Create Balance on Bandwidth Consumption using Network Coding in Wireless Sensor Networks

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Received: November 2019 Revised: January 2020 Accepted: March 2020

ABSTRACT:
In recent years, Network Coding (NC) has been used to increase performance and efficiency in Wireless Sensor Networks (WSNs). In NC, Sensor Nodes (SNs) of network first store the received data as a packet, then process and combine them and eventually send them. Since the bandwidth of edges between SNs is limited, management and balancing bandwidth should be used for NS. In this paper, we present an optimization model for routing and balancing bandwidth consumption using NC and multicast flows in WSNs. This model minimizes the ratio of the total maximum bandwidth to the available bandwidth in network's edges and we use the dual method to solve this model. We also use the Karush–Kuhn–Tucker conditions (KKT) to calculate a lower bound and find the optimal solution and point in optimization model. For this purpose, we need to calculate the derivative of the Lagrangian function relative to its variables, in order to determine the condition as a multi-excited multi-equation device. But since the solution of equations KKT is centralized and for WSNs with a large number of SNs, it is very difficult and time consuming and almost impractical, we provide a distributed and repeatable algorithm for solving proposed model in which instead of deriving derivatives, combination Sub-gradient method and network flow separation method are used, thus allow each SN locally and based on the information of its neighboring nodes performs optimal routing and balances bandwidth consumption in the network. The effectiveness of the proposed optimization model and the proposed distributed algorithm with multiple runs of simulation in terms of the number of Source SNs (SSNs) and Lagrange coefficient and step size have been investigated. The results show that the proposed model and algorithm, due to informed routing and NC, can improve the parameters of the average required time to find the route optimal, the total amount of virtual flow in network's edges, the average latency end-to-end of the network, the consumed bandwidth, the average lifetime of the network and the consumed energy, or not very weak compared to other models. The proposed algorithm also has great scalability, because computations are done distributed and decentralized, and there is an insignificant dependence between the SNs.

KEYWORDS: Wireless Sensor Networks; Consumption Bandwidth; Network Coding; Virtual Multicast Flow; Optimization Model.

1. INTRODUCTION

The tasks of each wireless sensor node include collecting, processing, and storing sensed data from the perimeter environment of WSNs and sending processed data by other SNs to sinks [1]. The limitations of WSNs include energy resources, memory, and bandwidth [2]. The maximum flow of transferable in WSNs is equal to the maximum amount of data that can be sent from a SSN to a set of destination SNs. In [3], the theorem Max-Flow Min-Cut and the method of calculating the maximum flow of transferable in graph theory are presented, which is equal to capacity of the minimum of cutting between the source and destination SNs. In traditional routing methods, the maximum flow of transferable cannot be reached, because they consider that the flow of data within the network is similar to the flow of fluids in the transmission pipelines, but using NC, hop nodes can process and combine imported packets and sent them as multicast and they use the maximum network capacity [4]. Since in WSNs, wireless channels have a multicasting feature, NC improves network performance. The left side Fig. 1 shows an example of a butterfly network that capacity and bandwidth of each its edge are equal to one, so that the maximum of flow of transferable between the SSN, S_1 and the destination nodes S_6 and S_7 is equal to two, so we can send maximum two packets in each unit to both destination nodes S_6 and S_7. However, using traditional routing cannot reach the maximum flow of two packets per unit time. For this

Paper type: Research paper
DOI: https://doi.org/10.29252/mjee.14.3.3
How to cite this paper: E. Kharati, **"Create Balance on Bandwidth Consumption using Network Coding in Wireless Sensor Networks"**, *Majlesi Journal of Electrical Engineering*, Vol.14, No. 3, pp. 23-38, 2020.

purpose, according the right side Fig. 1, the node S_4 instead of sending separately and appropriately packets P_1 and P_2, encodes and XOR them together as $P_1 \oplus P_2$ and sends to the node S_5 and node S_5 simultaneously sends them to both destination nodes S_6 and S_7. As a result, the maximum flow of two packets per unit time can be reached, using the NC.

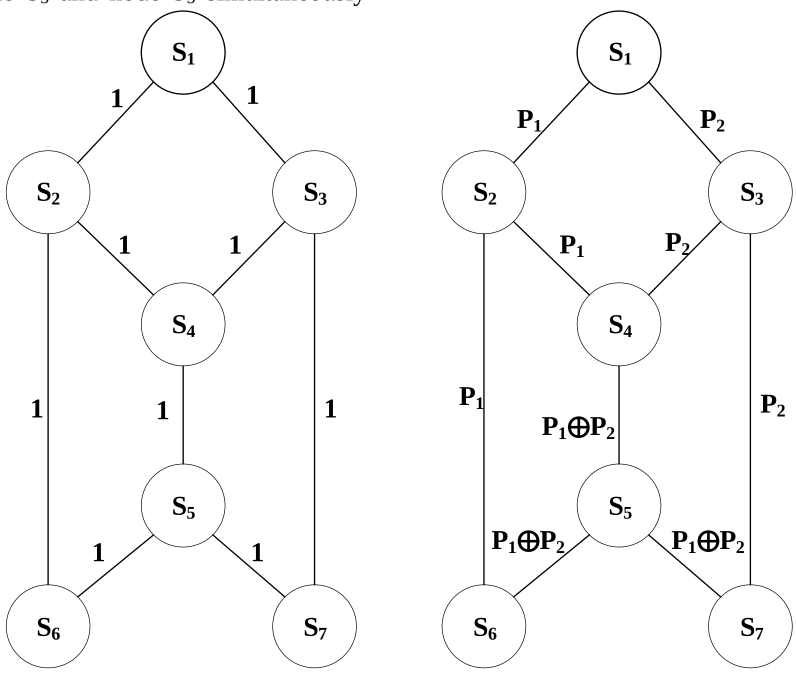

Fig. 1. A Butterfly Network
Left: with a maximum flow of two packets per time unit from the source node S_1 to the destination nodes S_6 and S_7 [5]
Right: How to achieve the maximum flow of two packets per unit time using NC [6].

In this paper, we present an optimization model for routing and balancing in bandwidth consumption using NC and multicast flows in WSNs. This model minimizes the ratio of the total maximum bandwidth to the available bandwidth in network's edges and uses the dual method to solve this model. We also use the KKT conditions to calculate a lower bound and find the optimal solution and point in optimization model. For this purpose, we need to calculate the derivative of the Lagrangian function relative to its variables, in order to determine the condition as a multi-excited multi-equation device. But since the solution of equations KKT is centralized and for WSNs with a large number of nodes, it is very difficult and time consuming and almost impractical, we provide a distributed and repeatable algorithm for solving a proposed model in which instead of deriving derivatives, the combination of Sub-gradient method and network flow separation method is used, thus allowing each node locally and based on the information of its neighboring nodes, it performs optimal routing and balances bandwidth consumption in the network.

