New Structure of Reactive Power Market by Considering Reactive Power Losses

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ABSTRACT

In this paper, a new reactive power market structure is studied and presented. Active power flow by itself causes active and reactive losses. Considering such losses after active power market clearing and in the reactive power market procedure without paying any costs is the main purpose of this paper. For this purpose, new methodology for reactive power structure is proposed which the reactive losses are considered in reactive power market. Therefore, this study attempts to improve the reactive power market and promote fair competition in reactive power generation by improving the market structure. Also, in this work, the cost payment function of synchronous generators, which has an important influence on reactive power market, is modified. In order to stimulate and describe the proposed methods in the implementation of reactive power market, Cigre 32 bus test system and the proposed methods were applied. As will be shown, the total payment by ISO will be reduced by using the proposed methods.

KEYWORDS: Deregulation, Power market, Ancillary services, Reactive power, Power losses.

1. INTRODUCTION

In recent decades, electrical grids have been restructured around the world and changed from the previous exclusively vertical state to the competitive one. This change has been achieved by the complete separation of generation and transmission activities and also development of competition in the generation sector. Such restructuring has led to the separation of different services which had been previously supplied by electricity companies. Although energy exchange is the main purpose of electricity markets, in order to have a secure and reliable electrical grid, ancillary services are vital and should be appropriately supplied. In most of the electricity markets, ancillary services are supplied by system operators via commercial contracts with the market participants.

Among the 6 ancillary services defined in Order No. 888 of Federal Energy Regulatory Commission (FERC)[1], supplying reactive power is one of the most important ancillary services, which has a very effective role in the secure operation of power systems. In a competitive electricity market, the appropriate components of reactive power market are formed by the appropriate selection of the following cases:

1) Market structure, 2) Payment mechanism, 3) Pricing model

Reactive power market structure is chosen according to environmental and political circumstances. Ancillary services of reactive power are usually separated from real power and an independent market is implemented for it. Nevertheless, in some references, by simultaneously performing active and reactive power markets, integrated optimization has been performed on these two costs [2]. In order to prevent the interference of reactive power market and energy market [3-5], independent markets are used for both powers. In this model of reactive power market, the output of active power market is used as the input of this market. Because of different constrains in a reactive power market, the amount of active power cannot be always constant in all generators and has to change in order to maintain the stability of the grid. As a result, one of the important issues in the separated active and reactive power markets is how to face this issue, which is directly related to the lost opportunity cost. In [6-8], by considering a combined objective function, a framework has been presented for optimization on all the active and reactive power costs. Reactive power may be implemented as real time, day-ahead, seasonal, or a combination of the mentioned states. In [3], [7], [9], daily market structure has been followed. In the day-ahead reactive power market, reactive power suppliers declare the amount of generated power as a curve for different hours to independent system operator (ISO). Because of the market sensitivity to load and grid circumstances, the day-ahead reactive power market can make the market power and raise the total cost of the reactive power. Being close to consumption time and, consequently, making more precise predictions about generation and consumption amounts and better allocation of reactive power are the advantages of the day-ahead market. In contrast, in [10-14], the reactive power market has been seasonally implemented. Long-term market implementation solves the problem of creating market power, but cannot precisely predict grid status at consumption time [15]. Proposed a three stage time frame for reactive power which in the first stage the ISO determines the technical requirements of the service considering different scenarios for the next year's period and at the next stage in day a head period after energy and frequency control service prices are determined, the ISO estimates the variable costs associated to the service by evaluating the contingencies required to apply a set of preventive reactive power and voltage control actions. The final stage consists of evaluating real variable costs, once these have been incurred, and added to the fixed costs to conform the total costs of the service.

An appropriate payment structure should be considered for ancillary service providers of reactive power while paying attention to technical (for example, local nature of reactive power, generators' capacity curve, etc.) and economical (generation cost of reactive power for generators including opportunity cost, sale type, market power, etc.) issues. A pay as bid market is proposed [16] and compared with the market clearing price market.

