Adaptive Second Order Terminal Backstepping Sliding Mode for Attitude Control of Quadrotor with External Disturbances

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ABSTRACT:
This paper proposes a backstepping terminal sliding mode control with adaptive algorithm which applied to Quadrotor for free chattering, finite time convergence and robust aims. First of all, dynamic equation of a quadrotor has been obtained based on Euler-Lagrangian equations with considering additional disturbance and uncertainty. Furthermore, a nonlinear control scheme has been proposed to deal against defined perturbations. In the proposed control scheme, instead of using regular control input, the derivative of the control input has been achieved from terminal second-layer sliding surface. An adaptive algorithm has been used to achieve the robust performance against external disturbances like wind effects. The adaptive control law estimates the upper bound of disturbance and uncertainty. Stability and robustness of the proposed controller have been proved by using the classical Lyapunov criterion. The simulation results demonstrate the validation of the proposed control scheme.

KEYWORDS: Adaptive algorithm, Backstepping method, Terminal second order sliding mode, Quadrotor.

1. INTRODUCTION

In the past few years, the interest in Unmanned Aerial Vehicle (UAV), especially quadrotor, has been grown strongly. The design of flight controller which can offer the accurate and robust performances to UAVs is an important step in the design of fully autonomous vehicles. Fixed wing UAVs have been used in surveillance and different missions for years, but their lack of stationary flight capability has shifted the focus of engineers to the vertical Take-Off and Landing (VTOL) vehicles offering ability to hover above a target. Quadrotor vehicle is one of the most important VTOL vehicles which can reach a stable hovering and flight using the equilibrium forces produced by four rotors. One of the advantages of the quadrotor configuration and flight is the maneuver possibility against other kind of UAVs.

Different control methods have been explored for the attitude and position control of quadrotor. The dynamic model used in this paper is based on the model which presented in [1]. Reference [2] has applied backstepping and conventional sliding mode which is easy to implement practically. Many researchers have worked on backstepping control for quadrotor stabilization [3]. Integral backstepping also has been used in [4-5]. Most of the important common techniques which have applied for a quadrotor are based on the sliding mode design which can suppress the effects of uncertainty and bounded disturbances. Sliding mode control has been used for ensuring desired tracking trajectories [6]. Robustness is an important issue about quadrotor flight in the outdoor situation. Reference [7] has proposed sliding mode control for flight control in the presence of disturbances. Nonlinear disturbance observer has been presented for a quadrotor in [8]. Super twisting algorithm has been applied for a quadrotor in [9]. Adaptive algorithms are used for online estimate of some parameters. Adaptive sliding mode control has been used for trajectory tracking under the underground effects and noisy sensors in [10]. PD-2 feedback controller has been applied for the compensation of the Coriolis and gyroscopic torques [11-12]. Direct approximate-adaptive control, using CMAC nonlinear approximation has been presented and robustness properties to disturbances and unknown payloads achieved [13]. Reference [14] has proposed robust adaptive control based on baseline control combining with model reference adaptive control for quadrotor application.

In this paper, an innovative control scheme is proposed for robust and finite time convergence of the quadrotor attitudes and altitude. This control algorithm decreases chattering effect and improves transient response. This paper proposes a new attitude and altitude control approach for a quadrotor based on second order sliding mode control theory which uses an adaptive tuning law.

The main attributes of proposed controller are robustness, finite time stabilization and free chattering control input. An adaptive tuning law is used for the controller to estimate the unknown but bounded system uncertainties and disturbances. The main contribution of this work is proposing integral backstepping which is combined with terminal second layer sliding mode and uses adaptive scheme to reach the mentioned aims. The authors emphasizes that the proposed control scheme is different with conventional adaptive backstepping sliding mode and numerical simulations have been carried out to the high performance of the proposed controller.

The paper is organized as follows. In Section 2, a brief description of the system model is given. Some assumptions and Lemmas are provided in Section 3. The proposed controller for attitude and altitude of the controller is designed in Section 4 and its stability is also proved. Simulation results and experiments are provided in Section 5. And finally conclusion is made.

2. DYNAMIC MODELLING

In this section, the basic state-space model of the quadrotor is described. The dynamics of the four rotors are much faster than the main system and thus neglected in this case. The generalized coordinates of the rotorcraft are $q = (x, y, z, \psi, \theta, \phi)$, where $\xi = (x, y, z) \in \Re^3$ represents the relative position of the rotorcraft with respect to an inertial frame and $\eta = (\psi, \theta, \phi) \in S^3$ are the three Euler angles representing the orientation of the rotorcraft, called yaw, pitch and roll of the vehicle. Considering kinetic and potential energy of system, let us define Lagrangian as $L(q, \dot{q}) = T_{rot} + T_{trans} - u$, where $T_{trans} = \frac{m}{2}\dot{\xi}^T\dot{\xi}$ is the translational kinetic energy, $T_{rot} = \frac{1}{2}\dot{\eta}^T J \dot{\eta}$ is the rotational kinetic energy, $u = mgz$ is the potential energy, z is the quadrotor altitude, g is acceleration of gravity, m is the mass of quadrotor and J is the auxiliary matrix expressed in terms of η.