The rest of this article is structured as follows. Section 2 includes introducing and investigating NC in WSNs. Section 3 involves presenting an optimization model using NC to balance the bandwidth consumption and find the optimal route. In section 4, a distributed and repeatable algorithm is proposed to solve the problem of choosing the path with NC, taking into account bandwidth constraints in WSNs. In section 5, simulation, comparison and evaluation of the effectiveness of the proposed model and algorithm are discussed and finally, in section 6, conclusions are made and suggestions for further research are presented.

2. RESEARCH BACKGROUND

The reference [4] introduced the COPE architecture to send coded packets in WSNs, which can significantly increase the performance. This architecture uses NC between the IP and MAC sessions, sending multicast packets, and XORs data flows together, so implementation is easy. In [5], authors used a combination of NC and topology control that increased efficiency in WSNs and, in reference [6], authors used the combination of NC and motion control of sink moving, which would increase efficiency in WSNs. In [7], it has been shown that the use of NC maximizes the transmission flowing in the network. In [8], it is showed that to reach the maximum multicast flow capacity for each destination, NC should be used. NC reduces traffic flow and shares all the nodes in sending packets, and as a result, traffic volume is balanced and therefore reduces energy consumption and increases the lifetime of WSNs. Also, because in NC, packets are sent from multiple paths, it increases reliability and security. Since the required energy to compute and combine packets in the middle SNs is much less than the required energy to send and receive packets, NC increases efficiency or performance and reduces bandwidth consumption, latency, complexity, and costs in the network. Fig. 2 shows a variety of methods NCs in WSNs [9].

The NC disadvantage, memory limitation, and buffer overflow in nodes of WSNs and increased traffic in the network bandwidth are due to sending packets from different paths. In NC, the hop nodes produce a series of encoded packets, and destination nodes must be able to detect and decode these packets by the Gaussian elimination method [10]. The complexity of the Gaussian elimination method is of degree three, and as the number of SSNs in the WSN is higher, NC is impossible and impractical. The other problem with Gaussian elimination method is the limited number of encoded packets, and if the number of encoded packets is less than the limited number, the number of decoded packets would be almost zero [11]. To overcome these problems and to perform optimal NC, we must use more nodes with more computational power or we limit the number of encoder hop nodes that selection of the minimum number of nodes with the ability NC is a NP-hard problem [9].

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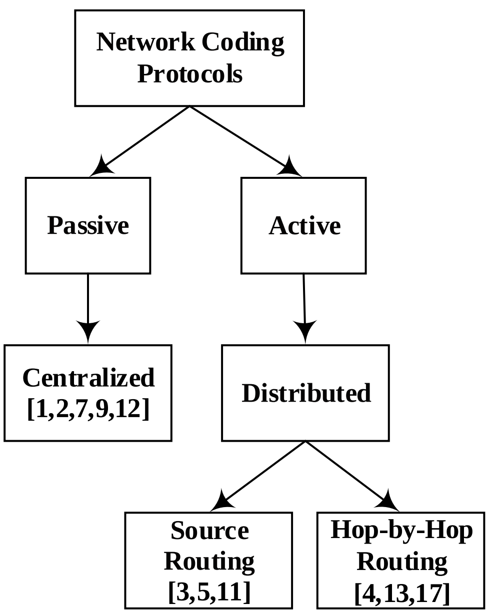

Fig. 2. Types of NC Methods in WSNs [9].

2.1. Contributions in this Article

The most important works done in this article compared with previous articles are as the following:

- The goal of proposed optimization model is to create balancing and reduce bandwidth consumption, which is the generalized Integer Linear Programming model (ILP) in [8, 12].
- The proposed model is independent of the density and deployment, the sending domain, and the energy model of the SNs, and its parameters include the rate of production and sending of data.
- The proposed algorithm with appropriate complexity, distributed and repeatable and based on the information of the neighboring nodes, balances the bandwidth consumption in network's edges, which greatly increases the scalability of WSNs.
- Investigating the effect of increasing the number of SSNs and Lagrange coefficient and step size in the proposed model and algorithm on the average required time to find the optimal path, the total amount of virtual flow in network's edges, the average latency of end-to-end network, the consumed bandwidth, the average lifetime of the network and consumed energy.

3. SYSTEM MATHEMATICAL MODEL

In this paper, in order to simplify problem and the proposed optimization model, we assume that environment is open and flat, and radio coverage is completely regular, which in the future can be used in real-world conditions, such as inside the building or under hard conditions, and irregular and intermittent radio coverage. For the modeling of WSN, such as articles [8], [12], we use the model of graph G=(N,E) and supergraph G=(N,A), which N is the finite set of nodes, E is the finite set of edges, and A is the set of superedges. An edge of a node such as i starts and ends with another node, such as j, and it is displayed with (i,). A superedge consists of a set of edges that starts with a node such as i and ends with $\mathcal{J}i$ or a set of neighbors or nodes that are within the sending domain of the node i represented by (i,). Sending data using NC involves two stages of coding and routing [4]. In the coding step, data is stored in packets in the hop nodes, and then their linear combination is sent to the output edges [5]. In the routing stage, the best subgraph is selected to send coded multicast packets. Optimal routing without using NC for multicast flows is a Np-hard problem [6], while using NC as an optimization problem [13]. In NC, there are two types of virtual and real flows, where virtual flow is an intermediate variable and is used to obtain real flow. We assume that $_{(i,j)}$ is the amount of virtual flow passing on the edge (i,j) and $R_{(i,\mathcal{J}i)}$ the real flow passing from node i to its neighbors or $\mathcal{J}i$. Table 1 contains the required symbols and definitions for the optimization model and proposed algorithm.

Table 1. Symbols used in the optimization model and proposed algorithm.

Symbol	Definition
$\mathcal{J}i$	A set of neighbors or nodes that are in the sending domain of node i
$a_{(i,j)}$	The amount of cost function or required energy to send a packet on the edge (i,j)
B_{Max}	The amount maximum of bandwidth for each edge in the network
$B_{(i,j)}$	The amount of available bandwidth on edge (i,j)
$V_{(i,j)}$	The amount of virtual flow on the edge (i,j)
$R_{(i,\mathcal{J}i)}$	The amount of real flow of the node i on the superedge $(i,\mathcal{J}i)$
Δ_i	The amount of fixed and non-negative of supply and demand in the node i
$x[n]$	A point of the feasible space at the step n
$\theta[n]$	Step size at point $x[n]$
$\omega[n]$	Sub-gradient of the Lagrangian function at the point $x[n]$
$x[n+1]$	The next step
$\hat{x}[n+1]$	Solution and optimal point

The optimization model below shows how to send a multicast flow based on NC at the lowest cost in the network [5], [8], [12].