Pricing model is another important point in managing the ancillary services of reactive power and should reflect the generation cost of the reactive power of different suppliers in a non-discriminative way. Besides, the pricing model should be such that the probable suppliers are encouraged to participate in this market. Pricing model refers to the allocation of reactive power costs for different participants. In [3], [9], [17], [18], the pricing model based on the capacity curve of power plants has been employed. In [19] nodal pricing schemes of reactive power is presented to improve reactive power markets.

In [20] by correcting the above-mentioned model, the model of payment cost function was tried to be completed in the reactive power absorption region. Moreover, the payment function was corrected considering the constraints due to the stability and end region heating limit. In [21], to simplify and avoid the complexities of the above model, the quadratic function was used for the payment function of the generators. Although this model facilitated the optimization procedure, it had less accuracy than the previous method. In [18], the cost curve was linearized and modelled as 4 working regions with different line slopes in order to avoid using non-linear functions. To continue the optimization trend, this linearization could remove most of the complexities associated with non-linear methods and was found as a fast and robust method. In [7], [22], by connecting the reactive power to the active one for generator, the cost function was extracted as a quadratic function. By neglecting the initial costs and generation losses of reactive power, in[23], only the lost opportunity cost was taken into consideration.

It is usually mandatory to generate some reactive power by generators in reactive power markets. There are different methods for determining this amount in different markets all over the world. The most conventional method is to use power coefficients for both reactive power absorption and generation regions and generators should supply this amount of reactive power. In this paper, a new method is proposed for considering mandatory generation range of units, which is based on the active power transaction amount between units and loads. Considering the above points, the main innovations of this paper can be represented as follows:

- Presenting new method for pricing model and, as a result, correct clearing of the reactive power market, which is done by modifying Q_{base} (minimum generated reactive power).
- Correct calculation of maximum reactive power for each generator according to the amount of consumed reactive power of units.

In the second section, payment cost function is investigated and modified. In the third section, modelling reactive power losses in the reactive power market is studied. Reactive power market clearing according to the mentioned cases in the two previous sections is studied in the fourth section. In the fifth section, the stimulation results are presented and, finally, in the sixth section, the conclusion is made. Furthermore, the procedure of finding the amount of transaction between generators and customers is explained in Appendix.

2. CORRECTING PAYMENT COST FUNCTION

Reactive power market structure is developed on the EPF of suppliers for their services. The reactive power capacity curve of a generator is shown in Figure 1. In this figure, Q_{base} is the required reactive power of the generator for its accessories. If the working point is within the curve, for example (P_A ,Q_{base}), then the unit can increase its reactive generation from Q_{base} to Q_a without re-changing P_A, which can increase losses in windings and, as a result, raises the cost of losses. If the generator operates in the limiting curve, any kind of increase in Q requires a reduction in P. Assuming that working point A is defined by ($P_{A,}Q_A$) in the curve, if the reactive power of the unit is exceeded, e.g. Q_B, it is necessary for the working point to move backward to B($P_{B,}Q_B$),where $P_{B<}P_A$. This issue indicates that the unit has to reduce its real power. Reduced generation of generator is called lost opportunity cost.

According to this curve, cost function of generators is shown as in Figure 2. In this figure:

$a_{0:}$ Availability of offer price

m_1: Operational offer price for operating in the excited mode (reactive power absorption), $Q_{Min} \le Q \le 0$, \$/MVarh

m_2: Operational offer price for operating in the region $Q_{base} \le Q \le Q_A$, \$/MVarh

m_3Q: Probable offer price for operation in the region $Q_A \le Q \le Q_B$,(\$/MVarh) (MVarh)

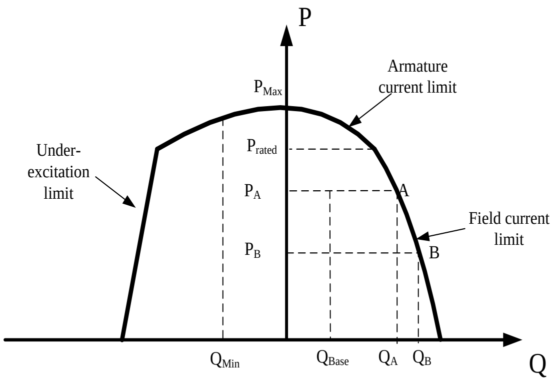

Fig. 1. Capacity curve of synchronous generator [9]

