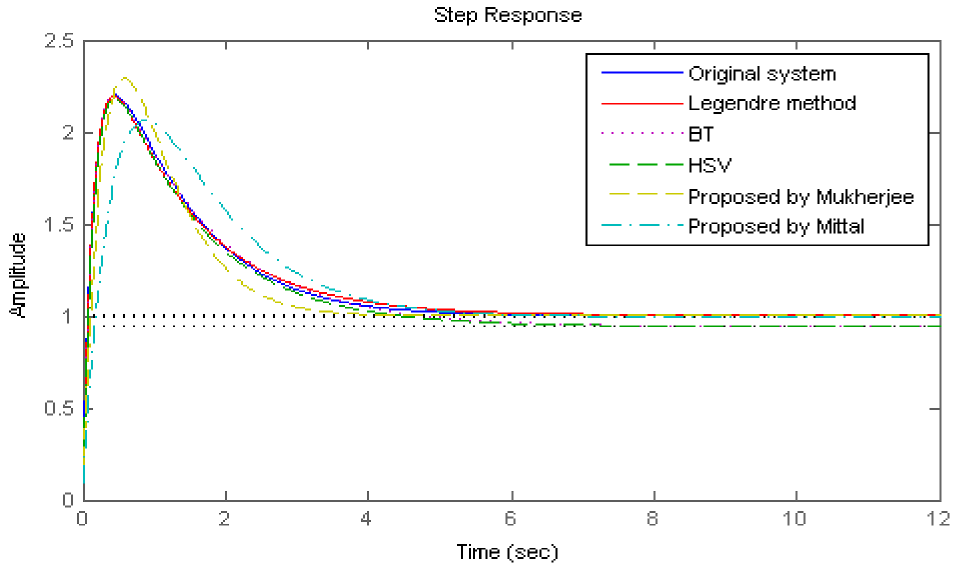

Fig. 3. Step response of full order and reduced order model by the proposed method and other methods for test system 1

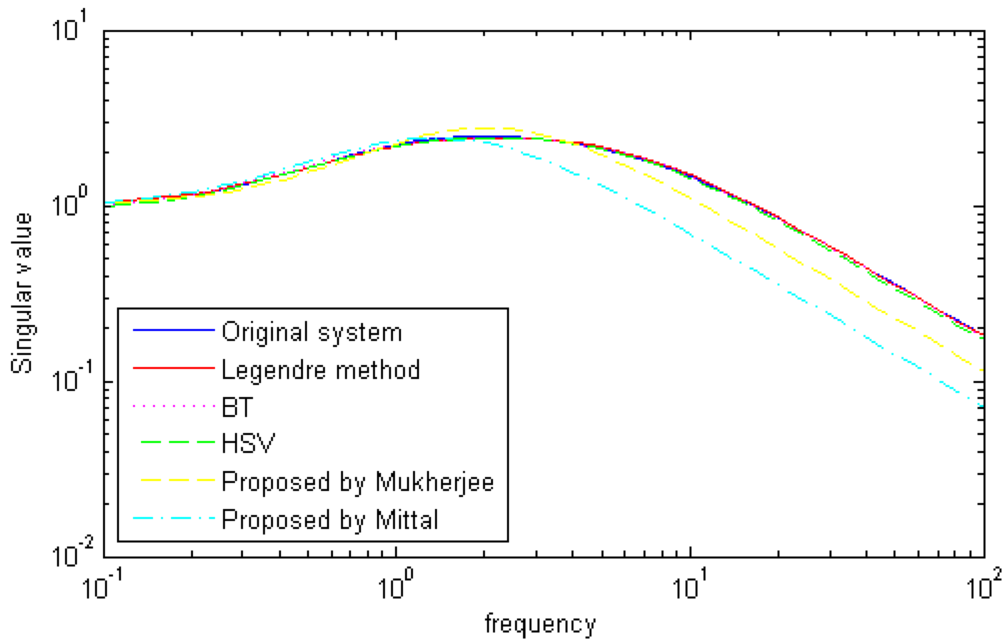

Fig. 4. The frequency response of full order and reduced order model by the proposed method and other methods for test system 1

These figures show that the achieved results from the proposed method are very similar to original system comparing to other methods.
The steady-state gains of full and reduced systems should be equal since it is a very important requirement for model reduction. Also, the frequency responses of full and reduced systems are very similar, which will make the stability and performance characteristics of both systems to be the same.
Furthermore, some specifications such as steady state value, rise time, settling time and maximum overshoot are compared in Table 1. Also, H_∞ norm of the error between the step responses of full order and reduced

order models ($e = |y - y_r|$) is given in Table 1. It is clearly seen that the specifications of reduced order model that is achieved by the proposed method are close to the specifications of original system.