$$minimize$$

$$f(R) = \sum_{i\in N}\sum_{j\in\mathcal{J}i} R_{(i,\mathcal{J}i)}.a_{(i,j)} \quad (1)$$

$$\forall\, i,j \in N, (i,j) \in E, j \in \mathcal{J}i$$

$$Subject\ to:$$

$$R_{(i,\mathcal{J}i)} \geq \sum_{j\in\mathcal{J}i} V_{(i,j)} \quad (2)$$

$$\forall\, i,j \in N\,, (i,j) \in E, j \in \mathcal{J}i$$

$$V_{(i,j)} \geq 0 \quad (3)$$

$$\forall\, i,j \in N\,, (i,j) \in E$$

$$R_{(i,j)} \leq B_{(i,j)} \quad (4)$$

$$\forall\, i,j \in N\,, (i,j) \in E, j \in \mathcal{J}i$$

$$\sum_{j\in\mathcal{J}i} V_{(i,j)} - \sum_{j\in\mathcal{J}i} V_{(j,i)} = \begin{cases}\Delta_i\ if\ i\ is\ source\\ -\Delta_i\ if\ i\ is\ sink\\ 0\ otherwise\end{cases} \quad (5)$$

$$\forall\, i,j \in N\,, (i,j) \in E, j \in \mathcal{J}i$$

In this model, the objective function (1), minimizes the cost of real flows, or $f(R)$ on all network nodes, where $a_{(i,j)}$ is the amount of cost function or required energy to send a packet over the edge (i,j) which depends on the sending domain of node. Constraint (2) states that the real flow that passes from a node is always greater than the value of all the virtual flows passing through that node, and since the antenna of all nodes is oriented in a general direction, by sending a flow of node i, all neighbors i or $\mathcal{J}i$ receive the flow. Constraint (3) states that real and virtual flows are positive. Constraint (4) states that the maximum real flow passing through an edge should be less than or equal to the bandwidth at that edge. Constraint (5) states the law of the survival of multicast flows as a single-flow, where the difference between the total input and output flows in the hop SNs is zero and in the Source Sensor Nodes (SSNs) is equal to fixed and non-negative value supply and demand or Δ_i and in the destination nodes is equal to -Δ_i.

In [14], a similar optimization model has provided the above constraints for sending the multicast flow with minimum energy consumption in a subgraph of WSN, for this purpose the value of the cost function $a_{(i,j)}$ is considered equal to the square of the distance between the two nodes. If the sending domain of a node is less than the distance between two nodes, then the flow between two nodes will not be established. In [15], authors have proved that the above optimization model is a linear optimization model with exponential execution time, and solution of this model is an optimal value of $_{(i,i)}$. In this paper, because the bandwidth in network's edges is limited, in order to reduce energy consumption and cost, the objective function and the main cost of the problem are the balance of bandwidth consumption. If B_{Max} is the maximum bandwidth for each network's edge and $_{(i,)}$ is the amount of available bandwidth in the edge (i,j), then we consider cost function as $a_{(i,j)}=B_{Max}/B_{(i,j)}$. Therefore, the edges with freer bandwidth will have more priority to be selected for routing based on NC. Also, due to the use of multicasting in SNs, we change the constraint (4) as $R_{(i,\mathcal{J}i)} \leq \sum_{j\in\mathcal{J}i} B_{(i,j)}$. In addition, in order to be able to replace only the value of virtual flow with the value of real flow in other constraints, we modify the constraint (2) as $R_{(i,\mathcal{J}i)} = \max_{j\in\mathcal{J}i}\{V_{(i,j)}\}$. However, because this constraint is discrete, then it should be converted into a continuous form. For this purpose, we use the norm approximation l^m as following [2].

$$R_{(i,\mathcal{J}i)} = \max_{j\in\mathcal{J}i}\{V_{(i,j)}\} = \lim_{n\to\infty}\left(\sum_{j\in\mathcal{J}i}\left(V_{(i,j)}\right)^n\right)^{1/n} \approx \left(\sum_{j\in\mathcal{J}i}\left(V_{(i,j)}\right)^n\right)^{1/n} \quad (6)$$

Also, for the proposed model to be convex and in the form of a linear minimization problem, we place constraints on an unequal side. According to described above, the proposed convex mathematical optimization model for sending multicast flows based on NC and balancing bandwidth consumption in network's edges is as follows. We call this model as an optimization model of generate balance in bandwidth consumption using NC or BNCOM.

$$minimize$$

$$f(R) = \sum_{i\in N}\sum_{j\in\mathcal{J}i} R_{(i,\mathcal{J}i)}.\frac{B_{Max}}{B_{(i,j)}} \quad (7)$$

$$\forall\, i,j \in N, (i,j) \in E, j \in \mathcal{J}i$$

$$Subject\ to:$$

$$R_{(i,\mathcal{J}i)} = \left(\sum_{j\in\mathcal{J}i}\left(V_{(i,j)}\right)^n\right)^{1/n} \quad (8)$$

$$\forall\, i \in N, (i,j) \in E, j \in \mathcal{J}i$$

$$-V_{(i,j)} \leq 0 \quad (9)$$

$$\forall\, i,j \in N, (i,j) \in E$$

$$R_{(i,\mathcal{J}i)} - \sum_{j\in\mathcal{J}i} B_{(i,j)} \leq 0 \quad (10)$$

$$\forall\, i,j \in N, j \in \mathcal{J}i$$

$$\sum_{j\in\mathcal{J}i} V_{(i,j)} - \sum_{j\in\mathcal{J}i} V_{(j,i)} - \Delta_i = 0 \quad (11)$$

$$\forall\, i,j \in N, (i,j) \in E, j \in \mathcal{J}i$$

3.1. Dual Model

Linear minimization problems are subclasses of convex optimization problems and there are several ways to solve them [16]. One of the methods for solving mathematical optimization problems is using a dual model [17]. For this purpose, and calculating a lower bound for the model, the Lagrangian convex function is required and a coefficient for each constraint [18]. Since the variable $_{(i,i)}$ is in terms of the auxiliary variable $V_{(i,j)}$, it can be replaced in the objective function and an optimization model with a objective function in terms of the variable $V_{(i,j)}$ can be obtained. The Lagrangian function for the objective function and the constraints of the above model is as follows, where V is the variable of virtual flow optimization in the network and α, β, γ and δ are Lagrange coefficients in terms of variables i and j for constraints.

$$L(V,\alpha,\beta,\gamma,\delta) = \sum_{i\in N}\sum_{j\in\mathcal{J}i} R_{(i,\mathcal{J}i)}.\frac{B_{Max}}{B_{(i,j)}} + \sum_{i\in N}\alpha_i.\left(\sum_{j\in\mathcal{J}i}\left(V_{(i,j)}\right)^n\right)^{1/n} + \sum_{i\in N}\beta_i.\left(R_{(i,\mathcal{J}i)} - \sum_{j\in\mathcal{J}i} B_{(i,j)}\right) + \sum_{i\in N}\sum_{j\in\mathcal{J}i}\gamma_{(i,j)}.\left(-V_{(i,j)}\right) + \sum_{i\in N}\sum_{j\in\mathcal{J}i}\delta_{(i,j)}.\left(V_{(i,j)} - V_{(j,i)} - \Delta_i\right) \quad (12)$$

Since the objective function and the conditions of the proposed optimization model are convex and separable, the dual of this model is also convex and has a unique solution. As a result, the proposed model above is defined as follows.