Different functions have been presented for the expected cost function of units[20]. Based on the classification of reactive power generation costs, a total expected payment function and, as a result, a proposed structure can be mathematically formulated as shown below:

$$EPF_i = a_{o,i} + \int_{Q_{Min}}^{0} m_{1i} \times dQ_i + \int_{Q_{base}}^{Q_A} m_{2i} \times dQ_i + \int_{Q_A}^{Q_B} (m_{3i} \times Q_i) \times dQ_i \qquad (1)$$

$$J_{Payment} = \sum_i (\rho_0 \times W_i - \rho_1 \times W_{1,i} \times Q_{1,i}) + \sum_i (\rho_2 \times W_{2,i} \times (Q_{2,i} - Q_{base,i}) + \rho_2 \times W_{3,i} \times Q_{A,i} + \frac{1}{2}\rho_3 \times W_{3,i} \times (Q_{3,i}^2 - Q_{A,i}^2) \qquad (2)$$

Thus far, the most complete function proposed for payment cost function is the one shown above, but this relation has some problems. As mentioned previously, Q_{base} shows the amount of reactive power required for the domestic consumption of power plants. So, this amount of reactive power should not appear in the amount of generated reactive power of the unit for injection into the grid and should be omitted from the maximum reactive power generated in the units. Thus:

$$Q_{A,i}^{new} = Q_{A,i} - Q_{base,i} \qquad (3)$$

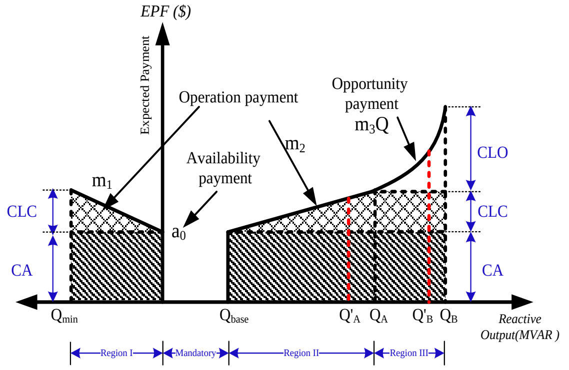

Fig. 2. Structure of reactive offer by suppliers

3. CONSIDERING REACRIVE POWER LOSSES IN COAST FUNCTION

Since the generators with high power exchange with distant loads have a more contribution in losses, the existing markets are not appropriate for market clearing. In other words, a power plant with high active power generation must have a more contribution in reactive power losses and receive power for the reactive power generation exceeding this amount. On the other hand, a power plant close to the consumer does not need reactive power generation to compensate for the reactive power losses and can generate more reactive power. However, in the previous methods, this generator must always generate reactive power within a certain range without considering the load and only receives payment power for the reactive power exceeding this amount. For this objective, first, allocation of reactive power losses as a result of active power flow must be done. In accordance with the described method in [24], reactive power losses can be obtained as follows:

1) Obtaining current of all the branches from the solved load flow;

$$\bar{I}_k = I_{kx} + jI_{ky},\ k = 1,2,....,N_B \quad (4)$$

where N_B is the total number of branches and I_{kx} and I_{ky} show the real and imaginary parts of the mixed current.

2) Assuming the inactivation of T^i transaction, load flow is implemented again and the current of the all the branches is obtained:

$$\bar{I}_k^{Ti} = I_{kx}^{Ti} + jI_{ky}^{Ti},\ k = 1,2,....,N_B,\ i = 1,2,.....,N_T \quad (5)$$

where N_T is the total number of transaction. In this step, the generator is kept active while its active power is equal to zero.