The dynamic of quadrotor is obtained from Euler-Lagrangian equations with considering external forces $F = (F_\xi, \tau)$ as follows:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = F \tag{1}$$

By neglecting body forces because of their small amplitude, we have

$$\hat{F} = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix} \tag{2}$$

The main thrust is described as below

$$u = f_1 + f_2 + f_3 + f_4 \tag{3}$$

Where $f_i = k_i \omega_i^2$, $i = 1, \dots, 4$ and k_i are positive constants and ω is the angular speed of each motor. Then F_ξ can be rewritten as $F_\xi = R\hat{F}$, where R is translational matrix which van be shown as following:

$$R = \begin{pmatrix} c\theta\, c\psi & s\psi\, s\theta & -s\theta \\ c\psi\, s\theta\, s\phi - s\psi\, c\phi & s\psi\, s\theta\, s\phi + c\psi\, c\phi & c\theta\, s\phi \\ c\psi\, s\theta\, c\phi + s\psi\, s\phi & s\psi\, s\theta\, c\phi - c\psi\, s\phi & c\theta\, c\phi \end{pmatrix} \tag{4}$$

Where c and s denote cos and sin respectively.

The generalized torque for the η variables are defined as

$$\tau = \begin{pmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{pmatrix} \tag{5}$$

Where

$$\tau_\psi = \sum_{i=1}^{4} \tau_{M_i} \tag{6}$$

$$\tau_{M_i} = c(f_1 - f_2 + f_3 - f_4) \tag{7}$$

$$\tau_\theta = (f_2 - f_4)l \tag{8}$$

$$\tau_\phi = (f_3 - f_1)l \tag{9}$$

Thus, the control distribution from the four actuator motors of the quadrotor is given by

$$\begin{pmatrix} u \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -l & 0 & l & 0 \\ 0 & l & 0 & -l \\ c & -c & c & -c \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} \tag{10}$$

Where l is the distance from the motors to the center of gravity and c is a constant known as force-to-moment scaling factor.

Totally, the dynamic model of quadrotor can be obtained by the following equations.

$$m\ddot{\xi} + \begin{pmatrix} 0 \\ 0 \\ \mathrm{mg} \end{pmatrix} = F_\xi \tag{11}$$

$$J\ddot{\eta} + \dot{J}\dot{\eta} - \frac{1}{2}\frac{\delta}{\delta\eta}(\dot{\eta}^T J\, \dot{\eta}) = \tau \tag{12}$$

$$\bar{V}(\eta, \dot{\eta}) = \dot{J}\dot{\eta} - \frac{1}{2}\frac{\delta}{\delta\eta}(\dot{\eta}^T J\, \dot{\eta}) \tag{13}$$

$$J\ddot{\eta} + \bar{V}(\eta, \dot{\eta}) = \tau \tag{14}$$

$$\bar{V}(\eta, \dot{\eta}) = \left(\dot{J} - \frac{1}{2}\frac{\delta}{\delta\eta}(\dot{\eta}^T J\,)\right)\dot{\eta} = C(\eta, \dot{\eta})\dot{\eta} \tag{15}$$

By substituting F_ξ with equation (11), the state-space of translation motion with external disturbances in z-axis can be obtained as following

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = -\begin{pmatrix} 0 \\ 0 \\ \mathrm{g} \end{pmatrix} + \frac{1}{\mathrm{m}}\begin{pmatrix} -s\theta \\ c\theta\, s\phi \\ c\theta\, c\phi \end{pmatrix} u + d(t) \tag{16}$$

The external disturbances has been considered in z-axis, because of the purpose of this paper is just altitude control in terms of position control. $d(t)$ can be considered as acceleration of external wind $d_z(t)$ which can effect on position control.

Moreover, the dynamic model of quadrotor in terms of rotation with considering uncertainty is written as

$$\begin{pmatrix}\ddot{\phi}\\ \ddot{\theta}\\ \ddot{\psi}\end{pmatrix} = f(\phi,\theta,\psi) + g(\phi,\theta,\psi)\tau + \Delta f(\psi,\theta,\phi) \qquad (17)$$

Where Δf is a vector of uncertainty can simply added to rotational equations and we have

$$f(\phi,\theta,\psi) = \begin{pmatrix}\dot{\theta}\dot{\psi}\left(\frac{I_y-I_z}{I_x}\right) - \frac{J_p}{I_x}\dot{\theta}\Omega\\ \dot{\phi}\dot{\psi}\left(\frac{I_z-I_x}{I_y}\right) - \frac{J_p}{I_y}\dot{\phi}\Omega\\ \dot{\phi}\dot{\theta}\left(\frac{I_x-I_y}{I_z}\right)\end{pmatrix} \qquad (18)$$

$$g(\phi,\theta,\psi) = \begin{pmatrix}\frac{l}{I_x} & 0 & 0\\ 0 & \frac{l}{I_y} & 0\\ 0 & 0 & \frac{l}{I_z}\end{pmatrix} \qquad (19)$$

Where $\Omega = \omega_2 + \omega_4 - \omega_1 - \omega_3$, $I_{x,y,z}$ are body inertia and J_p is propeller rotor inertia.

