State Estimation for a Class of Singular Systems Using Shifted Legendre Polynomials

Zahra Razavinasab[1], MohammadAliVali[2] ,Mahmoud Samavat[1]
1- Department of Electrical Engineering, ShahidBahonar University, Kerman, Iran.
Email: z.razavinasab@eng.uk.ac.ir(Corresponding author), msamavat@mail.uk.ac.ir
2- Department of Mathematics, ShahidBahonar University, Kerman, Iran.
Email: mvali@mail.uk.ac.ir

Received : January 2015 Revised :March 2016 Accepted :June 2016

ABSTRACT:
For linear time-invariant continuous-time singular systems, two new simple approaches (The Kronecker method and the recursive method) are developed in order to estimate the states from the system input-output information via shifted Legendre polynomials (SLP), and a simple observer in the descriptor form. Sufficient conditions for the existence of the present observer are given. These two methods make easy the system of state equations by turning it into the solution of a set of linear algebraic equations. The advantage of these algorithms is their easy implementation in a digital computer, and also solutions can be obtained for any length of time. Further, these approaches include the filtering and the smoothing effect which can reduce the influence of zero-mean measurement noise on estimation. Simulation results of a given numerical example demonstrate the effect of the proposed approaches.

KEYWORDS: Descriptor observer, Shifted Legendre polynomials, Singular systems, State estimation.

1. INTRODUCTION

Singular or descriptor systems describe a wide class of systems. These types of systems are encountered in many areas, such as electrical, social, economical, chemical, mineral, power systems, etc. Therefore a lot of works have been devoted to the analysis and design of techniques of these systems in the past several decades (see, e.g. [1], [2], [3] and [4]).

The problem of designing state observers and signal estimators has a great theoretical and practical importance in the area of control design and signal processing. Luenberger observers [5] and observers in descriptor form [2] are two kinds of many approaches designing an observer for linear time invariant singular systems. Considerable interest (see, e.g. [6], [7], [8] and the references therein) has been shown in studying observer problems of linear singular systems.

A number of orthogonal functions or polynomial series such as block-pulse functions (BPF) [9], shifted Chebyshev polynomials of first kind (SCP1) [10], shifted Legendre polynomials (SLP) [11] and wavelets [12] have been considered to estimate the states for standard (conventional) systems. Orthogonal functions approach has inherent filtering property [12] as it involves integration process which has the smoothing effect [11]. In order to solve the estimation problems for singular systems without using orthogonal functions, some efforts were made in [13], [14] and [15].

In this paper, motivated by the basic idea given in [11] for the state estimation of linear time invariant (LTI) systems via SLP, we extend the method to LTI singular systems. First a simple descriptor observer is used for estimating the states from the system input-output information. Among other observers designed for singular systems, it can be seen that this descriptor observer has a very simple form. Then these estimated states are analyzed via SLP by presenting two new powerful computational methods (The Kronecker method and the recursive method). This paper is organized as follows. Section 2 is dedicated to the preliminaries which involve the properties of SLP and the integration operational matrix of SLP. Also a form of descriptor observer and sufficient conditions for the existence of this observer are given in this section. Section 3 proposed two approaches for the state estimation of LTI singular systems by the use of SLP and descriptor observer which are introduced in section 2. In Section 4, an illustrative example is given to show the applicability and accuracy of the methods. Furthermore, the influence of zero-mean measurement noise on estimation is examined. Concluding remarks are presented in Section 5.

2. PRELIMINARIES

2.1. Properties of the shifted Legendre polynomials

A set of SLP, denoted by $\{\varphi_i(t)\}$ for i = 0,1,2, …, $m-1$, is orthogonal with respect to the weighting function $w(t)=1$, over the interval $[t_0, t_f]$. These polynomials are given by the following recursive formula [11]

$$\varphi_{i+1}(t) = \frac{(2i+1)}{(i+1)}\varphi\varphi_i(t) - \frac{i}{(i+1)}\varphi_{i-1}(t) \tag{1}$$

