Optimal Control of Nonlinear Systems using Multi-Layer Perceptron Neural Network and Adaptive Extended Kalman Filter

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ABSTRACT:
In this paper we present a nonlinear optimal control method based on approximating the solution of Hamilton-Jacobi-Bellman (HJB) equation. Value function is approximated as the output of Multilayer Perceptron Neural Network (MLPNN). Parameters of MLPNN are weights and biases of each layer that form structure of the proposed neural network. These parameters are unknown thus we apply an Adaptive Extended Kalman Filter to approximate unknown parameters. In so doing, the problem of solution of HJB equation is converted to estimation of MLPNN parameters. Also, convergence of the estimation error of MLPNN parameters is proven. Two examples have been brought to show the merits of the proposed approach and to compare the obtained results by applying the multilayer Perceptron and the Radial Basic Function Neural Network (RBFNN).

KEYWORDS: Neural Network, RBF, MLP, Kalman Filter, Optimal Control.

1. INTRODUCTION

Because optimization problems have a basic tool in all areas of applied mathematics, engineering etc. Optimization has been developing in all directions at an astonishing rate during the last few decades [5]-[12]-[15]-[16]-[17]-[18]-[19]. Additionally, in engineering field, the optimal control design for linear time invariant (LTI) systems include the solution of algebraic Riccati equation (ARE) generally. For nonlinear systems, there is one special extension including state dependent Riccati equation technique that provides high performance control but for most nonlinear systems, the optimal control design requires the solution of Hamilton-Jacobi-Bellman (HJB) equation [1]-[2]. Furthermore, we know that some optimization problems have not analytical solution like HJB, thus we have to use approximate techniques to solve them.

One method of approximation of HJB is based on power series. The basic idea is to approximate the value function as truncated power series and to find the corresponding terms of the series by fitting it in HJB equation [3]-[14]. Another approximate method has been developed to start with a stabilizing controller for a given system and then converges point-wise to optimal control. The basis of this technique, a successive Galerkin approximation has been proposed for generalized Hamilton-Jacobi-Bellman (GHJB) equation and it has been shown that the convergence of the successive approximation for optimal control. Also in [1]-[2] several approximation methods are proposed. The difficulty with successive Galerkin and other similar methods is the choice of basis function. It is important for the convergence of the solution to optimal control that the difficulty might be resolved by applying neural network as basis function [4]. Furthermore, neural network can be used for estimating these problems, in [6], nonlinear H_∞ control using radial basic function (RBF) has been reported, the proposed method is on the basis of estimation of value function using nonlinear RBF neural network (RBFNN) by gradient method and offline training.

In [7], authors presented a nonlinear optimal control technique based on approximating the solution of HJB equation by RBFNN and adaptive extended Kalman filter. In [8], authors analyzed the neural networks approach applied to the estimation of chlorophyll concentration in coastal waters and discussed the use of two types of neural networks including the radial basic function neural network and multilayer Perceptron. Thus, studying [8] was a motive to consider estimation of HJB for value function by MLPNN and compare the obtained result with RBFNN. Then in the present paper, HJB equation is solved using Multi-Layer Perceptron (MLP) for optimal value function. The performance of MLP depends on its weights and biases in its layers. However, the weights and biases in hidden layer and output layer are unknown and appear nonlinearly, an adaptive extended Kalman filter method is developed to train the neural network MLP, online with good accuracy. Additionally, convergence of the estimation error of MLPNN parameters has been proven by using the proposed Lyapunov's functions in a Lemma.

The paper is organized as follows: Section 2 states the problem. MLP neural and approximation is described in section 3. Section 4 presents MPL neural network training by adaptive extended Kalman Filter. In section 5, two examples are brought and in these, our proposed method is compared with the obtained results of [7]. Finally, our conclusion is given in section 6.

2. STATEMENT OF PROBLEM

Consider nonlinear time invariant system with

$$\dot{x} = f(x) + g(x)u$$
$$where \quad x \in \Omega \subset R^n, u \in R^m, f : R^n \to R^n$$
$$g : R^n \to R^n \times R^m \tag{1}$$

$x_0 = 0$ is assumed an equilibrium point and $f(x_0) = 0$. This assumption does not decrease the generality of the problem. Also consider the cost function

$$V(x) = \int_0^\infty (x^T Q x + u^T R u) dt \tag{2}$$

Where $Q \in R^{n\times n}, R \in R^{m\times m}$ are two positive definite matrices. Admissible control is u^* which minimizes the performance of index (2).