To avoid repetition in terms of attitude and altitude control state-space model of Euler angles (Roll, Pitch and Yaw) is added to height of quadrotor and it is considered as follows

$$\begin{gathered}\dot{x}_{2i-1} = x_{2i}\\ \dot{x}_{2i} = f_i(x) + g_i(x)u_i + \Delta f_i + d_i\ ,\ i = 1,\dots,4\\ y_i = x_{2i-1}\end{gathered} \qquad (20)$$

Where $x_{2i-1} = (\phi,\theta,\psi,z)^T$, $x_{2i} = (\dot{\phi},\dot{\theta},\dot{\psi},\dot{z})^T$, $\Delta f_i = (\Delta f_\phi, \Delta f_\theta, \Delta f_\psi, 0)^T$, $d_i(t) = (0,0,0,d_z(t))^T$ and $g_i(x) = \left(\frac{l}{I_x}, \frac{l}{I_y}, \frac{l}{I_z}, \frac{c\theta\, c\phi}{m}\right)^T$ for $i = 1,\dots,4$.

3. PRELIMINARY

3.1. Lemmas

Following Lemmas are required for some mathematical aspects of controller design:

Lemma1. [15] For $x_i \in \mathbb{R}$, $i = 1, 2, \dots, n$, $0 < p \le 1$ is a real number, then the following inequality holds:

$$(|x_1| + |x_2| + \cdots + |x_n|)^p \le |x_1|^p + \cdots + |x_n|^p \qquad (21)$$

Lemma 2. [16] Assume that a continuous, positive definite function V(t) satisfies the following differential inequality:

$$\dot{V}(t) \le -c\, V^{\xi}(t) \quad \forall t \le t_0 \quad V(t_0) \ge 0 \qquad (22)$$

Where $c > 0$, $0 < \xi < 1$ are two constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\xi}(t) \le V^{1-\xi}(t_0) - c(1-\xi)(t-t_0), \quad t_0 \le t \le t_1 \qquad (23)$$

and $V(t) \equiv 0 \quad \forall t \ge t_1$ with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\xi}(t_0)}{c(1-\xi)} \qquad (24)$$

3.2. Assumptions

The following assumption is defined for uncertainty and disturbance of system.

A1. All states of quadrotor system are measurable.

A2. The derivative of the external disturbances $\dot{d}_i(t)$ and uncertainties $\frac{d}{dt}(\Delta f_i)$ are bounded and maximum bound of them is defined by D_i for $i = 1,2,3,4$.

4. CONTROLLER DESIGN AND STABILITY ANALYSIS

The aim of developing a sliding mode controller is to achieve robustness against uncertainty. The sliding mode control can be used upon regulatory variables to bring the new variables to the equilibrium state. However, high frequency chattering in the control input is the problem which can cause instability to the system by exciting unmodeled dynamics and even leads to breakdown. Defining second layer sliding mode by terminal sliding mode let control input obtained by integrating the discontinuous signal of conventional sliding mode. In order to achieve this continuous control signal, integral backstepping method is combined with second terminal sliding mode control. Moreover, adaptive method is used to achieve robustness and better response.

First, the whole system dynamic is redefined based of some error variables. The error equation is defined as $e_i = x_i - x_d$ and the integral variable as $\dot{\zeta}_i = e_i$ is added to error dynamics of the system and it will be known as integral augmented state. x_d is the desired command of each state.

The control scheme of proposed control is designed in the following:

The error dynamic of system can be rewritten as following equations

$$\begin{gathered}\dot{\zeta}_{2i-1} = e_{2i-1}\\ \dot{e}_{2i-1} = e_{2i}\\ \dot{e}_{2i} = f_i(e) + g_i(e)u_i + \Delta f_i + d_i\ ,\ i = 1,\dots,4\end{gathered} \qquad (25)$$

Derivative of error equation is

$$\dot{e}_{2i-1} = \dot{x}_{2i-1} - \dot{x}_{(2i-1)d} = x_{2i} - \dot{x}_{(2i-1)d} \qquad (26)$$

Let x_{i+1} be a virtual input which should be stabilized.

Lyapunov candidate is introduced as

$$V_1 = 1/2 \textstyle\sum_{i=1}^{4}(\zeta_{2i-1}^2 + e_{2i-1}^2) \qquad (27)$$

Following state feedback control proposed

$$x_{2i} = -\zeta_{2i-1} + \dot{x}_{(2i-1)d} - \alpha_i e_{2i-1} \qquad (28)$$

Where α_i are positive constants.

Then the derivative of V_i is obtained as below

$$\dot{V}_1 = \sum_{i=1}^{4}\left(\zeta_{2i-1}\,.\,\dot{\zeta}_{2i-1} + e_{2i-1}.\,\dot{e}_{2i-1}\right)$$

$$\dot{V}_1 = \textstyle\sum_{i=1}^{4}\left(e_{2i-1}.\left(\zeta_{2i-1} + x_{2i} - \dot{x}_{(2i-1)d}\right)\right) \qquad (29)$$

Substituting equation (28) to derivative of Lyapunov function, it yields following

$$\dot{V}_1 \le -\textstyle\sum_{i=1}^{4}\alpha_i\,.\,e_{2i-1}^2 \qquad (30)$$

Here, it is assumed that variable z_{2i-1} converges to zero in short finite time via following designed sliding mode method. Where, errors of virtual inputs is defined as $z_{2i-1} = x_{2i} - \mu_i$, where $\mu_i = -\zeta_{2i-1} + \dot{x}_{(2i-1)d} - \alpha_i e_{2i-1}$ and closed loop system is obtained as $\dot{x}_{2i-1} = z_{2i-1} + \mu_i = z_{2i-1} - \zeta_{2i-1} + \dot{x}_{(2i-1)d} - \alpha_i e_{2i-1}$.