For $i = 1,2,3,...$,with

$$\varphi = \frac{2(t-t_0)}{t_f - t_0} - 1 \tag{2}$$

$$\varphi_0(t) = 1, \quad \text{and } \varphi_1(t) = \varphi \tag{3}$$

Afunction $f(t)$ that is square integrable on $t \in [t_0, t_f]$ can be represented in terms of SLP as

$$f(t) \cong \sum_{i=0}^{m-1} f_i\varphi_i = f^T\Phi(t) \tag{4}$$

where

$$f = [f_0, f_1, \dots, f_{m-1}]^T \tag{5}$$

$f_i$in (4) is given by

$$f_i = \frac{(2i+1)}{(t_f - t_0)}\int_{t_0}^{t_f} f(t)\varphi_i(t)dt \tag{6}$$

and

$$\Phi(t) = [\,\varphi_0(t), \varphi_1(t), \dots, \varphi_{m-1}(t)]^T \tag{7}$$

is called SLP vector. The integration of the SLP vector can be approximated by

$$\int_{t_f}^{t} \varphi(\tau)d\tau \approx P_{SLP}\,\Phi(t) \tag{8}$$

where

$$P_{SLP} = \frac{(t_f - t_0)}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ \frac{-1}{3} & 0 & \frac{1}{3} & 0 & \dots & 0 & 0 \\ 0 & \frac{-1}{5} & 0 & \frac{1}{5} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{1}{2m-3} \\ 0 & 0 & 0 & 0 & \dots & \frac{-1}{2m-1} & 0 \end{bmatrix}, \tag{9}$$

in whichP_{SLP} is called the integration operational matrix of SLP.

2.2. Descriptor observer for singular systems

A linear time-invariant singular system is considered as follows

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \quad x(0) = x_0, \end{aligned} \tag{10}$$

where$x(t)$ is the n-state vector, $u(t)$ is the r-control vector, $y(t)$ is the p-output vector. $E$is a singular matrix, i.e.,$\det(E)=0$. E and $A \in R^{n\times n}$, $B \in R^{n\times r}$, and $C \in R^{p\times n}$ are the known real matrices satisfying following assumptions [16], [17].

Assumption A1: $\det(sE-A) \not\equiv 0$, $\forall s \in c$ except a finite number of s.

Assumption A2: rank $\begin{bmatrix} sE-A \\ C \end{bmatrix} = n$, $\forall s \in c_+$.

Assumption A3: rank $\begin{bmatrix} E & 0 \\ A & E \\ C & 0 \end{bmatrix} = n + \text{rank}(E)$.

A1 guarantees that the plant (10) is solvable, i.e., the solution to the plant (10) exists and is unique for sufficiently piecewise smooth input functions u and consistent initial values. A2 implies that the triple (E, A, C) is finite detectable. $c$denotes the complex plane and c_+ denotes closed and right-half complex plane. A3 insures the triple (E, A, C) is impulsive observable.

Consider the descriptor observer as follows:

$$\begin{aligned} E\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + K(y - C\hat{x}) \\ &\triangleq \tilde{A}\hat{x}(t) + Bu(t) + Ky(t) \end{aligned} \tag{11}$$

where$\tilde{A} = A - KC$and $\hat{x}$ is the estimated vector of order n. There exists a matrix K such that the pair $(E, A-KC)$ is internally proper and stable if and only if (E, A, C) is impulsive observable and finite detectable [16].

We have the error dynamic in the form of:

$$E\dot{e} = (A - KC)e, \tag{12}$$

where $e = x - \hat{x}$.

As it was discussed above, a matrix K can be chosen such that the error equation (12) or equivalently the pair $(E, A-KC)$ isregular, internally proper andstable if and only if (E, A, C) is impulsive observable and finite detectable.

With Assumptions A2 and A3, e is free of impulsive behavior and the eigenvalues of the observer, given by the roots of characteristic polynomial $\det(sE - A + KC)$, lie at chosen locations in the left-half complex plane. Thus, e goes to zero as t tends to infinity at a rate determined by the chosen roots of characteristic polynomial. The degree of characteristic polynomial is given by:

$$\deg(\det(sE - A + KC)) = rank(E), \tag{13}$$

which is equivalent to the assumption A3. A distinguishing feature of this observer is reconstructing the states of the plant (10) without any prior knowledge of initial conditions.