$$minimize: L(V, \alpha, \beta, \gamma, \delta)$$
$$\text{Subject to} \tag{13}$$
$$\forall\ i\in N,\ j\in \mathcal{J}i,\ \alpha_i\geq 0,\ \beta_i\geq 0,\ \gamma_{(i,j)}\geq 0,\ \delta_{(i,j)}\geq 0$$

If $\widehat{\Psi}$ is the optimal value of the proposed model and $\hat{V}$ is the optimal value of the variable V, then $\widehat{\Psi} \leq L(\hat{V}, \alpha, \beta, \gamma, \delta)$. Also, if $\widehat{\Phi}$ is the duality optimal value of the proposed model, then $\widehat{\Phi} = L(\hat{V}, \hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\delta})$, where $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ and $\hat{\delta}$ are the optimal values of the Lagrangian coefficients α, β, γ and δ. The difference between $\widehat{\Psi}$ and $\widehat{\Phi}$ is the dual distance [20]. In general, for convex and non-convex problems, $\widehat{\Psi} \leq \widehat{\Phi}$, but if the convex optimization problem is achievable with at least one point, then $\widehat{\Psi} = \widehat{\Phi}$, which in this case is known as the strong duality or slater conditions [21]. In this paper, because we want to find the best optimal point for the problem, slater conditions or strong duality must be met. That is, the solution and the optimal point of the proposed optimization model and the dual model of the model should be the same. For this purpose, we use the Karush-Kuhn-Tucker conditions (KKT).

3.2. Karush-Kuhn-Tucker Conditions (KKT)

The KKT conditions are used to find the optimal solution and optimal point in the optimization model. For this purpose, it is necessary to first calculate the derivative of the Lagrangian function relative to its variables and coefficients, in order to obtain the condition as a multi-excited multi-equation device, solving this device leads to finding the optimal solution and point for the variable V or the amount of virtual flow in the network. The derivative of the Lagrangian function relative to its variables and coefficients is as follows:

$$\frac{\partial L(V, \alpha, \beta, \gamma, \delta)}{\partial V_{(i,j)}} = \frac{B_{Max}}{B_{(i,j)}} \cdot \frac{\partial R_{(i,\mathcal{J}i)}}{\partial V_{(i,j)}} + \alpha_i \cdot \frac{\partial \left(\left(\sum_{\forall j\in \mathcal{J}i} \left(V_{(i,j)}\right)^n \right)^{1/n} \right)}{\partial V_{(i,j)}} + \beta_i \cdot \frac{\partial \left(R_{(i,\mathcal{J}i)} - \sum_{j\in\mathcal{J}i} B_{(i,j)} \right)}{\partial V_{(i,j)}} + \gamma_{(i,j)} \cdot \frac{\partial \left(-V_{(i,j)}\right)}{\partial V_{(i,j)}} + \delta_{(i,j)} \cdot \frac{\partial \left(V_{(i,j)} - V_{(j,i)} - \Delta_i \right)}{\partial V_{(i,j)}} \Rightarrow$$

$$\frac{\partial L(V, \alpha, \beta, \gamma, \delta)}{\partial V_{(i,j)}} = \frac{B_{Max}}{B_{(i,j)}} \cdot \frac{\partial R_{(i,\mathcal{J}i)}}{\partial V_{(i,j)}} + \alpha_i \cdot \frac{1}{n} \cdot \left(V_{(i,j)}\right)^{n-1} \cdot \left(\sum_{j\in\mathcal{J}i} \left(V_{(i,j)}\right)^n \right)^{(1/n)-1} + \beta_i \cdot \frac{\partial R_{(i,\mathcal{J}i)}}{\partial V_{(i,j)}} - \gamma_{(i,j)} + \delta_{(i,j)} \tag{14}$$

$$\frac{\partial R_{(i,\mathcal{J}i)}}{\partial V_{(i,j)}} = \frac{1}{n} \cdot \left(V_{(i,j)}\right)^{n-1} \cdot \left(\sum_{j\in\mathcal{J}i} \left(V_{(i,j)}\right)^n \right)^{(1/n)-1} \tag{15}$$

$$\frac{\partial L(V, \alpha, \beta, \gamma, \delta)}{\partial \alpha_i} = \left(\sum_{j\in\mathcal{J}i} \left(V_{(i,j)}\right)^n \right)^{1/n} \tag{16}$$

$$\frac{\partial L(V, \alpha, \beta, \gamma, \delta)}{\partial \beta_i} = R_{(i,\mathcal{J}i)} - \sum_{j\in\mathcal{J}i} B_{(i,j)} \tag{17}$$

$$\frac{\partial L(V, \alpha, \beta, \gamma, \delta)}{\partial \gamma_{(i,j)}} = -V_{(i,j)} \tag{18}$$

$$\frac{\partial L(V, \alpha, \beta, \gamma, \delta)}{\partial \delta_{(i,j)}} = V_{(i,j)} - V_{(j,i)} - \Delta_i \tag{19}$$

The multi-excited multi-equation device of this model is as follows. Since the proposed model is convex, there is an optimal and minimal solution for the variable V according to the Lagrange coefficients α, β, γ and δ, which must satisfy the following conditions.

$$\frac{\hat{\alpha}_i}{n} \cdot \left(\hat{V}_{(i,j)}\right)^{n-1} \cdot \left(\sum_{j\in\mathcal{J}i} \left(\hat{V}_{(i,j)}\right)^n \right)^{(1/n)-1} + \left(\hat{\beta}_i + \frac{B_{Max}}{B_{(i,j)}} \right) - \hat{\gamma}_{(i,j)} + \left(\hat{\delta}_{(i,j)} - \hat{\delta}_{(j,i)} \right) = 0 \tag{20}$$

$$\hat{\alpha}_i \cdot \left(\left(\sum_{j\in\mathcal{J}i} \left(\hat{V}_{(i,j)}\right)^n \right)^{1/n} - \sum_{j\in\mathcal{J}i} B_{(i,j)} \right) = 0 \tag{21}$$

$$-\hat{V}_{(i,j)} \leq 0 \tag{22}$$

$$\hat{\gamma}_{(i,j)} \cdot \hat{V}_{(i,j)} = 0 \tag{23}$$

$$\sum_{j\in\mathcal{J}i}\hat{V}_{(i,j)}-\sum_{j\in\mathcal{J}i}\hat{V}_{(j,i)}-\hat{\Delta}_i=0 \qquad (24)$$

$$\alpha_i\geq 0,\beta_i\geq 0,\gamma_{(i,j)}\geq 0,\delta_{(i,j)}\geq 0 \qquad (25)$$

4. DISTRIBUTED ALGORITHM FOR SOLVING OPTIMIZATION MODEL

In the previous section, we presented an optimization model for routing and balancing in bandwidth consumption using NC and multicast flows in WSNs. This model minimizes the ratio of the total maximum bandwidth to the available bandwidth in network's edges and uses the dual method to solve this model. We also used the KKT conditions to calculate a lower bound and find the optimal solution and point. For this purpose, we need to calculate the derivative of the Lagrangian function relative to its variables and coefficients, so that conditions are considered as a multi-excited multi-equation device. Solving this device leads to finding the optimal solution for the variable V or the amount of virtual flow in the network. The solution of the KKT conditions for the WSNs with a large number of nodes is very difficult and time consuming and almost impractical because it requires the data of all nodes gathered at a central node and the calculations performed, and then sending the answer of these equations to other nodes. In this section, to solve this problem, we provide a distributed and repeatable algorithm for solving a proposed model in which instead of derivatives, the combination of sub-gradient method and method of separation of network flows is used. Then each node is calculated locally and based on the information of its neighboring nodes and these equations and the optimal routing is made and then it is decided which packets pass on what paths to balance bandwidth consumption in the SNs.