The proposed controller is improved based on finite time approach, terminal second order sliding mode and adaptive control. This controller provides the complete compensation of the uncertainty and disturbance of the quadrotor. Since the virtual control and the stabilizing function derived are obviously not equal. First-layer of sliding surface is defined in terms of integral sliding which can converge the errors of virtual inputs to zero. So conventional sliding surface [17] with integral state is defined as following equation [18]

$$s_i = \left(\frac{d}{dt} + \eta_i\right)^{n-1} \int z_{2i-1} \tag{31}$$

Where n is chosen 2, η_i are positive constants and then, it is obtained as

$$s_i = z_{2i-1} + \eta_i \int_o^t z_{2i-1}\, dt \tag{32}$$

The derivative of sliding surface is obtained as

$$\dot{s}_i = \dot{z}_{2i-1} + \eta_i z_{2i-1} \tag{33}$$

and $\dot{z}_i$ is

$$\begin{aligned}\dot{z}_{2i-1} &= \dot{x}_{2i} - \dot{\mu}_i \\ &= f_i(x) + g_i(x)u_i + \Delta f_{\mathrm{i}} + d_i - \dot{\mu}_i \\ &= z_{2i}\end{aligned} \tag{34}$$

Using the derivative of control input for the development of the control law, leads the system to reduce the chattering phenomena. Terminal sliding surfaces are chosen as [19] to guarantee the sliding surfaces converge to zero in finite time and achieve the second-order sliding mode control. In this paper a second-layer terminal sliding mode surface is considered as follows

$$\sigma_i = \dot{s}_i + \beta_i |s_i|^{\gamma_i} sign(s_i) \tag{35}$$

Where β_i are positive constants and $0.5 < \gamma_i < 1$.

When second-layer sliding surface converge to zero each integral sliding surface in Eq. (35) will converge to zero in finite times [20] which the time is

$$t_{r_i} = \frac{|s_i(0)|^{1-\gamma_i}}{\beta_i\,(1-\gamma_i)} \tag{36}$$

While the second-layer terminal sliding surfaces converge to zero, it guarantees the finite time convergence of first sliding surfaces. The derivative of second-layer is obtained as

$$\dot{\sigma}_i = \ddot{s}_i + \beta_i \gamma_i |s_i|^{\gamma_i - 1} \dot{s}_i \tag{37}$$

Where

$$\ddot{s}_i = \dot{z}_{2i} + \eta_i z_{2i} \tag{38}$$

The design of the proposed control scheme is

$$u_i = u_{ei} + u_{si} + u_{ni} \tag{39}$$

Where u_e is equivalent control which is obtained from the derivative of second terminal sliding mode (35), u_s is adaptive switching control which makes second-layer terminal sliding surface stable in finite time and u_n is designed to ensure finite time convergence of tracking error for states and their integrals. Actually, the control input is achieved from integrating discontinuous signal and eliminate chattering effect. Control signals are defined as following based on Eq. (20)

$$\dot{u}_{ei} = g_i^{-1}(x)(-\dot{f}_i(x) - (\dot{g}_i(x).u_i) - \ddot{\mu}_i - \eta_i z_{2i} - \beta_i \gamma_i |s_i|^{\gamma_i - 1} \dot{s}_i) \tag{40}$$

Where $g_i^{-1}(x) \neq 0$ from Eq. (20) and $\dot{g}_i(x)$ are zero for $i = 1,2,3$ in terms of constant inertia.

$$u_{si} = -\int g_i^{-1}.\ \hat{k}_i\, .sign(\sigma_i) \tag{41}$$

Where $\hat{k}_i$ are estimation for $k_i$which are upper bound of uncertainties or disturbance for $i = 1,2,3,4$. And the error between estimate value and real one is defined as $\tilde{k} = \hat{k} - k$. Estimation laws are

$$\dot{\hat{k}}_i = \upsilon_i.|\sigma_i| \tag{42}$$

where$0 < \upsilon_i < 1$.

$$u_{ni} = -(g_i^{-1}).\int \frac{b.\sigma_i}{\|\sigma\|^2}\, dt \tag{43}$$

$$\sigma = [\sigma_1, \sigma_2, \sigma_3, \sigma_4]^T \tag{44}$$

$$b = \sum_{i=1}^4 (b_{1i}|\zeta_{2i-1}| + b_{2i}|e_{2i-1}|) \tag{45}$$

The proposed control law in Eq. (40) and (41) with the adaptation laws in Eq. (42) will guarantee time occurrence of sliding motion, which is proved in the following theorem.