3. STATE ESTIMATION USING SLP

$\hat{x}(t)$, $u(t)$, $y(t)$ and $\hat{x}(0)$ in (10) can be expressed in terms of m-set SLP

$$\hat{x}(t) \approx \sum_{i=0}^{m-1} \hat{x}_i \varphi_i(t) = \hat{X}\varphi(t), \tag{14}$$

$$u(t) \approx \sum_{i=0}^{m-1} u_i \varphi_i(t) = U\varphi(t), \tag{15}$$

$$\hat{x}(0) \approx \hat{X}_0 \varphi(t), \tag{16}$$

where

$$\hat{X} = [\hat{x}_0, \hat{x}_1, \hat{x}_2, \dots, \hat{x}_{m-1}], \tag{17}$$

$$U = [u_0, u_1, u_2, \dots, u_{m-1}], \tag{18}$$

$$\hat{X}_0 = [\hat{x}_0, 0, 0, \dots, 0\,]. \tag{19}$$

$$Y = [y_0, y_1, y_2, \dots, y_{m-1}], \tag{20}$$

Integrating (10) once with respect to t, using the approximated values of $\hat{x}(t), u(t), y(t)$, $\hat{x}(0)$ and using the matrix of integrationP_{SLP}, gives

$$E\hat{X} - \tilde{A}\hat{X}P_{SLP} = E\hat{X}_0 + (BU + KY)P_{SLP} \tag{21}$$

3.1. Kronecker method

Applying the operation of Kronecker product ($\otimes$) [18] to (21) and rearranging the terms leads to:

$$vec(\hat{X}) = \left(I_m^T \otimes E - {P_{SLP}}^T \otimes \tilde{A}\right)^{-1} vec(D), \tag{22}$$

where

$$D \triangleq E\hat{X}_0 + (BU + KY)P_{SLP}, \tag{23}$$

and the operation of vec, stacks the columns of an appropriate matrix into a single column vector [18]. Finally,$\hat{x}(t)$ can be obtained using (14).

Solving (22) involves inversion of a matrix of $m \times n$ size, which becomes large as the value of m increases. On the other hand, more accurate results can be obtained with increasing the value of m. In the next section we will develop a new recursive method via SLP which will extend those given in [11] for singular systems.

3.2. Recursive SLP method

Substituting matrix P_{SLP} into (22) and rearranging the terms, gives

$$\begin{bmatrix} Z_{00} & Z_{01} & 0 & \dots & 0 & 0 \\ Z_{10} & Z_{11} & Z_{12} & \dots & 0 & 0 \\ 0 & Z_{21} & Z_{22} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & Z_{(m-2)(m-2)} & Z_{(m-2)(m-1)} \\ 0 & 0 & 0 & \dots & Z_{(m-1)(m-2)} & Z_{(m-1)(m-1)} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{m-2} \\ x_{m-1} \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_{m-2} \\ d_{m-1} \end{bmatrix}, \tag{24}$$

where

$$Z_{ij} = \begin{cases} \dfrac{2E}{(t_f - t_0)}, & if \ \ i = j = 1,2, \dots, m-1 \\ \dfrac{A}{(2i+3)}, & if \ i = 0,1,2, \dots, m-2, and \ \ j = i+1, \\ -\dfrac{A}{(2i-1)}, & if \ \ i = 1,2, \dots, m-1, and \ j = i-1, \\ \dfrac{2E}{(t_f - t_0)} - A, & if \ \ i = j = 0, \\ 0 & otherwise \end{cases} \tag{25}$$

$$d_i = \begin{cases} \dfrac{2E\hat{X}_0}{(t_f - t_0)} + B\left(u_0 - \dfrac{1}{3}u_1\right) + K(y_0 - \dfrac{1}{3}y_1) & if \ i = 0, \\ B\left(\dfrac{u_{i-1}}{2i-1} - \dfrac{u_{i+1}}{2i+3}\right) + K(\dfrac{y_{i-1}}{2i-1} - \dfrac{y_{i+1}}{2i+3}) & if \ i = 1,2, \dots, m \\ \dfrac{1}{2m-3}(Bu_{m-2} + Ky_{m-2}) & if \ i = m-1, \end{cases} \tag{26}$$