If optimal control exists, it can be obtained as follows [2]:

$$u^* = -\frac{1}{2} R^{-1} g(x)^T \nabla V^{*^T} \tag{3}$$

Where $\nabla = \frac{\partial}{\partial x}$ is gradient operator, V^* is the value function that satisfies HJB equation with boundary conditions $V^*(0) = 0$.

$$\varpi(\nabla V^*) = \nabla V^{*^T} f + x^T Q x - \frac{1}{4} \nabla V^{*^T} g R^{-1} g^T \nabla V^{*^T} = 0 \tag{4}$$

We can rewrite generalized Hamilton-Jacobi-Bellman (GHJB) equation as follows [1]-[2]:

$$\varpi(\nabla V^*, u^*) = \nabla V^{*^T} (f(x) + g(x)u^*) + x^T Q x + u^{*^T} R u^* = 0 \tag{5}$$

It is clear that the above mentioned equation is linear in terms of ∇V^T.

Also, we know that the solution of GHJB is analytically impossible thus we have to apply approximate technique using equations (3), (5) and the present method to solve GHJB.

3. MLP NEURAL NETWORK APPROXIMATION OF THE VALUE FUNCTION

GHJB equation can be approximated by MLP neural network (MLPNN). In this paper, the neural network has one hidden nonlinear layer and one linear output layer. More hidden layers can be applied but one hidden layer is sufficient in this problem. For more details about MLPNN refer to [10]. Fig. 1 shows the schematic diagram of MLPNN.

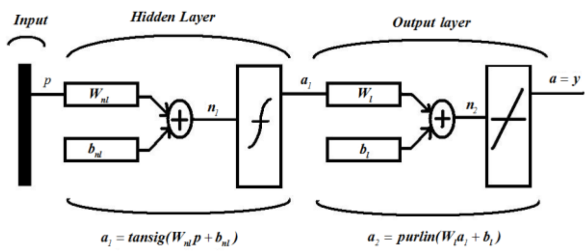

Fig. 1. Schematic diagram of MLPNN [10]

$$a = w_l F(w_{nl} p + b_{nl}) + b_l \tag{6}$$

Where w_l, w_{nl} are corresponding weight vectors and b_l, b_{nl} are biases vector in linear and nonlinear layer. p and a are respectively input and output of the proposed network. Consider active function in nonlinear layer as follows:

$$F(x) = \frac{1-e^{-x}}{1+e^{-x}} \tag{7}$$

The active function can be different from our selection. In Fig.2, the proposed active function is shown.

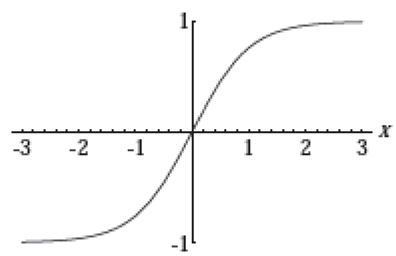

Fig. 2. active function of hidden layer.

Output is a linear layer

$$a = w_l F(x) + b_l \tag{8}$$

We approximate the unknown $V(x)$ using the output of MLPNN. Also we know value function must be positive definite and for this purpose, consider $V(x)$ as follows:

$$V(x)=\frac{1}{2}(x-x_0)\bar{P}(x-x_0)+\frac{1}{2}(a-a_0)(a-a_0) \quad (9)$$

where $a=w_l F(w_{nl}p+b_{nl})+b_l$. $\bar{P}$ is positive definite solution of Riccati equation corresponding to the linearized of (1), a_0 is related to equilibrium point. Clearly a is a function of w_l, w_{nl}, b_l, b_{nl}.

$$a-a_0=w_l^T\psi(w_l,w_{nl},b_l,b_{nl}) \quad (10)$$

Considering

$$\theta=[w_l \quad w_{nl} \quad b_l \quad b_{nl}] \quad (11)$$

By subsisting (10) in (9)

$$V(x)=\frac{1}{2}x^T\bar{P}x+\frac{1}{2}\psi^T(\theta)w_l w_l^T\psi(\theta) \quad (12)$$

If *V(x)* is (12), it can be identified that $V(x)>0$ for all $x\in R^n, x\neq 0 \quad and \quad V(x=0)=0$.

If parameters vector θ can be found so that the proposed approximation of the value function in (2) satisfies the GHJB equation.