Theorem1. Considering assumption (A2), if the system errors are controlled with control law (39) and the adaptation laws (42), then states of system ζ_i, e_i and sliding surface of virtual control σ_i will move toward zero in a finite time T_{ri} started in any initial point.

$$T_{ri} \le \frac{2\left(\sum_{i=1}^4 \zeta_{2i-1}^2(0) + e_{2i-1}^2(0) + \sigma_i^2(0) + \frac{1}{\Gamma_{\mathrm{i}}}\left(\hat{k}_i(0) - k_i\right)^2\right)^{1/2}}{\min\left(\varepsilon_i\sqrt{2}, \omega_i\sqrt{2\Gamma_{\mathrm{i}}}, b_1\sqrt{2}, b_2\sqrt{2}\right)} \tag{46}$$

Proof. Choose a positive definite function in the form of

$$V_2(t) = V_1 + \frac{1}{2}\sum_{i=1}^4 \left(\sigma_i^2 + \frac{1}{\Gamma_{\mathrm{i}}}\tilde{k}_i^2\right) \tag{47}$$

whereΓ_{i} are positive, $\tilde{k}_i = \hat{k}_i - k_i$ are estimate errors for $i = 1,2,3,4$ and $\dot{k}_i = 0$ because of their slow change. Taking the time derivative of $V_2(t)$,

$$\dot{V}_2(t) = \sum_{i=1}^4 (\dot{\zeta}_{2i-1}\zeta_{2i-1} + e_{2i-1}\dot{e}_{2i-1} + \sigma_i\dot{\sigma}_i + \frac{1}{\Gamma_{\mathrm{i}}}\dot{\hat{k}}_i\tilde{k}_i) \tag{48}$$

Substituting $\dot{V}_1$ from Eq. (29) and the adaptation laws have been shown in Eq. (42), into the Eq. (48), we have

$$\dot{V}_2(t) = \sum_{i=1}^4 (-c_i\, .e_i^2 + \sigma_i\dot{\sigma}_i + \frac{1}{\Gamma_{\mathrm{i}}}\upsilon_i.|\sigma_i|\, .\tilde{k}_i) \tag{49}$$

$$\dot{V}_2(t) = \sum_{i=1}^4 (-c_i\, .e_{2i-1}^2 + \sigma_i(\ddot{s}_i + \beta_i\gamma_i|s_i|^{\gamma_i-1}\dot{s}_i) + \frac{\upsilon_i}{\Gamma_{\mathrm{i}}}.|\sigma_i|\, .\tilde{k}_i) \tag{50}$$

According to the equation (38)

$$\dot{V}_2(t) = \sum_{i=1}^4 (-c_i\, .e_{2i-1}^2 + \sigma_i(\dot{z}_{2i} + \eta_i z_{2i} + \beta_i\gamma_i|s_i|^{\gamma_i-1}\dot{s}_i) + \frac{\upsilon_i}{\Gamma_{\mathrm{i}}}.|\sigma_i|\, .\tilde{k}_i) \tag{51}$$

Substituting Eq. (34) and taking derivative of it, make the following

$$\dot{V}_2(t) = \sum_{i=1}^4 (-c_i\, .e_{2i-1}^2 + \sigma_i\big(\dot{f}_i(x) + g_i(x)\dot{u}_i +$$

$\dot{g}_i u_i + \dot{d}_i + \dot{\Delta f}_i - \ddot{\mu}_i + \eta_i z_{2i} + \beta_i \gamma_i |s_i|^{\gamma_i - 1} \dot{s}_i \big) + \frac{\upsilon_i}{\Gamma_\mathrm{i}} . |\sigma_i| . \tilde{k}_i \big)$ (52)

using assumption (A2) and by introducing u_i,

$$\dot{V}_2(t) \leq \sum_{i=1}^{4} (-c_i . e_{2i-1}^2 + \sigma_i \left(-\hat{k}_i . sign(\sigma_i) - \frac{b . \sigma_i}{\|\sigma\|^2} + D_i \right) + \frac{\upsilon_i}{\Gamma_\mathrm{i}} . |\sigma_i| . \tilde{k}_i)$$

$$\dot{V}_2(t) \leq \sum_{i=1}^{4} (-c_i . e_{2i-1}^2 + \sigma_i \left(-\hat{k}_i . sign(\sigma_i) + k_i sign(\sigma_i) - k sign(\sigma_i) - \frac{b . \sigma_i}{\|\sigma\|^2} + D_i \right) + \frac{\upsilon_i}{\Gamma_\mathrm{i}} . |\sigma_i| . \tilde{k}_i) \tag{53}$$

using the fact $\sum_{i=1}^{4} \sigma_i \left(\frac{b . \sigma_i}{\|\sigma\|^2} \right) = 1$ and $-\hat{k}_i sign(\sigma_i) + k_i sign(\sigma_i) = -\tilde{k}_i sign(\sigma_i)$, it is obtained that

$$\dot{V}_2(t) \leq \sum_{i=1}^{4} (-c_i . e_{2i-1}^2 + \sigma_i \left(-\tilde{k}_i sign(\sigma_i) - k_i sign(\sigma_i) - \frac{b . \sigma_i}{\|\sigma\|^2} \right) + |\sigma_i| D_i + \frac{\upsilon_i}{\Gamma_\mathrm{i}} . |\sigma_i| . \tilde{k}_i) \tag{54}$$

$$\dot{V}_2(t) \leq \sum_{i=1}^{4} \left(-\tilde{k}_i |\sigma_i| - k_i |\sigma_i| + |\sigma_i| D_i - b + \frac{\upsilon_i}{\Gamma_\mathrm{i}} . |\sigma_i| . \tilde{k}_i \right) \tag{55}$$

$$\dot{V}_2(t) \leq \sum_{i=1}^{4} \left(-(k_i - D_i) |\sigma_i| + \tilde{k}_i \left(-|\sigma_i| + \frac{\upsilon_i}{\Gamma_\mathrm{i}} . |\sigma_i| \right) - b \right) \tag{56}$$