$\hat{X}$can be obtained as the following recursive relations:

$$R_i = \begin{cases} {Z_{ii}}^{-1} & if \ i = 0, \\ (Z_{ii} + Z_{i,i-1}R_{i-1}Z_{i,i-1})^{-1} & if \ i = 1,2\,, \dots, m-1. \end{cases} \tag{27}$$

$$v_i = \begin{cases} R_0 d_0 & if \ i = 0, \\ R_i\left(d_i - Z_{i,i-1}v_{i-1}\right), & if \ i = 1,2, \dots m-1. \end{cases} \tag{28}$$

$$\hat{x}_{m-1} = v_{m-1},$$
$$\hat{x}_i = -R_i Z_{i,i+1} x_{i+1} + v_i, fori = m-2, \dots, 2, 1, 0. \tag{29}$$

Even though the matrix E is singular, Z_{ii} and $Z_{ii} + Z_{i,i-1}R_{i-1}Z_{i,i-1}$ in (27) turn out to be non-singular. Using the proposed recursive method, the size of the matrix to be inverted is kept to n instead of $m \times n$ as in the case of Kronecker product method. Therefore in the case of recursive method, the size of the matrix becomes much smaller and we will have considerable computational advantages when compared with the Kronecker method. It is worthy to mention that the zero entries in P_{SLP} greatly simplify the solution procedures.

4. ILLUSTRATIVE EXAMPLE

Consider a system in the form of (10) with the following parameters:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \qquad A = \begin{bmatrix} -5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad (30)$$

$u(t) = cos(5t)$, (31)

For this system, $n = 3$ and $rank(E) = 2$. It is easy to verify that the Assumptions A1, A2 and A3 are met. We design a descriptor observer in the form of (11) to have both the eigenvalues at $s = -6$ by choosing

$$K = \begin{bmatrix} 1 & 1 \\ 0 & -5 \\ 0 & -1 \end{bmatrix}, (32)$$

Let $x(0) = [1 \quad -1 \quad 0]^T$, the original states of the plant are given by

$x_1 = e^{-5t}$, $x_2 = -\cos(5t)$, $x_3 = 0$,(33)

At first, the initial estimates for the states are taken as$x(0)$. The estimates of the states are obtained by using the proposed recursive method with $m = 7$. The results are shown in Figs. 1-3. It can be seen that the performance of the proposed algorithm on tracking the states of the system is excellent. Next, the initial estimates for the states are assumed to be $\hat{x}(0) = [0.5 \quad 0.5 \quad 0.5]^T$. In order to exhibit the performance of the proposed algorithm in noisy environments, system output corrupted with measurement Gaussian noise and signal to noise ratio (SNR) is equal to 10. Figs. 4-6 show that the tracking in this case is satisfactory despite the noisy environment. Note that both the Kronecker method and the recursive SLP method have exactly the same results.