Sub-gradient method is similar to the depth search algorithm, and it searches directly to find the optimal solution of problem in the area of the response, and generating a convergent sequence whose limit point is a local minimum. Sub-gradient method is used in nonlinear optimization models [19]. In the sub-gradient method, the value of the objective function does not always decrease, but it may increase. Any point that is generated during the run of the search algorithm is achievable and can be used as an intermediate solution. If the problem-solving process is stopped before it reaches the solution, then the final point is an achievable solution and is probably the best solution for the main problem. The convergence rate of the Newton method is more than the Sub-gradient method, but since the Sub-gradient method is simpler and less computational than the Newton method, often is used for wireless SNs with low-power [22]. In mathematics, Sub-gradient method is a concept that generalizes the derivative for irrelevant functions. The Sub-gradient value of a point such as x_0 of an open convex set is equal to the vector v, which for every point x holds the relation $f(x)$-$f(x_0)$≥v⊗(x-x_0), where f is a convex function on an convex open set and is not necessarily indeterminate at all points, and the operator ⊗ represents the interior multiplication. The set of all Sub-gradients x_0 for a convex function is called Sub-gradient in x_0 and is represented with $\partial(x_0)$, which is a non-null, closed, and convex set. The value of the Sub-gradient $\partial(x_0)$ is in the closed interval and non-null $[a,b]$, where $a=\lim_{x\to x_0^-}\frac{f(x)-f(x_0)}{x-x_0}$ and $b=\lim_{x\to x_0^+}\frac{f(x)-f(x_0)}{x-x_0}$ [23].

The proposed optimization model in section 3 of this paper is a convex model. If we assume that X is a set of optimal points and solutions, then the set X must be closed, convex and non-null. Assuming there is an optimal point for the proposed model, this proposed model can be solved using Sub-gradient method. In the Sub-gradient method, in step n, if $x[n]$∈X be an point of achievable space and $\theta[n]$ be the step size, and $\omega[n]$ be the Sub-gradient of the Lagrangian function at $x[n]$, then $x[n$+1]∈X is the next step point and $\hat{x}[n+1]$ will be the solution and optimal point. As a result, we consider nearest point to $\hat{x}[n+1]$ as the point [n+1] using the mapping function P_X, whose relations are as follows. Therefore, at each stage of the Sub-gradient, in the negative direction of Sub-gradient, the value of the function is reduced.

$$\hat{x}[n+1]=x[n]\text{-}\theta[n].\omega[n] \qquad (26)$$

$$P_X(\hat{x})=argmin\{\|x\text{-}\hat{x}\|: x\in X\} \qquad (27)$$

$$x[n+1]=P_X(\hat{x}[n+1]) \qquad (28)$$

The Lagrange optimization model presented in section 3 of this article for each SN such as i is as follows.

$$\begin{aligned}L_i(V,\delta)_i &= R_{(i,\mathcal{J}i)}.\frac{B_{Max}}{B_{(i,j)}} + \delta_i.\left(\sum_{j\in\mathcal{J}i}V_{(i,j)}-\sum_{j\in\mathcal{J}i}V_{(j,i)} - \Delta_i\right)\\ &= R_{(i,\mathcal{J}i)}.\frac{B_{Max}}{B_{(i,j)}}+\sum_{j\in\mathcal{J}i}V_{(i,j)}.\left(\delta_i[n]-\delta_j[n]\right) - \delta_i.\Delta_i\end{aligned} \qquad (29)$$

The sum of the above Lagrangian functions for all

SNs is equal to the Lagrangian function of the optimization model.

$$L(V,\delta)=\sum_{i\in N}L_i(V,\delta)_i = \sum_{i\in N}R_{(i,\mathcal{J}i)}.\frac{B_{Max}}{B_{(i,j)}} + \sum_{i\in N}\left(\left(\sum_{j\in\mathcal{J}i}V_{(i,j)}.\left(\delta_i[n] - \delta_j[n]\right)\right)-\delta_i.\Delta_i\right) \quad (30)$$

But the value of the objective function may not be reduced at some times, or the value $\hat{x}[n+1]$ does not belong to X. Therefore, it is necessary that the best value $[n]$ be stored in the variable $\hat{x}$ and the corresponding value of the Lagrangian function at this point and for that value is stored at the upper limit or UB as follows.

$$UB_i=L_i(V[n],\delta[n])_i = R_{(i,\mathcal{J}i)}.\frac{B_{Max}}{B_{(i,j)}} + \sum_{j\in\mathcal{J}i}V_{(i,j)}\left(\delta_i[n]-\delta_j[n]\right) - \delta_i[n].\Delta_i \quad (31)$$

At this stage, the Lagrangian function is decomposed and discrete into several smaller functions, and each function is assigned to a node. The calculation of each function in each node is performed simultaneously, and at each step, each SN must calculate and maintain both the value of virtual flow between the two nodes i and j in the step n or $_{(i,j)}[n]$ and the Lagrangian coefficient of the problem at n or $\delta_i[n]$ and then each SN exchanges the value of these variables with its neighbors and eventually is updated to improve the total of Lagrange functions. For this purpose, once we assume that value of variable $V_{(i,j)}[n]$ is constant, then we calculate and update the value of variable $\delta_i[n+1]$ and once we assume the value of variable $\delta_i[n]$ is constant, and then we calculate and update the value of variable $V_{(i,j)}[n+1]$. In the first case, assuming that the value of variable $_{(i,)}[n]$ be constant, the value of variable $\delta_i[n+1]$ is calculated and updated with the Sub-gradient method as following:

$$(\omega_\delta)_i[n]=\frac{\partial L_i(V,\delta)_i}{\partial\delta_i} = \sum_{j\in\mathcal{J}i}V_{(i,j)}[n] - \sum_{j\in\mathcal{J}i}V_{(j,i)}[n]-\Delta_i \quad (32)$$

$$\delta_i[n+1]=(\delta_i[n]+\theta_n[n](\omega_\delta)_i[n])^+ \quad (33)$$

Which, $\theta_\delta[n]$ is step size and $(\omega_\delta)_i[n]$ is Sub-gradient or derivative of the Lagrangian function relative to the variable δ_i in $x[n]$, and $()^+$ is the mapping function that calculated the nearest nonnegative point as $(a)^+=\begin{cases}a\ if\ a>0\\0\ if\ a\le 0\end{cases}$ [4].