According to the Reference [21], There always exists $k_i > 0$ such that $\tilde{k}_i > 0$. It yields from Eq. (56)

$$\dot{V}_2(t) \leq \sum_{i=1}^{4} \left(-(k_i - D_i) |\sigma_i| - \left(|\sigma_i| - \frac{\upsilon_i}{\Gamma_\mathrm{i}} . |\sigma_i| \right) . |\tilde{k}_i| - (b_1 |\zeta_i| + b_2 |e_i|) \right) \tag{57}$$

There always exists k_i and Γ_i such that $k_i > D_i$ and $\Gamma_\mathrm{i} > \upsilon_i$, which yields $\varepsilon_i > 0$ and $\omega_i > 0$. Where $\varepsilon_i = k_i - D_i$ and $\omega_i = |\sigma_i| - \frac{\upsilon_i}{\Gamma_\mathrm{i}} . |\sigma_i|$. Then

$$\dot{V}_2(t) \leq \sum_{i=1}^{4} (-\varepsilon_i |\sigma_i| - \omega_i . |\tilde{k}_i| - b_1 |\zeta_i| - b_2 |e_i|)$$

$$\dot{V}_2(t) \leq \sum_{i=1}^{4} \left(-\varepsilon_i . \sqrt{2} \frac{|\sigma_i|}{\sqrt{2}} - \omega_i . \sqrt{2\Gamma_\mathrm{i}} \frac{|\tilde{k}_i|}{\sqrt{2\Gamma_\mathrm{i}}} - b_1 \sqrt{2} \frac{|\zeta_i|}{\sqrt{2}} - b_2 \sqrt{2} \frac{|e_i|}{\sqrt{2}} \right) \tag{58}$$

And finally,

$$\dot{V}_2(t) \leq - \min\left(\varepsilon_i \sqrt{2}, \omega_i \sqrt{2\Gamma_\mathrm{i}}, b_1 \sqrt{2}, b_2 \sqrt{2} \right) \sum_{i=1}^{4} \left(\frac{|\sigma_i|}{\sqrt{2}} + \frac{|\tilde{k}_i|}{\sqrt{2\Gamma_\mathrm{i}}} + \frac{|\zeta_i|}{\sqrt{2}} + \frac{|e_i|}{\sqrt{2}} \right) \tag{59}$$

$$\dot{V}_2(t) \leq - \min\left(\varepsilon_i \sqrt{2}, \omega_i \sqrt{2\Gamma_\mathrm{i}}, b_1 \sqrt{2}, b_2 \sqrt{2} \right) . V_2^{1/2} \tag{60}$$

Therefore, from lemma (2), the error trajectory $e_i(t)$, the integral error $\zeta_i(t)$ will converge to zero, and errors of virtual control will converge to the second order sliding surface $\sigma_i(t) = 0$ in the finite time $T_{ri} \leq \frac{2\left(\sum_{i=1}^{4} \zeta_{2i-1}^2(0) + e_{2i-1}^2(0) + \sigma_i^2(0) + \frac{1}{\Gamma_\mathrm{i}} \left(\hat{k}_i(0) - k_i \right)^2 \right)^{0.5}}{\min\left(\varepsilon_i \sqrt{2}, \omega_i \sqrt{2\Gamma_\mathrm{i}}, b_1 \sqrt{2}, b_2 \sqrt{2} \right)}$. When second terminal sliding surface converge to zero, virtual control input converge to integral sliding surface $s_i(t) = 0$ in finite time (t_{ri}).

It is quite clear that Lyapunov function candidate like $V_3(t) = \frac{1}{2} \sum_{i=1}^{4} \left(\sigma_i^2 + \frac{1}{\Gamma_\mathrm{i}} \tilde{k}_i^2 \right)$ is also finite time stable. Therefore, variables z_{2i-1} converge to zero in small finite time and equation (30) can be obtained easily.

Remark1. It can be seen in equation (35) that $\dot{u}_i$ contains the term $|s_i|^{\gamma_i - 1} \dot{s}_i$ which has negative functional power $\gamma_i - 1$, for $i = 1,2,3,4$ and so the singularity may occur if $s_i(t) = 0$ and $\dot{s}_i(t) \neq 0$. However, once the system enters the sliding mode, this situation will never occur because when $\sigma_i(t)$ is equal to zero then, leads to $\dot{s}_i = -\beta_i |s_i|^{\gamma_i} sign(s_i)$ and the term change to $|s_i|^{\gamma_i - 1} \dot{s}_i = -|s_i|^{\gamma_i - 1} . \beta_i |s_i|^{\gamma_i} sign(s_i)$ and it obtains as $-\beta_i |s_i|^{2\gamma_{i-1}} sign(s_i)$; and it can be seen that if $0.5 < \gamma_i < 1$, this term will be nonsingular. Therefore, the singularity just may occur in the reaching phase.