3) As a result, contribution of each transaction Ti in branch k is as follows:

$$\bar{I}_{k,cont}^{Ti} = \bar{I}_k - \bar{I}_k^{Ti} \quad (6)$$

4) Considering the non-linear nature of the grid, when the transactions are implemented simultaneously, the obtained sum in step 3 will not be equal to the amount of step 1.

$$\bar{I}_k \neq \sum_{i=1}^{N_T} \bar{I}_{k,cont}^{Ti} \quad (7)$$

As a result, the following current adjustment coefficient is used to adjust the current obtained in step 3:

$$\bar{I}_k = CAF \sum_{i=1}^{N_T} \bar{I}_{k,cont}^{Ti} \quad (8)$$

5) The new adjusted currents are obtained:

$$\bar{I}_{k,adj}^{Ti} = CAF_k \times \bar{I}_{k,cont}^{Ti} \quad (9)$$

6) Reactive power losses for each transaction is obtained as shown below:

$$Q_{losses}^{Ti} = \sum_{k=1}^{NB} \left\{ \left[\left(I_{kx,adj}^{Ti}\right)^2 + \left(I_{ky,adj}^{Ti}\right)^2 + C_k^{\mathrm{Re}} \times \frac{\left(I_{kx,adj}^{Ti}\right)^2}{I_k^{\mathrm{Re},sum}} + C_k^{\mathrm{Im}} \times \frac{\left(I_{ky,adj}^{Ti}\right)^2}{I_k^{\mathrm{Im}e,sum}} \right] \right\} \times X_k \quad (10)$$

Where:

$$I_k^{\mathrm{Re},sum} = \sum_{i=1}^{NT} \left(I_{kx,adj}^{Ti}\right)^2,\ I_k^{\mathrm{Im},sum} = \sum_{i=1}^{NT} \left(I_{ky,adj}^{Ti}\right)^2 \quad (11)$$

$$C_k^{\mathrm{Re}} = \sum_{i=1}^{NT} \sum_{\substack{j=1 \\ i\neq j}}^{NT} \left(I_{kx,adj}^{Ti} \times I_{kx,adj}^{Tj}\right),\ C_k^{\mathrm{Im}} = \sum_{i=1}^{NT} \sum_{\substack{j=1 \\ i\neq j}}^{NT} \left(I_{ky,adj}^{Ti} \times I_{ky,adj}^{Tj}\right) \quad (12)$$

Statement (9) can be divided into two parts: 1) Reactive power losses caused by active power flow, and 2) Reactive power losses caused by reactive power flow. Considering that the objective of implementing reactive power market is to omit the losses by active power flow, therefore:

$$Q_{losses.p}^{Ti} = \sum_{k=1}^{NB} \left\{ \left[\left(I_{kx,adj}^{Ti}\right)^2 + C_k^{\mathrm{Re}} \times \frac{\left(I_{kx,adj}^{Ti}\right)^2}{I_k^{\mathrm{Re},sum}} \right] \right\} \times X_k \quad (13)$$

In these relations, X_k is the reactance of branch k.

4. REACRIVE POWER MARKET CLEARING

To clear the reactive power market, the units present their offers as (1). When a unit enters the third region, the power plant must reduce its active power; as a result, in this state, the lost opportunity cost must be allocated to it. On the other hand, because of load and generation imbalance in the amount of active power, the amount of imbalance is compensated in the in-spot market. This issue imposes another cost, in addition to the lost opportunity cost, on ISO, which has not been mentioned in the references. Here, to avoid the complexities related to the third region, this region is neglected and the units can only generate in two operational regions. In order to model the losses in the payment structure, these two methods are proposed:

In the second state, simultaneous with declaring the amount of reactive power won by every unit in energy power market, the amount of reactive power which should be compulsively generated by the unit is declared. Market modeling in this state will be as follows:

$$J_{Payment} = \sum_{i}^{NG} (\rho_0 \times W_{0,i} - \rho_1 \times W_{1,i} \times Q_{1,i}) + \sum_{i}^{NG} (\rho_2 \times W_{2,i} \times Q_{2,i}) \quad (14)$$

$$Q_{\min,i} \leq Q_{1,i} \leq 0$$
$$0 \leq Q_{2,i} \leq Q_{A,i}^{new} \quad (15)$$

4.1. Constraints in reactive power market

The aim of implementing reactive power market is to optimize the function (15) while different system constraints are as follows:

4.2. Power flow equations:

$$Pg_i^{con} - Pd_i = \sum_j V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) \tag{16}$$

$$Q_i - Qd_i + QC_i = -\sum_j V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i) \tag{17}$$