The gradient of value function (12) is

$$\nabla V=x^T\bar{P}+\psi w_l w_l^T\nabla\psi \quad (13)$$

Where $\nabla V=\frac{\partial V}{\partial x}$ and $\nabla\psi=\frac{\partial\psi}{\partial x}$. Substituting ∇V in the GHJB we will get:

$$v(\nabla V,u)=(x^T\bar{P}+\psi^T(\theta)\, w_l w_l^T\nabla\psi(\theta))\,(f(x)+g(x)u)+x^TQx+u^TRu \quad (14)$$

It can be rewritten

$$\Upsilon=b(t,\theta)+v \quad (15)$$

Where

$$\Upsilon=-x^TQx-u^TRu-x^T\bar{P}(f(x)+g(x)u) \quad (16)$$

Υ is a known measurable function of x and u, and

$$b(t,\theta)=\psi^Tw_lw_l^T\nabla\psi(\theta)(f(x)+g(x)u) \quad (17)$$

b is a known function of the unknown MLPNN parameter θ. $v=\varpi(\nabla V,u)$ is an equation error according to approximation of value function. We know that the Kalman filter estimates these parameters. The next section explains this estimation.

4. MLPNN TRAINING

Estimating the MLPNN parameters equation (15) can be appeared as a state estimation problem for associated parameters of the given system where the MLPNN parameters are unknown state to estimate. Furthermore, the basis of Kalman filter is a recursive mathematical equation. KF estimates the state process by minimizing the mean of square error. Notice that equation (15) is a nonlinear equation. Thus, we can use an extended Kalman filter [EKF].

$$\dot{\theta}=\omega \quad (18)$$

$$\Upsilon=b(t,\theta)+v \quad (19)$$

Where Υ *and* θ are corresponding measurable output and unknown states of the given system, ω and v are white noise disturbances with covariance matrices Q_f and R_f corresponding the states and output [11]-[13].

$$\dot{\hat{\theta}}=K_f(\Upsilon-b(t,\hat{\theta})) \quad (20)$$

$$K_f=WB^TR_f^{-1} \quad (21)$$

$$B=\left.\frac{\partial b}{\partial\theta}\right|_{\theta} \quad (22)$$

$$\dot{W}=2\alpha W+Q_f-WBR_f^{-1}B^TW \quad (23)$$

where $\alpha>0, Q_f>0, R_f>0$. To increase the region of convergence of EKF, an adaptive extended Kalman filter algorithm is used.

$$\tilde{\Upsilon}=\Upsilon-\hat{\Upsilon} \quad (24)$$

$$K_f=W\hat{C}^TR_f^{-1} \quad (25)$$

Furthermore, the corresponding adaptive output matrix $\hat{C}$ is adjusted as the following:

$$\hat{C}=[\lambda\tilde{\Upsilon}+\rho sign(\tilde{\Upsilon})]\hat{\theta}^T \quad (26)$$

$$\rho=\rho_0+\rho_1\int_0^t\tilde{\Upsilon} \quad (27)$$

$$Where \quad \lambda>0, \hat{C}(0)=\hat{B}(0)=\frac{\partial b(0)}{\partial\hat{\theta}} \quad and \quad \rho_0,\rho_1>0$$
$$\dot{W}=2\alpha W-W\hat{C}^TR_f^{-1}\hat{C}W+Q_f \quad (28)$$

Where $W(0)=W^T(0)>0$.

The convergence of the MLPNN parameters is illustrated in the following lemma:

Lemma1. We consider the dynamics of the MPLNN parameters (19), if unknown parameters are estimated according to (24)-(28), then the output and the parameter errors $\tilde{\Upsilon},\tilde{\theta}$ converge to zero.

Proof. Considering

$$\tilde{\theta}=\theta-\hat{\theta} \quad (29)$$

Substituting (29) in AEKF's formula, we get

$$\dot{\tilde{\theta}}=K_f\tilde{\Upsilon}+\omega \quad (30)$$

Applying power series expansion, $b=b(t,\theta)$ is exhibited as

$$b=b_0+C(t,\theta)\theta \quad (31)$$

Where $b=b(t,0)$ and $C(t,\theta)$ is a nonlinear vector. Also, the estimate of $b=b(t,\theta)$ can be as follows:

$$\hat{b}=b_0+C(t,\hat{\theta})\hat{\theta} \quad (32)$$

Where $\hat{b}=b(t,\hat{\theta})$, we define $\tilde{b}=b-\hat{b}=C\theta-\hat{C}\hat{\theta}$. Then we have:

$$\dot{\tilde{b}}=\dot{b}-\dot{\hat{b}}=C\dot{\theta}+\dot{C}\theta-\dot{\hat{C}}\hat{\theta}-\hat{C}\dot{\hat{\theta}}$$
$$=C\omega+\dot{C}\theta-\hat{C}W\hat{C}^TR_f^{-1}\tilde{\Upsilon}-\dot{\hat{C}}\hat{\theta} \quad (33)$$