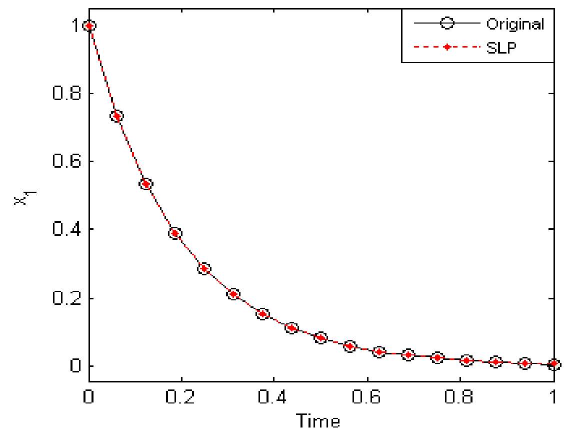

Fig. 1. x_1 and its estimate with known initial states

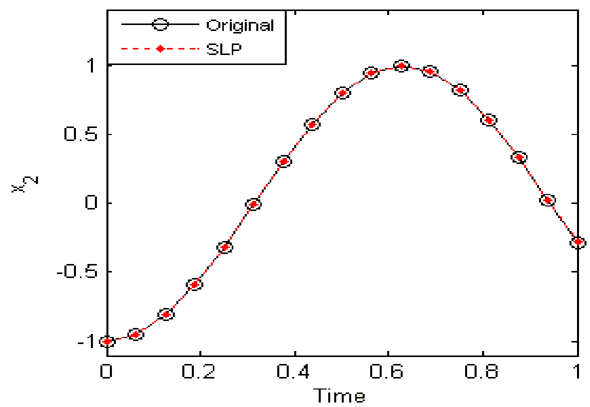

Fig. 2. x_2 and its estimate with known initial states

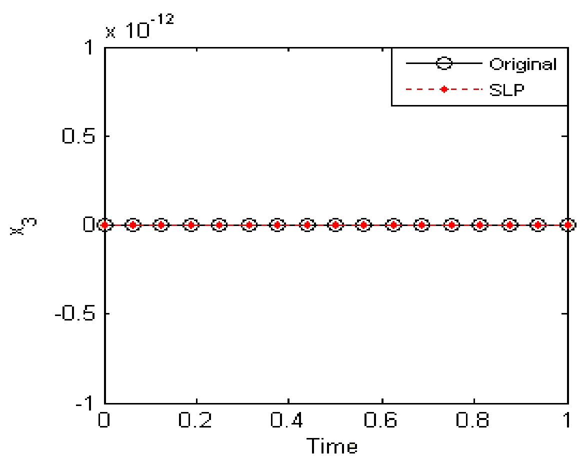

Fig. 3. x_3 and its estimate with known initial states

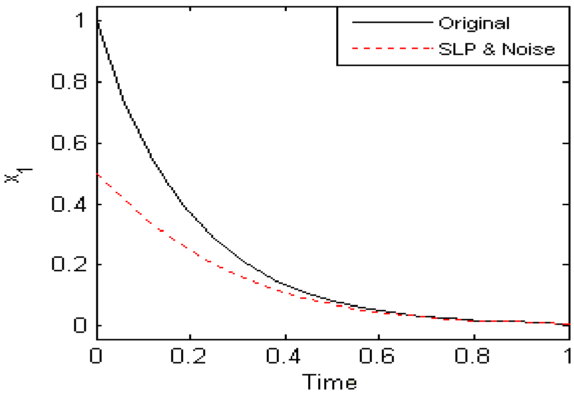

Fig. 4. x_1 and its estimate in noisy environment

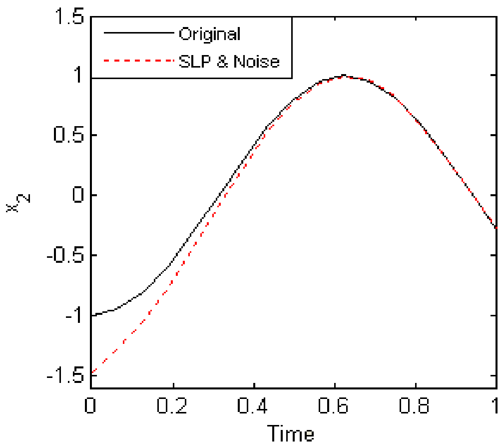

Fig. 5. x_2 and its estimate in noisy environment

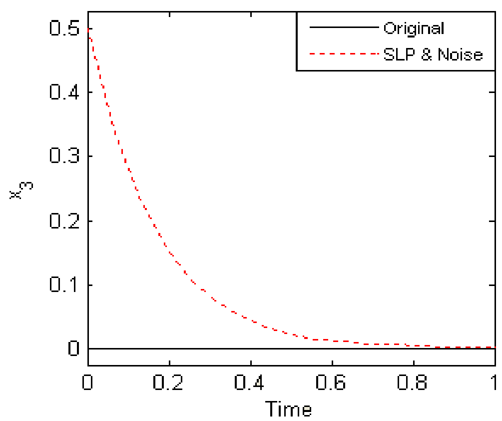

Fig. 6. x_3 and its estimate in noisy environment

5. CONCLUSION

Two algorithms for the state estimation of singular systems have been developed by using SLP and descriptor observer under sufficient conditions. The approaches make easy the system of state equations by turning it into the solution of a set of linear algebraic equations. Furthermore, the proposed algorithms include filtering and smoothing effect which can reduce the effect of zero-mean measurement noise on estimation. Using the recursive SLP method, which is the extension of the method given in [11] for singular systems, the size of the matrix to be inverted is reduced. Hence, it makes this method more attractive computationally than the Kronecker method. Finally, the SLP approach can be developed to estimate the system states and disturbance vector for descriptor systems with both input disturbances and output disturbances by using the descriptor estimator introduced in [16].