4.3. Operational constraints of generators

$$Q_i = Q_{1,i} + Q_{2,i} \tag{18}$$

$$\begin{gathered} W_{1i} \times Q_{Min,i} \le Q_{1i} \le 0 \\ W_{2i} \times Q_{loss,i} \le Q_{2i} \le W_{2i} \times Q_{A,i}^{new} \end{gathered} \tag{19}$$

$$W_{1,i} + W_{2,i} \le 1 \tag{20}$$

$$\begin{gathered} \begin{cases} Q_{A,i} = \sqrt{\left(\dfrac{V_{t,i}E_{af,i}}{X_{s,i}}\right)^2 - (Pg_i)^2} - \dfrac{V_{t,i}^2}{X_{S,i}} \text{ if } Pg_i < Pg_{r,i} \\ Q_{A,i} = \sqrt{MVA_i^{rated} - (Pg_i)^2} \text{ if } Pg_i > Pg_{r,i} \end{cases} \\ Q_{A,i}^{new} = Q_{A,i} - Q_{base,i} \\ Q_{base,i} = 0.1 \times Q_{\max,i} \end{gathered} \tag{21}$$

In Relation (22), V_t is terminal voltage of generator, X_S is synchronous reactance, and E_{af} is internal voltage of synchronous generator, obtained from the following relation:

$$E_{af,i} = \frac{X_{S,i}}{V_{t,i}} \sqrt{Pg_{r,i} + \left(Qg_{r,i} + \frac{V_{t,i}^2}{X_{s,i}}\right)^2} \tag{22}$$

4.4. Determining market prices:

Market prices are separately selected for every reactive power component. The following constraints assure that maximum offer prices are acceptable for a set of given offers:

$$W_{0.i} = W_{1.i} + W_{2.i} \quad \forall i \tag{23}$$

$$W_{0.i} \times a_{o,i} \le \rho_0 \quad \forall i \tag{24}$$

$$W_{1.i} \times m_{1,i} \le \rho_1 \quad \forall i \tag{25}$$

$$W_{2.i} \times m_{2,i} \le \rho_{2a,} \quad \forall i \tag{26}$$

4.5. Constraints of reactive power generation:

$$Q_{Min,i} \le Q_i \le Q_{B,i} \tag{27}$$

$$QC_{Min,i} \le QC_i \le QC_{\max,i} \tag{28}$$

4.6. Security constraints:

$$V_i^{Min} \le V_i \le V_i^{Max} \quad \forall i \tag{29}$$

$$Pg_{slack} \le Pg_{slack}^{\max} \tag{30}$$

According to the mentioned points, the implementation flowchart of the proposed reactive power market can be shown as Figure 3. In this figure, all the market implementation parts are similar and the first and second market types are separated using a different color (red). As can be observed, after determining the amount of active power of the units, reactive power losses are calculated. After determining the losses considering the market type, the offer prices as well as minimum and maximum generating reactive power of the units are presented to ISO. Considering these amounts, ISO clears the market and determines the generation reactive power of the units and the market prices.

5. SIMULATION RESULTS

The proposed reactive power market was tested on a CIGRE 32-bus network (Figure 4). This network had 19 synchronous generators, 1 synchronous condenser placed on bus 4041, and 22 loads. Bus 4011 was considered the reference bus and the rest were PQ bus. The active power of the units determined in the energy markets is shown in Table 1. The information and specifications of the network, including line impedance, maximum and minimum active and reactive power generated in generators, capacity of capacitors, and installed reactors in the network were presented in [25]. For the simulation, as mentioned previously, first, reactive power losses caused by active power flow were obtained and, then, according to this information and other information of the network, the reactive power market was implemented. In this simulation, in order to get reactive power losses, MTLAB software was used and also GAMS software was employed for optimized implementation of reactive power market[26], [27]. The two mentioned software were linked for convenience.