By taking derivative from (19) and using (26)-(27) and substituting (33). We get:

$$\dot{\tilde{\Upsilon}} = C\omega + \dot{C}\,\theta - \hat{C}W\hat{C}^T R_f^{-1}\tilde{\Upsilon} - \dot{\hat{C}}\,\hat{\theta} + \dot{v} \quad (34)$$

$$\dot{\tilde{\Upsilon}} = C\omega + \dot{C}\,\theta - \hat{C}W\hat{C}^T R_f^{-1}\tilde{\Upsilon} - \left[\lambda\tilde{\Upsilon} + \rho sign(\tilde{\Upsilon})\right]\hat{\theta}^T + \dot{v} \quad (35)$$

Now, by selecting Lyapunov function $V_\Upsilon = \tilde{\Upsilon}^2$, its derivative must be $\dot{V}_\Upsilon \le -\kappa\tilde{\Upsilon}^2$, $\kappa \ge 0$.

Then we will seek the above condition. Thus we have:

$$\dot{V}_\Upsilon = 2\tilde{\Upsilon}^T\dot{\tilde{\Upsilon}} \quad (36)$$

Equation (35) is substituted in (36) and two variables δ_1 and δ_2 are introduced as:

$$\delta_1 = C\omega + \dot{C}\,\theta + \dot{v} \quad (37)$$

$$\delta_2 = max\left(\left|\hat{C}W\hat{C}^T R_f^{-1}\right|\right) \quad (38)$$

We have

$$\dot{V}_\Upsilon = 2\tilde{\Upsilon}^T(\delta_1 - \hat{C}W\hat{C}^T R_f^{-1}\tilde{\Upsilon} - \left[\lambda\tilde{\Upsilon} + \rho sign(\tilde{\Upsilon})\right]\hat{\theta}^T\hat{\theta} + \dot{v} \quad (39)$$

Thus $\kappa \le 2\delta_2 - \dfrac{\delta_1}{\left|\tilde{\Upsilon}_{max}\right|}$ (40)

Since $C\theta + v$ is bounded and $\tilde{\Upsilon} = \tilde{b} + v = C\theta - \hat{C}\hat{\theta} + v$ converges to zero, because $\hat{C}\hat{\theta}$ has to be bounded and has to converge to $C\theta + v$ then it is obvious $\tilde{\Upsilon}$ converges to zero.

Regarding convergence of $\tilde{\theta}$, consider Lyapunov function as follows:

$$V_\theta = \tilde{\theta}^T\tilde{\theta} + \hat{C}^T\hat{C} \quad (41)$$

We will search $\mu \ge 0$ such that

$$\dot{V}_\theta \le -\mu\tilde{\theta}^2 \quad (42)$$

by replacing (26) in (41), we get:

$$\begin{aligned}\dot{V}_\theta &= 2\tilde{\theta}^T\dot{\tilde{\theta}} + 2\hat{C}^T\left[\lambda\tilde{\Upsilon} + \rho sign(\tilde{\Upsilon})\right]\hat{\theta}^T \\ &= 2\tilde{\theta}^T(-K_f\tilde{\Upsilon} + \omega) + 2\hat{C}^T\left[\lambda\tilde{\Upsilon} + \rho sign(\tilde{\Upsilon})\right]\hat{\theta}^T\end{aligned} \quad (43)$$

By sorting $\dot{V}_\theta$, we get:

$$\dot{V}_\theta = 2(-\tilde{\theta}^T K_f + 2\hat{C}^T\lambda)\tilde{\Upsilon} + 2\rho sign(\tilde{\Upsilon})\tilde{\theta}^T + 2\tilde{\theta}\omega \quad (44)$$

According to the previous proof, the first term converges to zero and by choosing

$$\mu \le \frac{2\rho}{|\theta_{max}|} + \frac{2\omega}{|\theta_{max}|} \quad (45)$$

We are sure that $\tilde{\theta}$ converges to zero.