In the second case, if the value of variable $[n]$ be constant, the value of variable $V_{(i,j)}[n+1]$ independently and separately is calculated and updated by each SN such as i for each edge (i,j) as follows,

$$V_{(i,j)}[n+1] = \underset{0\le x\le C_i}{argmin}\left\{\begin{array}{c}\frac{B_{Max}}{B_{(i,j)}}.\left(\sum_{t\in\mathcal{J}i}\left(\left(V_{(i,t)}[n]\right)^n+x^n\right)\right)^{1/n}\\ +x.\left(\delta_i[n]-\delta_j[n]\right)\end{array}\right\} \quad (34)$$

Where, variable x is the decision variable, and $argmin$ means to find the minimum value for the decision variable x and the values of variables $\delta_i[n]$ and $V_{(i,t)}[n]$ are constant, where variable $V_{(i,t)}$ is similar to variable $V_{(i,j)}$ except that the value of virtual flow from the node i to the receiver or destination is different, such as node t. This equation is a convex single-valued problem where has low computational complexity for each SN.

Because the proposed algorithm randomly selects Sub-gradient of the function or $(\omega_\delta)[n]$, the condition of algorithm termination may never occur even with the optimal point $\hat{x}[n]$ and the algorithm does not converge. For the convergence of the proposed algorithm, we must establish the relation $\lim_{n\to\infty}\frac{\|V[n+1]-V[n]\|}{\|V[n+1]\|}=0$. The convergence of the Sub-gradient algorithm depends on the choice of step size or θ. If the step size decreases in each step, the convergence of the algorithm increases, so that we first select the step size as an average value, and gradually reduce its value, approaching the optimal solution of the problem. It is proved that if the step size has the three conditions $\theta[n]>0\ (i)$, $\sum_{n\in N}\theta[n]=\infty\ (ii)$, $\lim_{n\to\infty}\theta[n]^2=0\ (iii)$, the algorithm ends either at the optimal point or after a limited number of steps [23], the value of UB_i will be close to $L_i(\hat{V},\hat{\delta})_i$. The condition (i) ensures that the step size must be positive and the condition (ii) ensures that the amount of step size or $[n]$ does not decrease very quickly, and the repetitions number is not very low, and condition (iii) ensures that increasing the repetitions number increases the reaching probability to the optimal solution and the step size can be reduced to zero. For example, if the step size be as $\theta[n] = \frac{l}{k+n}$ with l and k positive, then the series will be divergent, and the larger value of l, the step size will be larger and the larger value of k, the step size will be smaller [24]. Fig. 3 shows the flowchart of the proposed distributed and repeatable algorithm based on NC for routing and balancing the bandwidth consumption in network's edges. Fig. 4 also shows the proposed algorithm, which is a distributed algorithm based on NC and to balance the bandwidth consumption in the network. We call this algorithm BNCDA. This algorithm is executed by each SN and at each stage generates a string of points for routing the encoded data, and each point is calculated based on previous points. As the number of steps to reach the optimal solution is less, the convergence of the proposed algorithm BNCDA will be better.

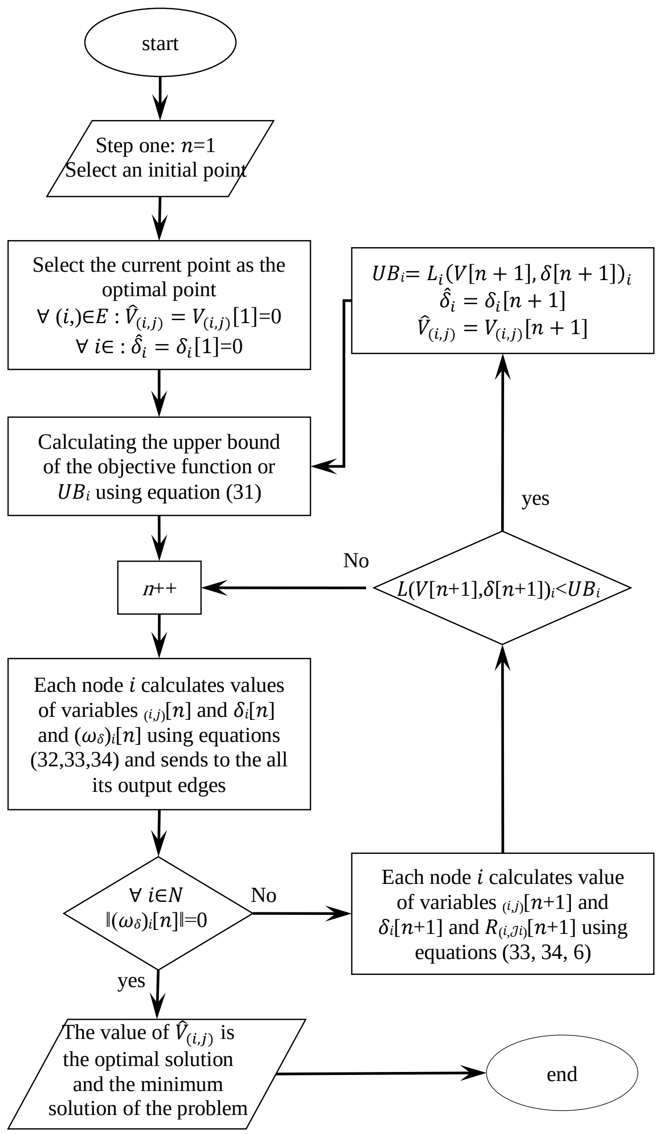

Fig. 3. Flowchart of Proposed Distributed Algorithm BNCDA.

Algorithm BNCDA
Begin
n←1;
Select an initial point and
Select the current point as the optimal point;
$\forall$ $(i,j)\in E$: $\hat{V}_{(i,j)} = V_{(i,j)}[1] = 0;$
$\forall$ $i\in N$: $\hat{\delta}_i = \delta_i[1] = 0;$
Calculating the upper bound of

the objective function or UB_i using equation (31);
while $\forall\ i\in N\ \|(\omega_\delta)_i[n]\|$!= 0
Begin
Each node calculates the values of
variables $V_{(i,j)}[n]$ and $\delta_i[n]$ and $(\omega_\delta)_i[n]$
using equations (32) and (33) and (34) and
Sends its output to all the network's edges;
n++;
Each node calculates the value of
The variables $_{(i,)}[n+1]$ and $\delta_i[n+1]$ and $R_{(i,\mathcal{J}i)}[n+1]$
using equations (33) and (34) and (6);
if $L_i(V[n+1], \delta[n+1])_i < UB_i$, then
Begin
update the optimal values of $\hat{V}_{(i,j)}$ and $\hat{\delta}_i$ and UB_i as
$\hat{V}_{(i,j)} = V_{(i,j)}[n+1];$
$\hat{\delta}_i = \delta_i[n+1];$
$UB_i = L_i(V[n+1], \delta[n+1])_i;$
End if
End while
return the value of $\hat{V}_{(i,j)}$ as the optimal and minimum solution value;
End.