Table1. Comparison of methods for test system1

Method	Steady state value	Overshoot (%)	Rise time	Settling time	ISE	Infinity norm of error	Infinity norm of model
Original system	1	120	0.0569	4.82	-	-	2.4747
Legendre	1	119	0.0572	5.35	0.0028	0.0397	2.4266
BT	0.94	134	0.0529	5.97	0.0493	0.0596	2.4301
HSV	0.944	132	0.0556	5.48	0.0481	0.0559	2.4321
Proposed by Mukherjee	1	129	0.0856	3.35	0.0569	0.3361	2.7514
Proposed by Mittal	0.995	107	0.141	5.47	0.2692	0.7696	2.4118

Also, the plot of $e = |y - y_r|$ is illustrated in Fig. 5 for reduced systems. This figure illustrates that the obtained error by the proposed method in this paper is less than other methods.

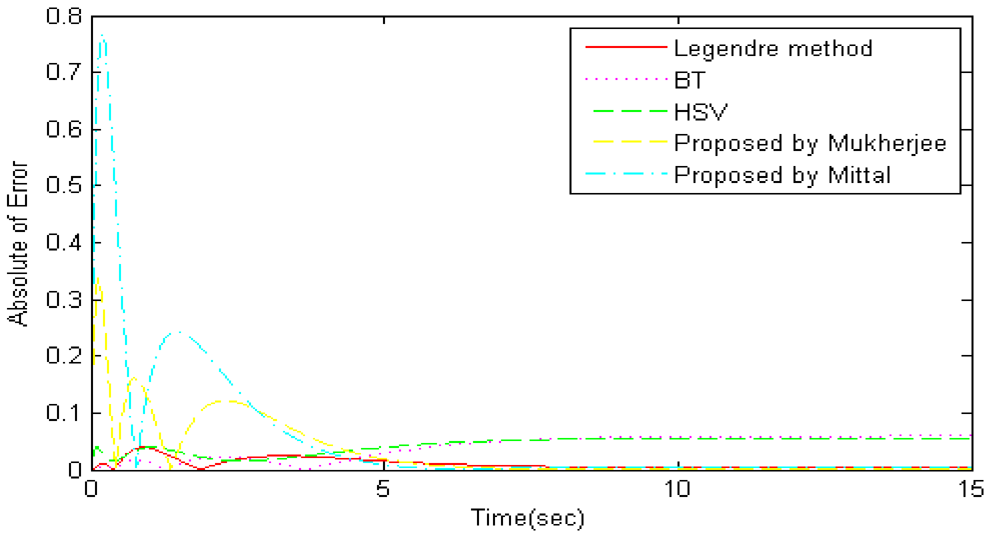

Fig. 5. The plot of $e = |y - y_r|$ for test system 1

Test system 2: In [41], a procedure is presented to obtain a reduced order system by Routh-Pade approximation using Luus-Jaakola algorithm. To compare the proposed method with Luus-Jaakola algorithm, the system given in [41] is adopted which is a third-order system:

$$G = \frac{8s^2 + 6s + 2}{s^3 + 4s^2 + 5s + 2} \quad (23)$$

Based on the explanations given for test system 1, the obtained reduced system by the proposed method is as follows:

$$G_{Legendre} = \frac{7.8922s + 4.7858}{s^2 + 3.6104s + 4.7874} \quad (24)$$

The step response of the original system and the obtained reduced model are shown in Fig. 6. In this figure, the responses of the system with second-order primary reduced models obtained by other methods are also included for comparison. Also, the plot of $e = |y - y_r|$ is given in Fig. 7.

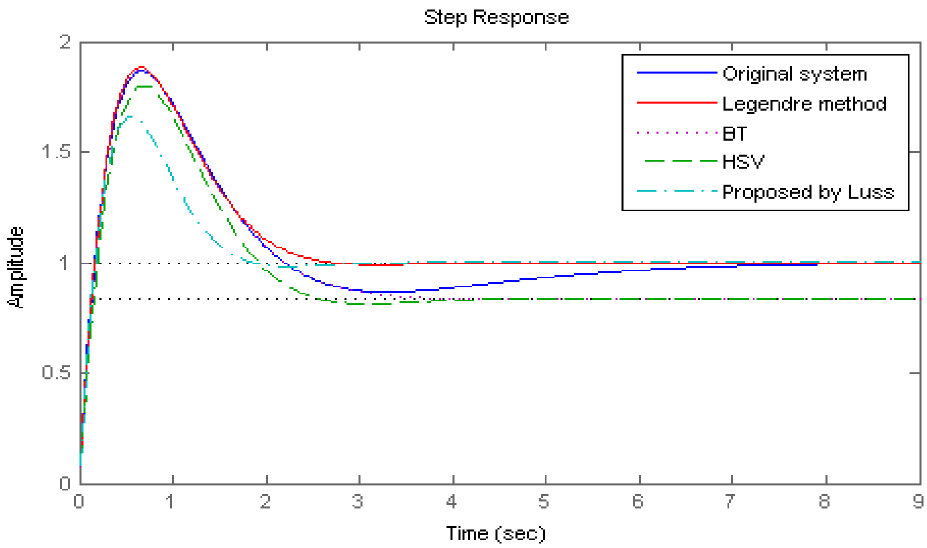

Fig. 6. Step response of full order and reduced order model by the proposed method and other methods for test system 2