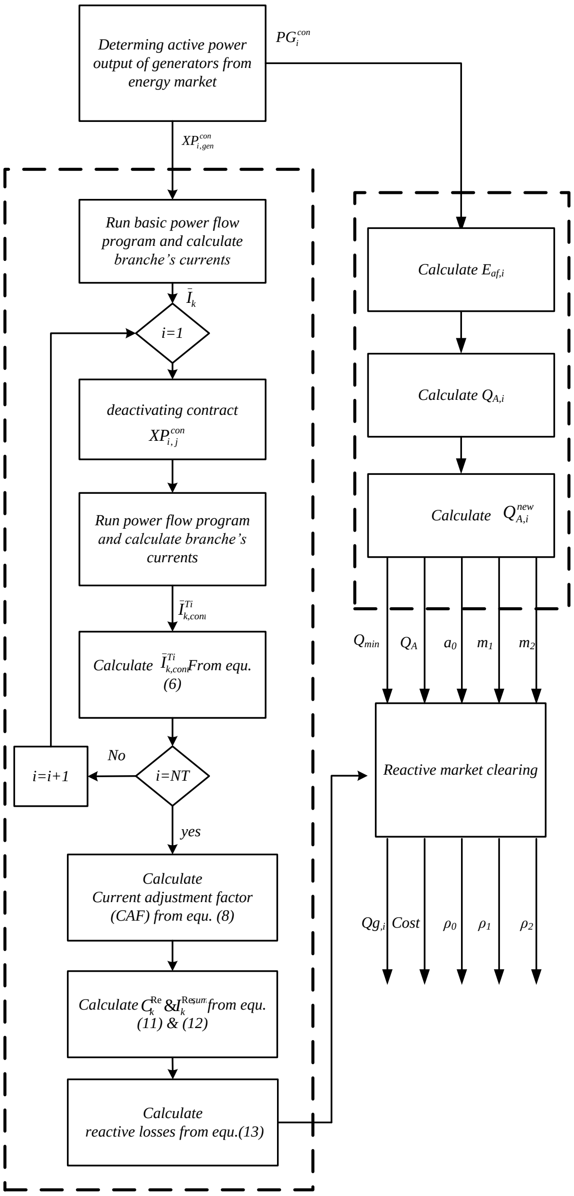

Fig. 3. Flowchart of the proposed reactive power market

Every participant in the reactive power market presents three components to the market, which are shown in Table 2. Minimum and maximum voltage limits considered for PQ and PV buses were 0.95 and 1.05 as well as 0.95 and 1.10, respectively.

In Table 1, the amount of active power by every generator winning in the energy market and the amount of reactive power losses caused by this energy exchange are shown.

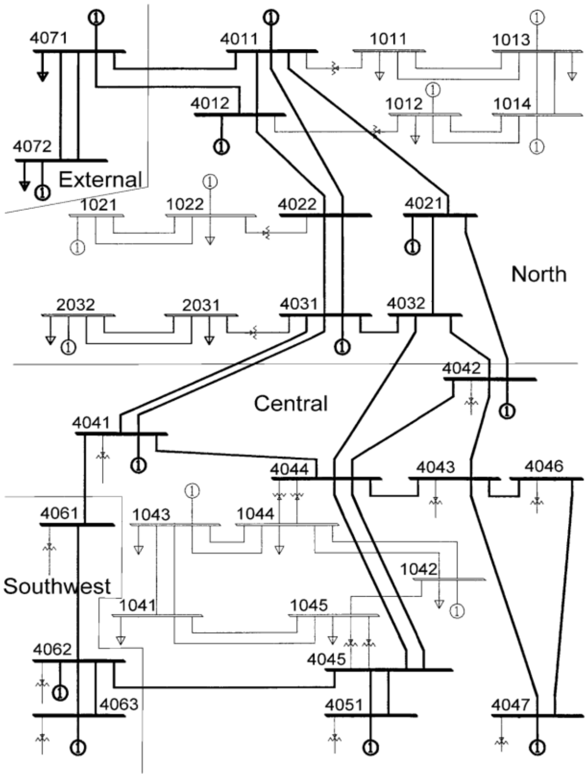

Fig. 4. Nordic 32-bus grid

Reactive power market clearing is an MINLP problem. This model is solved using GAMS, which is strong software for solving these problems, and SBB solver. Considering the offer prices and the presented market model, the market prices and total prices by ISO for both offer markets will be as in Table 3. As can be observed in this table, although a more amount of power was generated compulsively in the second state, the total market cost was less in the first state than the second one and it was more cost-effective, which could be because of the non-linear equations and network complexities.