5. SIMULATION

SISO nonlinear systems have been brought in this section. We apply the proposed algorithm on the first order nonlinear and the third order nonlinear system. Also, we will apply the proposed method and compare the obtained results of cost function with RBF neural networks.

Example 1. Consider the first order nonlinear time invariant regulation problem

$$\dot{x} = x^3 + u \quad (46)$$

Our aim is to design optimal control for (46) to minimize the following cost function:

$$J = \int_0^\infty (x^2 + 2x^4 + u^2)\,dt \quad (47)$$

The optimal control is given by [3],

$$u^* = -x - 2x^3 \quad (48)$$

By using the proposed method and 3 neurons with the introduced active function, we obtain the results that are shown in the following figures. Fig.3 shows state system, Fig.4 shows real cost function and estimated cost function by MLPNN and estimated cost function by RBFNN. The blue curve is a real cost function and the black and red curve are estimated cost functions by MLPNN and RBFNN respectively. By comparison of the estimated cost function by MLPNN and RBFNN, we observe that these two mentioned neural networks estimate the cost function of first order system well.

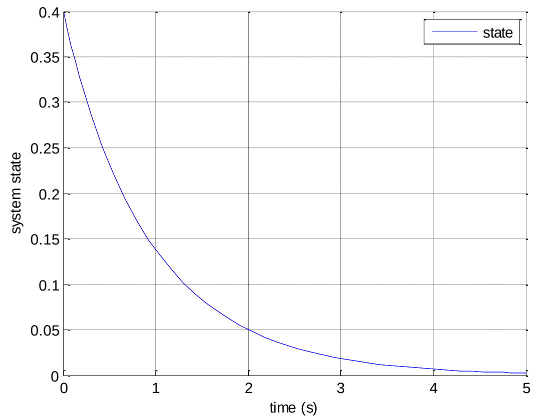

Fig.3. System state

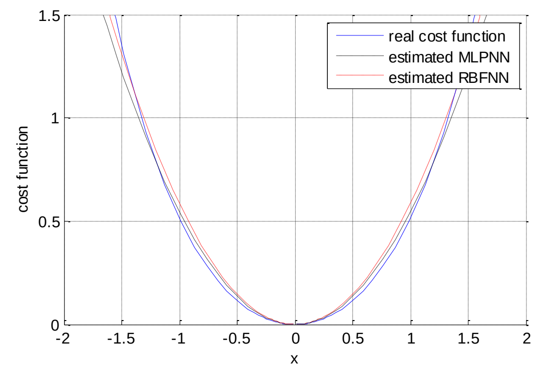

Fig.4. Performance cost functions

Example 2. Consider the problem of regulating the voltage of power generator that is third order system with equation below [9]

$$\begin{aligned}&\dot{\delta} = w(t)\\&\dot{w}(t) = -\frac{D}{2H}w(t) + \frac{w_0}{2H}(P_m(t) - P_c(t))\\&\dot{P}_c(t) = \frac{-1}{T'_{do}}P_c(t)\\&+\frac{1}{T'_{do}}\{\frac{V_s}{x_{ds}}sin(\delta(t))[k_c u_f + T'_{do}(x_d - x'_d)\frac{V_s}{x'_{ds}}w(t)sin(\delta(t))]\\&+T'_{ds}w(t)cot(\delta(t))\end{aligned} \quad (49)$$

Where $\delta(t)$ is the power angle, $w(t)$ is relative speed, $k_c = 1$ is gain of excitation amplifier, $u_f(t)$ is input, $D=5.0$ *and* $H=4$ are constant of per unit damping and the per unit inertia respectively, $w_0 = 100\pi, P_m = 0.9$ are corresponding synchronous machine speed and mechanical input power. $T_{do} = 6.9$ is the direct axis transient short circuit time, $V_s = 0.83 + i\,0.38$ is bus voltage, $x_d = 1.863,\ x'_d = 0.257$ are corresponding direct axis reactance and the direct axis transient reactance of generator, $T'_{do} = \frac{x'_{ds}}{x_{ds}}T_{do}$. $P_m(t)$ is assumed constant and $P_c(t) \to P_m$. We defined following constants:

$$\begin{aligned}&a_1 = \frac{-D}{2H}, a_2 = \frac{w_0}{2H}, a_3 = -\frac{1}{T_{do}}\\&a_4 = \frac{V_s^2(x_{ds} - x'_{ds})}{x_{ds}x'_{ds}} \quad and \quad a_5 = \frac{k_c V_s}{T'_{do}x_{ds}}\end{aligned} \quad (50)$$