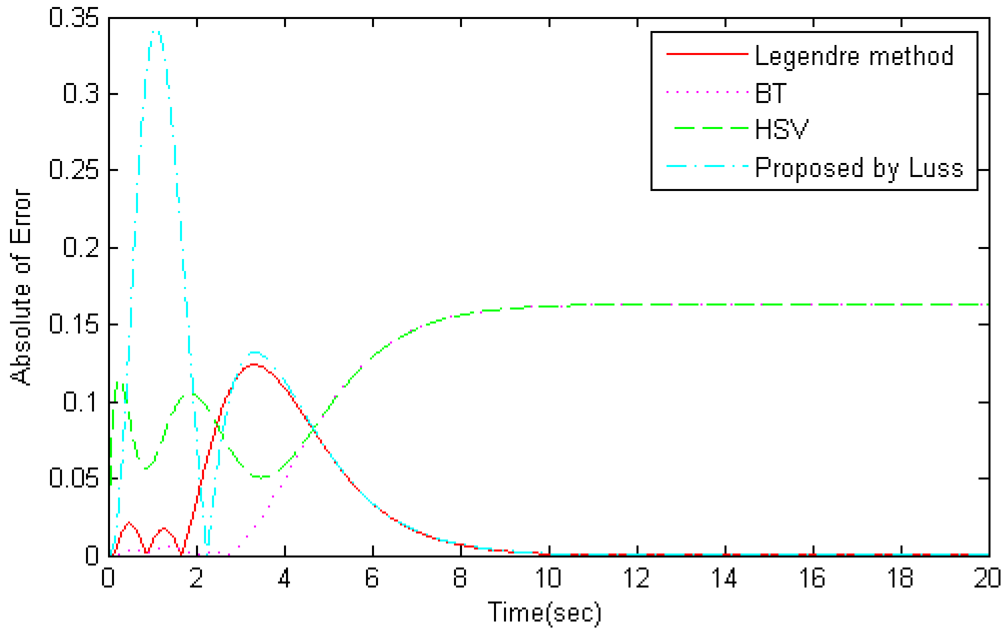

Fig. 7. The plot of $e = |y - y_r|$ for test system 2

Furthermore, maximum overshoot, rise time, settling time, steady state value, ISE and H_∞ norm of $e = |y - y_r|$ are shown in Table 2. Once again, the results obtained confirm that a satisfactory approximation has been achieved. It is clearly seen that the specifications of reduced order model that is achieved by the proposed method are close to the specifications of the original system, and better than other methods.

Table1. Comparison of methods for test system1

Method	Steady state value	Overshoot (%)	Rise time	Settling time	ISE	Infinity norm of error	Infinity norm of model
Original system	1	86.5	0.129	6.74	-	-	2.3001
Legendre	1	88.2	0.128	2.44	0.0338	0.1240	2.2713
BT	0.836	123	0.103	3.15	0.3802	0.1635	2.2790
HSV	0.836	115	0.118	3.44	0.4043	0.1635	2.2743
Proposed by Luss	1	66.1	0.13	1.71	0.1404	0.3425	1.9772

Test system 3: the third test system is a multivariable system given in [32]:

$$[G(s)]=\begin{bmatrix}\frac{2(s+5)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)}\\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)}\end{bmatrix}=\frac{1}{D(s)}\begin{bmatrix}a_{11}(s) & a_{12}(s)\\ a_{21}(s) & a_{22}(s)\end{bmatrix} \quad (25)$$

Where

$$\begin{aligned}D(s)&=\\ &s^6+41s^5+571s^4+3491s^3+10060s^2+13100s+6000\\ a_{11}(s)&=2s^5+70s^4+762s^3+3610s^2+7700s+6000\\ a_{12}(s)&=s^5+38s^4+459s^3+2182s^2+4160s+2400\\ a_{21}(s)&=s^5+30s^4+331s^3+1650s^2+3700s+3000\\ a_{22}(s)&=s^5+42s^4+601s^3+3660s^2+9100s+6000\end{aligned} \quad (26)$$