To solve this problem, the approach which was proposed in Reference [22] is used.

$$|s_i|^{\gamma_i - 1} \dot{s}_i = \begin{cases} |s_i|^{\gamma_i - 1} \dot{s}_i, & if\ s_i(t) \neq 0\ and\ \ \dot{s}_i(t) \neq 0 \\ |\sigma_i|^{\gamma_i - 1} \dot{s}_i, & if\ s_i(t) = 0\ and\ \ \dot{s}_i(t) \neq 0 \\ 0 & if\ s_i(t) = 0\ and\ \ \dot{s}_i(t) = 0 \end{cases} \tag{61}$$

Remark2. For 3D trajectory tracking, as usually θ_d, ϕ_d and ψ_d have been used for the control of the quadrotor motion in x-y plane. In this paper, the attitude and altitude of the quadrotor have been controlled, but the relation between attitudes and position of the quadrotor can simply be calculated.

5. SIMULATION

Simulation studies have been performed in order to test the proposed control strategy when the quadrotor helicopter goes to special height and rotate around one of its axis and then returns to initial height. Structural uncertainty and external time varying disturbances have been considered. The experiment is designed as following

The quadrotor start from initial attitude $(0.1,0.1,0.1)^T$rad and initial altitude $z_0 = 0$ m begin and at 7s later it goes to 10 m in height and start to rotate around x-axis as sinusoidal function for 23 s and finally, return to initial height.

MATLAB/SIMULINK software is used to solve algebraic equations by ODE45. Table.1 shows

quadrotor parameter and controller parameters have been designed as following.
$\beta_1 = 2, \beta_2 = 2, \beta_3 = 2, \beta_4 = 20$
$\gamma_1 = 3/5, \gamma_2 = 3/5, \gamma_3 = 3/5, \gamma_4 = 3/5$

$\eta_1 = 20, \eta_2 = 2, \eta_3 = 2, \eta_4 = 20$
$\alpha_1 = 10, \alpha_2 = 2, \alpha_3 = 2, \alpha_4 = 3$
$b_{1i} = 1, b_{2i} = 1 \quad \text{for } i = 1,2,3,4$

Following nonlinear functions have been chosen for uncertainty and disturbances.

$$\Delta f_{\mathrm{i}} = \sin(x_{2i}) \text{ for } i = 1,2,3. \tag{62}$$

$$d_4 = d_z(t) = 20\sin(t) \text{ for } 20s < t < 30s. \tag{63}$$

Table 1. Quadrotor parameters

Parameter description	Parameter	Value
Mass of the quadrotor	m	0.65 (kg)
Distance between the mass center and rotors	l	0.23 (m)
Gravitational acceleration	g	9.81(m/s^2)
Moment of inertia around the x-axis	I_x	0.0075 kg m^2
Moment of inertia around the y-axis	I_y	0.0075 kg m^2
Moment of inertia around the z-axis	I_z	0.013 kg m^2
Moment of inertia around the propeller axis	J_p	0.000065 kg m^2

Fig. 1 shows trajectory tracking for height control in presence external disturbances which described in Eq. (62) and occurs in 20 seconds. Time response of quadrotor height shows robustness and suitable transient properties.
Disturbances is compensated soon as soon adaptive coefficient grows in Fig. 2. Control inputs have been illustrated in Fig. 3 and 4 show less sensitivity of response to chattering effect that usually happen in equilibrium point of sliding surface.
Fig. 5 and 6 show transient state response and steady state of Euler angles. Finally controller parameters estimateion have been shown in Fig. 7 in presence of uncertainties. Fig. 8 illustrates the tracking response of angular velocities, as it is obvious they have been converged to their desired value in the presence of uncertainty.