Fig. 4. The pseudo code of the proposed distributed algorithm BNCDA.

5. SIMULATION AND PERFORMANCE EVALUATION

In this section, we evaluate the efficiency and performance of the proposed model BNCOM in section 3 and the proposed distributed algorithm BNCDA, in section 4 of this article in terms of bandwidth consumption and the total amount of virtual flow in network's edges or V based on parameters of step size or θ, Lagrange coefficient or δ in different stages and then we compare them with optimization models SIPNec [8] and OPT [12]. The model SIPNec uses NC and to send encoded data uses hop and mule nodes, but does not take into account the bandwidth balancing. The model OPT, before sending encoded data, performs data routing based on sending domain of SNs in a conscious and uniform manner and only is based on optimal routing and does not use NC and bandwidth balancing. The problem of this model is that if one SN does not sense the event on the path, the entire path will be lost. The above optimization models are obtained through a MILP model which have the best performance in their objective function and their main parameters are properly set.

Simulations in the MATLAB environment have been run on a computer with a 5-core processor 2.5 GHz Intel and 6 Gigabytes of RAM, and we use AMPL software to model optimization models and use the CPLEX software and the Pulp Library to solve optimization models. Also, to solve the model BNCOM, we can solve the equations KKT conditions. To implement and run the algorithm BNCDA, we use the Python language and we use the same software MATLAB to solve the equations and display the graphs. The measured values are resulted from an average of 20 runs and simulations, or repeat the simulation so that energy of the first SN is terminated, and therefore the results are 95% confident and 5% accurate.

We assume in the network simulation model that each SN has maximum four neighbors, and the SNs are synchronous, and how transferred and accessed of SNs to the bandwidth is without interference, and the location of occurrence of events is random and uniform, and the starting point for moving sinks is at the network center and the channel is ideal and the transmission between the SNs is coordinated and controlled from the media access control layer or MAC and any collisions or data errors do not occur. Also, in the simulation of this paper, like resources [6, 8, 12], we assume that 50 SNs are randomly located on a 100×100 square meters as grid and the initial energy of each SN is e_0=10 Jules and sending domain of each SN equal to r=10 meters and the maximum initial bandwidth of each edge is B_{Max}=10 KB and the size of each packet is 10 B and the buffer size of each SN is 100 KB and the production rate of packet at each SN is 10 packets per second. Fig. 5 shows the both of rate available bandwidth and consumed bandwidth in Bytes on the network during dynamic implementation of the algorithm BNCDA.

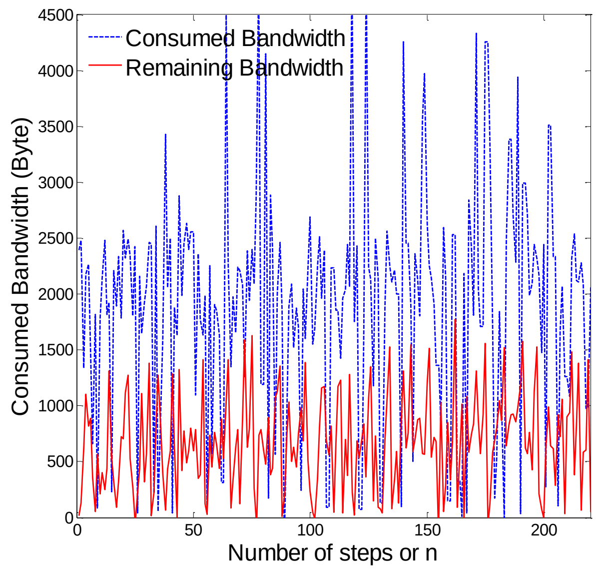

Fig. 6. The rate available bandwidth and consumed bandwidth of network during run of the proposed distributed algorithm BNCDA

5.1. Evaluation and Investigation of Effect of Step Size or θ on the Total Amount of Virtual Flow in

Network's Edges or V in the Proposed Distributed Algorithm BNCDA

As discussed in section 4, convergence of the Sub-gradient algorithm depends on step size selection. If step size at each step is reduced, then convergence of the algorithm BNCDA increases, so that we first select the step size as an average value and gradually decrease the amount of it approaching the optimal problem. If the step size is too large, the algorithm BNCDA is diverted from the optimal point and the series diverges as consistent and leads to imbalance in bandwidth consumption. Also, if the step size is too small, it will get stuck at the local optimal points and will imbalance in bandwidth consumption before the end of the network life too, because the number of repetitions decreases and the probability of reaching the main optimal solution decreases.

Fig. 6 shows the effect of increasing step size on the main variable of the problem or V or the total amount of virtual flow in network's edges, in the algorithm BNCDA for the four sets of SNs. These four sets of SNs are in terms of primary energy or e_0, the sending domain or r, the Lagrangian coefficient or δ, and the step size or θ. The points shown represent the optimal step size for each set, which is calculated from equations (26 and 33) using MATLAB software. As we can see, increasing the step size from the initial value to the points shown, the total amount of virtual flow in network's edges has increased, indicating the compatibility between the theoretical formulas and experimental results. However, after the points shown, the total amount of virtual flow in network's edges can still increase, but is not always guaranteed, as theoretical shown in section 4 of this paper.

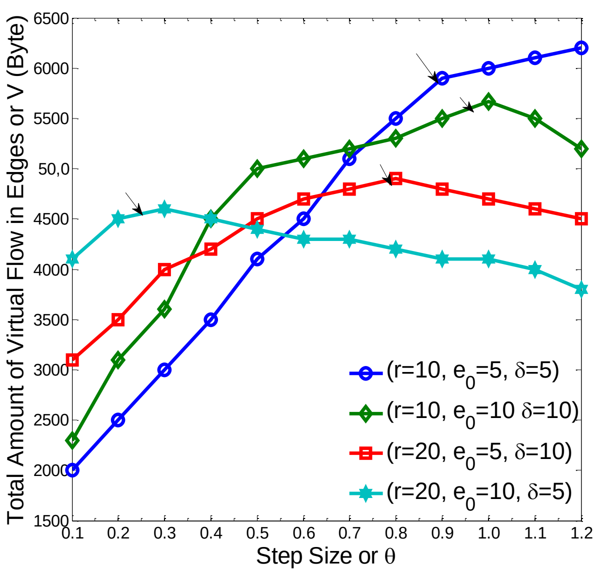

Fig. 6. Effect of step size or θ on the total amount of virtual flow in network's edges or V in the proposed distributed algorithm BNCDA and step size points for 4 different sets of SNs.