Based on the explanations given for test system 1, the obtained reduced system by the proposed method is as follows:

$$[G_{Legendre}(s)]=\frac{1}{\tilde{D}(s)}\begin{bmatrix}b_{11}(s) & b_{12}(s)\\ b_{21}(s) & b_{22}(s)\end{bmatrix} \quad (27)$$

Where

$$\begin{aligned}\tilde{D}(s)&=s^2+2.602s+1.243\\ b_{11}(s)&=1.480s+1.243\\ b_{12}(s)&=1.774s+0.291\\ b_{21}(s)&=0.668s+0.622\\ b_{22}(s)&=1.955s+1.241\end{aligned} \quad (28)$$

The comparison of the proposed method with the one proposed by Parmar in [32] is shown in Figs. 8-9, which illustrate a better performance of the proposed method.

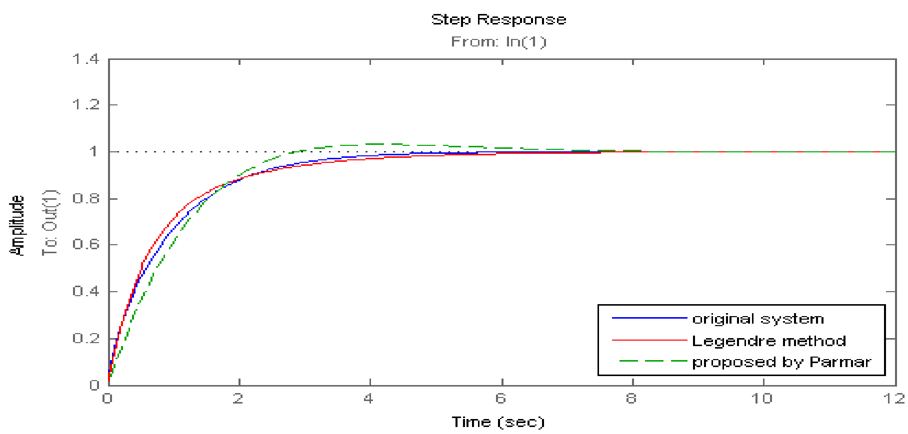

Fig. 8. Step response of full order and reduced order model by the proposed method and the one proposed by Parmer for test system 3 (input1- output1)

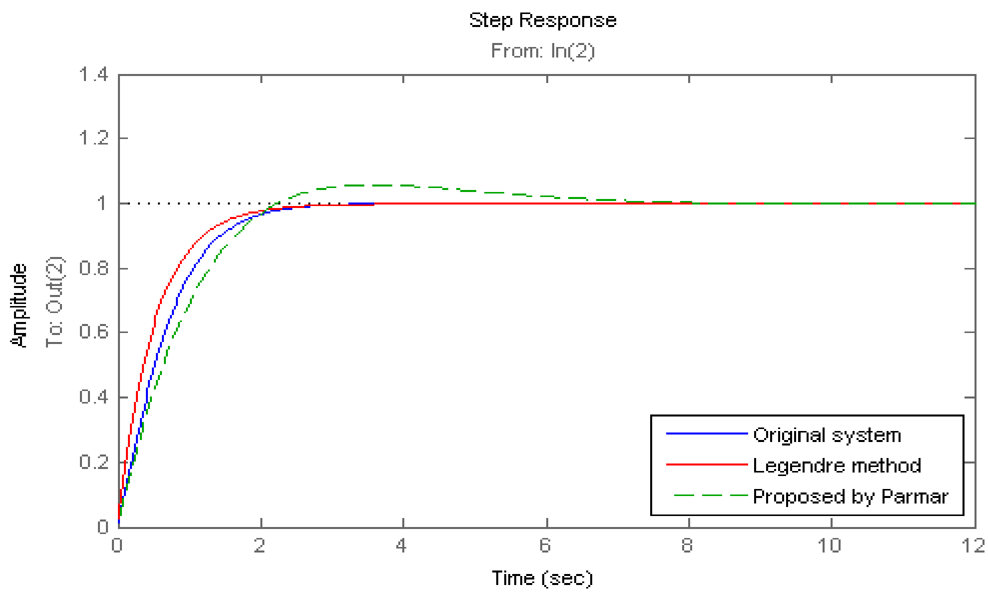

Fig. 9. Step response of full order and reduced order model by the proposed method and the one proposed by Parmer for test system 3 (input2- output2)

6. CONCLUSION

In this paper, an approach based on orthogonal polynomials using Legendre expansion and harmony search is investigated for order reduction. Routh array is applied to determine the stability conditions. To present the accuracy and efficiency of the method, three systems are reduced. The proposed method was compared with some order reduction techniques where the results obtained showed that the proposed approach has high accuracy which results in an adequate low-order model that retains the characteristics of full order model.

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