So equations convert to:

$$\begin{aligned}&\dot{\delta} = w(t)\\&\dot{w}(t) = a_1 w(t) + a_2(P_m(t) - P_c(t))\\&\dot{P}_c(t) = a_3 P_c(t) + w(t)P_c(t)cot(\delta(t)) +\\&a_4 w(t)sin^2(\delta(t)) + a_4 sin\,\delta(t)u_f\end{aligned} \quad (51)$$

By applying change of variables

$$\begin{aligned}&x = \delta - \delta_0\\&z = P_c - P_m\\&u_f = \frac{u - a_3 P_m}{a_5 sin\,x + \delta}\end{aligned} \quad (52)$$

The result is:

$$\begin{aligned}&\dot{x} = y\\&\dot{y} = a_1 y - a_2 z\\&\dot{z} = a_3 z + y(z + P_m)cot(x + \delta_0) + a_4 y\,sin^2(x + \delta) + u\end{aligned} \quad (53)$$

The objective is to obtain u minimizing the following cost function.

$$J = \int_0^\infty (x^2 + y^2 + z^2 + u^2)dt \quad (54)$$

Using feedback linearization from [9]

$$\begin{aligned}&u_{fl} = -y(z + P_m)cot(x + \delta_0)\\&-a_4 y\,sin^2(x + \delta_0) + k_1 x + k_2 y + k_3 z\end{aligned} \quad (55)$$

Where k is Kalman gain associated with linear system

$$A = \begin{bmatrix}0 & 1 & 0\\0 & a_1 & -a_2\\0 & 0 & a_3\end{bmatrix}, B = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

Q and R must be positive definite matrices

$Q = I_{(3)}$, $R=1$

Therefore the initial control is [9]

$$\begin{aligned}&u = -y(z + P_m)cot(x + \delta_0)\\&-a_4 y\,sin^2(x + \delta_0) + z + 1.0725y - 9.6993z\end{aligned} \quad (56)$$

Five neurons are used in hidden layer for this estimation. Fig.5 shows system states and Fig.6 illustrates the real cost function and two estimated cost functions by MLPNN and RBFNN. To estimate the cost function by RBFNN, we use 11 neurons in the hidden RBF layer. It is obvious that MLPNN has had a better performance than RBFF for estimation of cost function. Additionally, in this estimation, less neurons are used in MLPNN compared to RBFNN, and this presents the preference of MLPNN related to RBFNN to estimate the cost function.

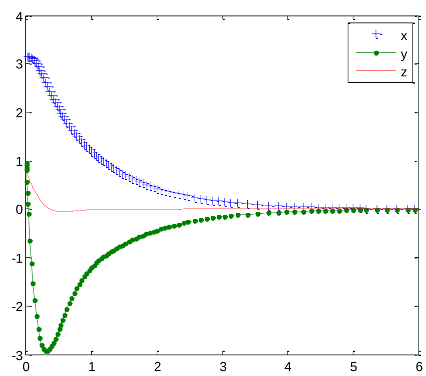

Fig. 5. System states

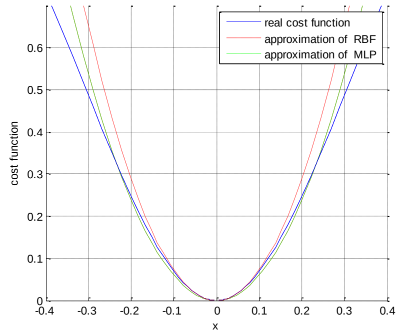

Fig. 6. Performance cost functions

6. CONCLUSION

We have shown an approach to obtain the optimal control law for nonlinear systems by applying neural networks. The proposed method solves the generalized Hamilton-Jacobi-Bellman (GHJB) equation by estimating value function using multilayer Perceptron neural network (MLPNN). The proposed neural network is trained by an Adaptive Extended Kalman Filter (AEKF) which estimates the MLPNN parameters online. We proved estimation parameter errors converge to zero by the Lemma. The method has been applied to the first order and the third order nonlinear system. Also our results have been compared with RBFNN neural network and the exact optimal control. As shown that the applied neurons number in MLPNN were less than the RBFNN. It causes decrease of calculations. But by increasing of neurons number in RBFNN, we could reach the close estimation of MLPNN so we come to this conclusion that MLPNN gives a better estimation to us.

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