Table 1. Amount of active power transaction and its corresponding reactive power losses

Bus number	Contracted active power	Allocated reactive losses
4071	2.27	0.0749
1013	2.96	0.3940
4012	3.78	0.1701
1012	3.88	0.1720
1014	3.5	0.5086
4072	22.41	3.900
1021	2.54	0.0899
1022	1.06	0.0063
4021	1.44	0.0139
2032	3.61	0.0818
4031	1.61	0.0217
4042	3.28	0.0865
4041	0	0
1043	0.92	0.0156
1042	1.8	0.0279
4062	2.90	0.0565
4063	5.87	0.1888
4051	3.25	0.1214
4047	5.37	0.3159

Table 2. Offers of generators in the reactive power market

Bus number	Offered prices		
	a_0	m_1	m_2
4071	0.4	0.41	0.41
4011	0.77	0.75	0.75
1013	0.50	0.54	0.54
4012	0.43	0.41	0.41
1012	0.42	0.42	0.42
1014	0.69	0.68	0.68
4072	0.96	0.86	0.86
1021	0.65	0.77	0.77
1022	0.88	1.03	1.03
4021	0.91	1.29	1.29
2032	0.73	1.12	1.12
4031	0.85	1.17	1.17
4042	0.90	1.26	1.26
4041	0.73	1.03	1.03
1043	0.77	0.90	0.90
1042	0.50	0.65	0.65
4062	0.76	1.05	1.05
4063	0.90	1.16	1.16
4051	0.50	0.76	0.76
4047	0.92	1.11	1.11

Table 3. Reactive power market clearing

	Availability Price, a_0	Operation Price, ρ_1	Operation Price, ρ_2	Total Cost
prices	0.96	0.00	1.29	32.6464

The reactive power won by every generator in the market and also maximum reactive power generated by every generator are presented in Table 4. The amounts of reactive power won by every generator in both markets are given in this table. As can be observed, a less number of generators could win in the market in the second type.

6. CONCLUSION

In this paper, a new method was presented for reactive power market clearing, in which the reactive power losses caused by active power implementation were considered and, thus, new methods were presented for reactive power market clearing. In this method, ISO calculated the amount of reactive power losses after implementing the energy market and these amounts were used in the reactive power market. ISO declared the reactive power losses simultaneously with the amount of active power won by every generator in the energy market. Since a large amount of the reactive power losses was compensated for using by this new method, then the payment costs by market were reduced. Consequently, the proposed method not only could improve justice among the market participants, but also could reduce the payment cost by ISO. Also, the mentioned method, due to less allocation of losses to the producers with fewer transactions in the energy market or those who exchange power with their close consumers, could encourage producers to effectively participate in the reactive power market.

Table 4. Amount of reactive power generated in each power plant

	Bus number	Q_{min}	Q_{base}	Q_A	Q_A^{new}	Q_g
1	4071	-0.84	0.17	3.66	3.49	0
2	4011	-1.67	0.333	9.98	9.64	0
3	1013	-1.00	0.2	4.04	3.84	0
4	4012	-1.33	0.267	5.61	5.35	5.348
5	1012	-1.33	0.267	5.49	5.22	2.306
6	1014	-1.17	0.233	4.62	4.39	0
7	4072	-7.5	1.5	29.92	28.42	3.9
8	1021	-1.00	0.2	4.63	4.43	1.449
9	1022	-0.67	0.134	1.93	1.79	1.792
10	4021	-0.50	0.1	2.08	1.98	1.978
11	2032	-1.42	0.283	6.56	6.27	2.79
12	4031	-0.59	0.117	2.54	2.42	2.419
13	4042	0.00	0.233	4.98	4.75	0
14	4041	-2.00	0.3	3.00	2.70	0
15	1043	0.00	0.067	1.45	1.39	0
16	1042	0.00	0.133	2.95	2.82	0
17	4062	0.00	0.2	4.13	3.93	0
18	4063	0.00	0.4	8.16	7.76	0
19	4051	0.00	0.233	5.02	4.78	0
20	4047	0.00	0.4	8.89	8.49	1.4

Appendix

Calculating active power transaction amount between the generator and load

In the bilateral electricity market, it is assumed that the sale and purchase transactions between independent producers and consumers have been signed in advance. A bilateral exchange matrix "XP" which shows all the combinations of transactions between the parties is made to approximately show this exchange. In this section, the model of this bilateral exchange is described in detail. The sum of all the transactions by the generator is equal to the total generation by the party to that generator.