REFERENCES

[1] J. D. Cobb,"**Controllability, observability and duality in singular systems**," *IEEE Trans. Autom. Control*, vol. 12, pp. 1076–1082, 1984.

[2] L. Dai, **Singular Control Systems**, *Berlin, Springer*, 1989.

[3] M. Alma, M. Darouach, "**Adaptive observers design for a class of linear descriptor systems**," *Automatica*, vol.50, pp. 578-583, 2014.

[4] Z. Razavinasab, M. Samavat, M. A. Vali, "**Analysis of singular transistor circuits using the orthogonal functions**", *Far East J. of Applied Mathematics*, 61, pp. 117-135, 2012.

[5] M. Hou,P.C. Muller, "**Design of a class of Luenberger observers for descriptor systems**", *IEEE Trans. Automatic Control*, vol. 40, pp. 133–136, 1995.

[6] M. Darouac, M.Boutayeb, "**Design of observers for descriptor systems**", *IEEE Trans. Autom. Control*, vol. 40, pp. 1323–1327, 1995.

[7] A.G.Wu, G.R. Duan,"**IP observer design for descriptor linear systems**", *IEEE Trans. Circuits Syst. II*,vol. 36, pp. 1423–1431, 2006.

[8] M. Darouach, "**Functional observers for linear descriptor systems**", *17th Mediterranean Conf. Control & Automation, Greece, Makedonia Palace, Thessaloniki, June 24-26*, pp. 1535-1539, 2009.

[9] B.M.Mohan, S. K. Kar, "**State estimation using Block-Pulse functions**", *India Conf., Annual IEEE., India, Kanpur, 27 January*, pp. 280-285, 2009.

[10] J.H.Chou, I.R.Horng, "**State estimation using continuous orthogonal functions**", *Int. J. Systems Sci.*,vol. 17, pp. 1261-1267, 1986.

[11] B. M.Mohan,S.K. Kar, "**State Estimation using Shifted Legendre Polynomials**", *the Third int. Conf.Industrial and Information Systems, IEEE Region 10, India, Kharagpur, December 8-10*, pp. 1-6, 2008.

[12] Z. Razavinasab, M.A. Vali,M., Samavat, "**Wavelet analysis of linear optimal control systems incorporating observers**", *Int. J.*

Innovative Computing, Information and Control, vol. 8, pp. 3215-3222, 2012.

[13] K. B. Datta,B. M. Mohan, "**Orthogonal Functions in Systems and Control**", *World Scientific*, 1995.

[14] M. Zasadzinski, M. Darouach, "**State estimation for a class of singular systems**", *Int. J. Systems Sci.*, vol. 23, pp. 517-530, 1992.

[15] M. Darouach, M. Zasadzinski, D. Mehdi, "**State estimation of stochastic singular linear systems**", *Int. J. Systems Sci.*, vol. 24, pp. 345-354, 1993.

[16] Z. Gao, D., Ho, "**State/Noise Estimator for Descriptor Systems With Application to Sensor Fault Diagnosis**", *IEEE Trans. On Signal Proc.,* vol.54, pp. 1316-1326, 2006.

[17] L. Elizabeth, L. Yip, R.F., Sincovec, "**Solvability, controllability, and observability of continuous descriptor systems**", *IEEE Trans. Automatic Control*, vol. 26s, pp.702-706, 1981.

[18] J.W.Brewer,. "**Kronecker products and matrix calculus in system theory**", *IEEE Trans. on Circuits and Systems*, cas-25, (9), pp. 772-781, 1978.