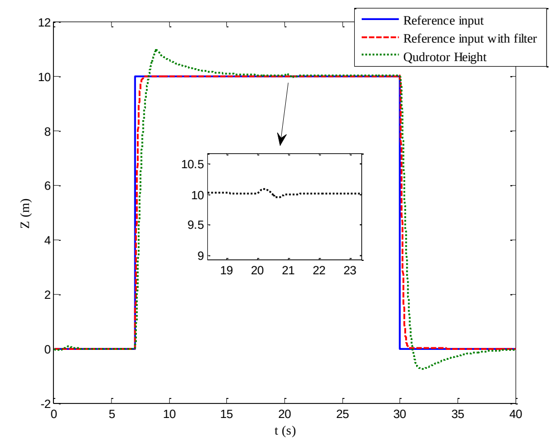

Fig. 1. Time responses of quadrotor height trajectory tracking

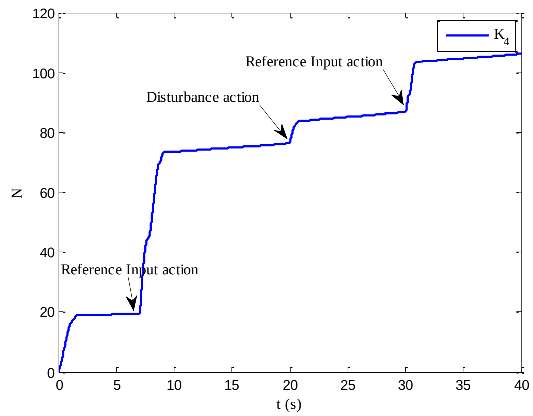

Fig. 2. Time response of k_4 estimation

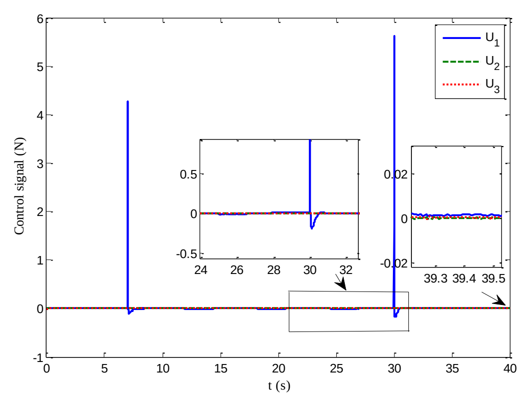

Fig. 3. The control inputs for attitude control

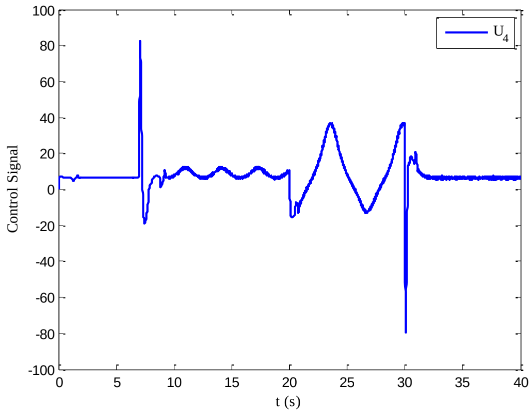

Fig. 4. Control signal for height control

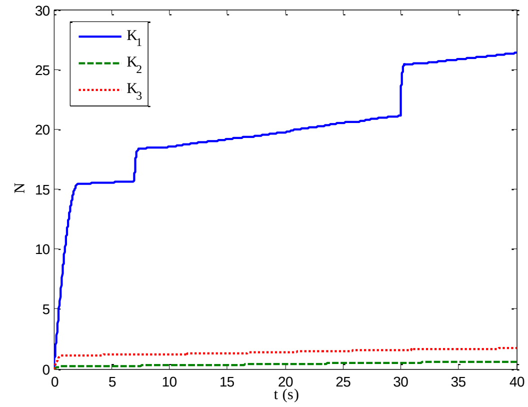

Fig. 7. The estimated parameters $\hat{k}_1$, $\hat{k}_2$ and $\hat{k}_3$.

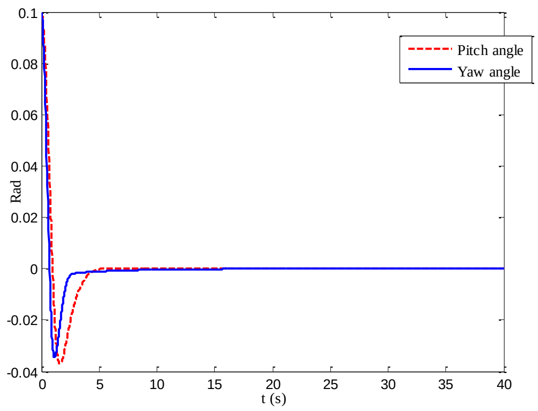

Fig. 5. Time response of the Pitch and Yaw angle.

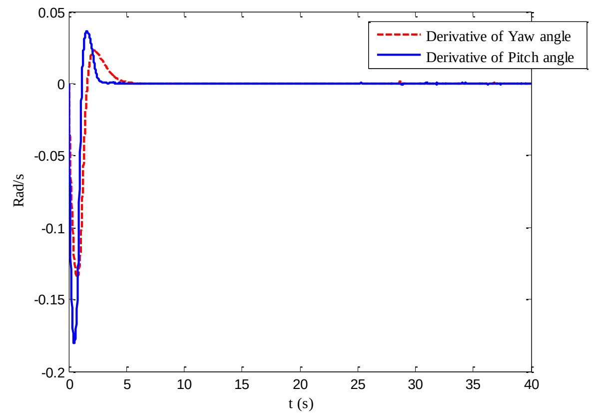

Fig. 8. Time response of the angular velocities for Yaw and Pitch angles.

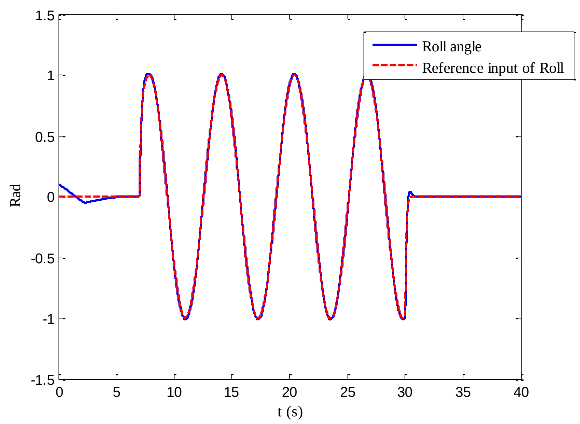

Fig. 6. Time response of the Roll angle.

6. CONCLUSION

In this paper a novel nonlinear control approach has been proposed for the quadrotor attitude and altitude control. Euler angles and the height of the quadrotor have been stabilized globally to track reference inputs with finite time approach. Integral backstepping sliding mode has been combined with terminal sliding mode and adaptive algorithms to achieve the robust and accurate performance. Unknown bounds of limited uncertainty and disturbance have been estimated via adaptive control while a special flight scenario is considered for the quadrotor.

The proposed controller demonstrates robust performance and also improves the transient response of the system's output. Adaptive gains have been converged to upper values of the disturbances and the effects of disturbances have been quickly compensated.

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