The bilateral transaction between generator j and consumer I is modeled according to the following steps:

- Consumption per bus is given as Pd_i
- A random number with normal flow RPg_i is allocated per generator in (0.8 $Pmax_j$, $Pmax_j$) region, where $Pmax_j$ is upper limit of generator in bus j. To correspond total generation to total demand, Pj^{con} is adjusted per generator as follows [28]:

$$Pg_j^{con} = \frac{RPg_j}{\sum_j RPg_j} * \sum_i Pd_i \tag{31}$$

- Therefore, sum of all Pg_j^{con} will equal sum of Pd_i .
- A random number with normal flow RTr_{ij} is allocated per transaction between a generator and a consumer in (0, Pg_j^{con}) region.
- Size change of the random number RTr_{ij} for the corresponding demand per bus. Here, an initial amount is obtained from the transaction:

$$TR_{i,j} = \frac{RTr_{i,j}}{\sum_j RTr_{i,j}} * \sum_i Pd_i \tag{32}$$

- Final exchanged power transaction XP_{iJ}^{con} will be finally as follows:

$$XP_{i,j}^{con} = TR_{i,j} + (Pg_j^{con} - \sum_i TR_{i,j}) * \frac{TR_{i,j}}{\sum_i TR_{i,j}} \tag{33}$$

According to the above relation, it can be inferred that the number of transactions will be N_G*N_L which is equal to N_T. It can be proved that these random transactions satisfy two rules: (A) Sum of all transactions with one consumer is equal to Pd_i, and (B) Sum of all transactions with one generator in bus j is equal to Pg_j^{con} [28].

Symbols

$a_{0:}$ Availability of offer price

m_1: Operational offer price for operating in region $Q_{Min} \leq Q \leq 0$, \$/MVarh

m_2: Operational offer price for operating in the region $Q_{base} \leq Q \leq Q_A$, \$/MVarh

m_3Q: Probable offer price for operation in the region $Q_A \leq Q \leq Q_B$, MVarh)/(\$/MVarh)

i,j: Indices of buses

Pg^{con}: Real power generation by transaction

Q: Reactive power generation per bus

Qd: Reactive power demand per bus

V: Bus voltage

Y: Element of admittance matrix of the grid

θ: Appropriate angle for Y

ρ_0: The Uniform availability price

ρ_1: The Uniform operating price for absorbing reactive

ρ_2: The Uniform operating prices for producing reactive power

ρ_3: The uniform opportunity price for reactive power

W_0 The binary variables for the discrete selection of a reactive power component if it is selected from any Region

W_1 The binary variables for the discrete selection of a reactive power component from Region-I

W_2 The binary variables for the discrete selection of a reactive power component from Region-II

W_3 The binary variables for the discrete selection of a reactive power component from Region-III

$XP_{i,j}^{con}$ The contracted real power transactions by load at bus i and generator j.

X_S Synchronous reactance

X Reactance of a transmission line

V_t Voltage at the generator terminal bus

QMin Lower limit of reactive power generation

QBase Reactive power required by generator for its auxiliary equipment

QC Reactive power support from shunt capacitors, p.u.

$\bar{I}_k$ Current of the branch k

N_B Total number of branches

I_{kx} The Real part of the I_k.

I_{ky} The Imaginary part of the I_k.

N_T Total number of transaction

$\bar{I}_{k,cont}^{Ti}$ Contribution of transaction Ti in branch k

CAF Current adjustment coefficient

$\bar{I}_{k,adj}^{Ti}$ New adjusted current

Q_{losses}^{Ti} Reactive power losses for each transaction

$I_{kx,adj}^{Ti}$ Real part of I_{adj}^{Ti}

$I_{ky,adj}^{Ti}$ Imaginary part of I_{adj}^{Ti}

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