5.2. Evaluation and Investigation of the Effect of Lagrangian Coefficient δ on the Total Amount of Virtual Flow in Network's Edges or V in the Proposed Distributed Algorithm BNCDA

Fig. 7 shows the effect of the Lagrangian coefficient δ on the total amount of virtual flow in network's edges or V in the algorithm BNCDA for four different sets of SNs. It has been observed that with increasing the Lagrangian coefficient δ at each stage, due to the higher balance in bandwidth consumption, the total amount of virtual flow in network's edges increases to one point, but after that point, the total amount of virtual flow in network's edges decreases. The reason is that since the position of the SNs is constant, the larger the Lagrange coefficient δ, the greater the number of SNs can distribute its encoded data in the network, and thus the amount of virtual flow in network's edges decreases. Eventually, with the excess increase of the Lagrange coefficient δ, the total amount of virtual flow in network's edges reaches zero in the algorithm BNCDA. Theoretically, finding the optimal value of the Lagrange coefficient δ requires a lot of calculations.

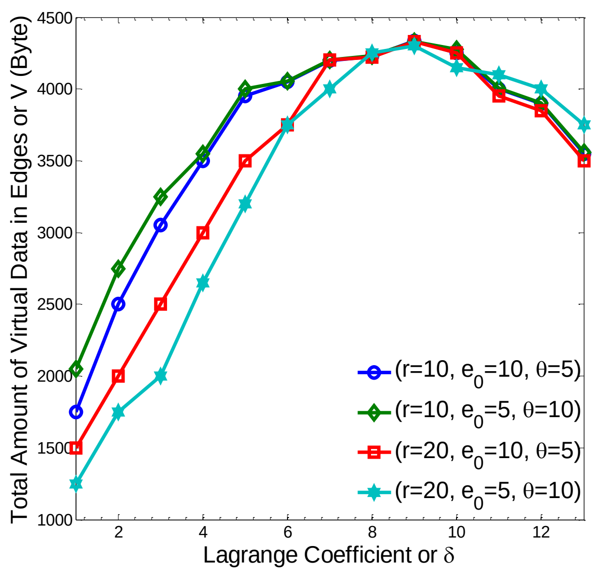

Fig. 7. The effect of the Lagrangian coefficient δ on the total amount of virtual flow in network's edges or V in the proposed distributed algorithm BNCDA for four different sets of SNs

5.3. Evaluation and Investigation of the Total Consumed Bandwidth in the Network

Figs 8 and 9 show the results of the evaluation and investigation of the effect of the number of sensor source nodes or SSNs on the network bandwidth consumption in bytes, respectively, for sending domain of SNs 10 and 20 meters. In the 4 SSNs state, four source sensor nodes are located at the four corners of the network and produce sent data, and other hop nodes code the data. The 5 SSNs state, like the 4 SSNs state, except that the fifth SSN is located at the center of the network. The number of neighboring nodes of a SSN with a sending domain 10 meters, between 2 and 4 and with sending domain 20 meters, is between 5 and 12. It is observed that increasing number of SSNs, more data is generated and more bandwidth is consumed. Using NC reduces the number of sending and data traffic and, as a result, reduces bandwidth consumption in the network. In the model BNCOM, since routing the encoded data is the edge with more free bandwidth, so the least bandwidth is consumed in the network. It has also been observed that with increasing the number of SSNs, the difference between model OPT and other optimization models is further determined. Increasing the sending domain of SSNs, the number of neighbor nodes of an SSN increases and bandwidth consumption increases on the network. However, with the increase in the number of neighbors, the opportunity NC increased in the network, and as a result, in the models BNCOM and SIPNec and the algorithm BNCDA that use NC, it reduces bandwidth consumption in the network than model OPT that does not have NC.

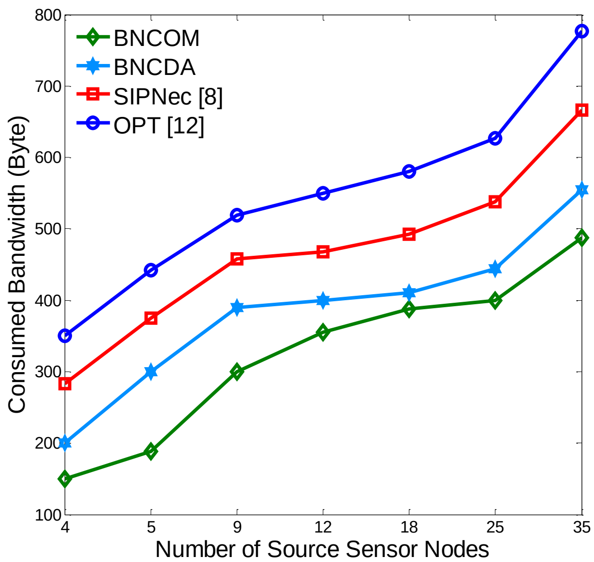

Fig. 8. Effect of the number SSNs on total consumed bandwidth in the network in Byte with sending domain of 10 m.

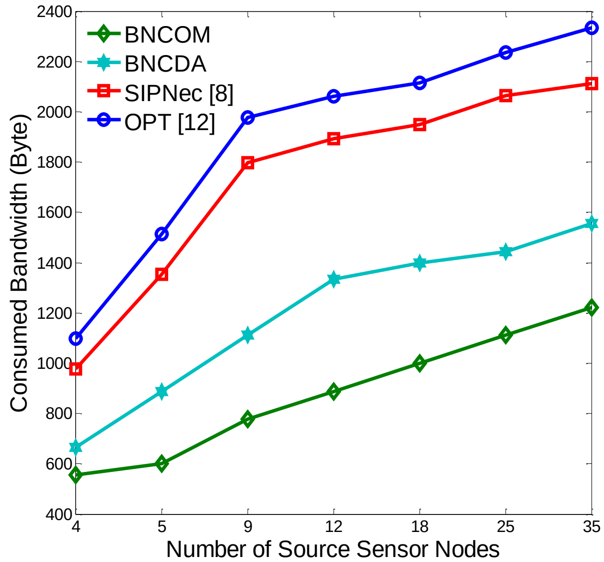

Fig. 9. Effect of the number SSNs on total consumed bandwidth in the network in Byte with sending domain of 20 m.

5.4. Evaluation and Investigation of the Total Consumed Energy in the Network

Figs. 10 and 11 show the results of the evaluation and investigation of effect of SSNs on the total consumed energy in milli-joule, respectively, for sending domain 10 and 20-meter SNs. The smaller the number of SSNs, the less data is generated and the less energy is consumed, and with the increase in the number of SSNs, more data is generated and more energy is consumed, but more NC can be used, which reduces the number of sending and data traffic and consequently reducing energy consumption in the network. Increasing the sending domain of SNs, the number of neighbors increases and energy consumption increases, but with the increase in the number of neighbors, the opportunity NC is increased and, as a result, reduces energy consumption in the algorithm BNCDA than un-NC methods. It has also been observed that increasing the number of SSNs, the differences between optimization models that use NC are further characterized than to model OPT that does not have NC. The model SIPNec also does not take into account the balance in bandwidth consumption, but has a weaker performance than the model BNCOM.

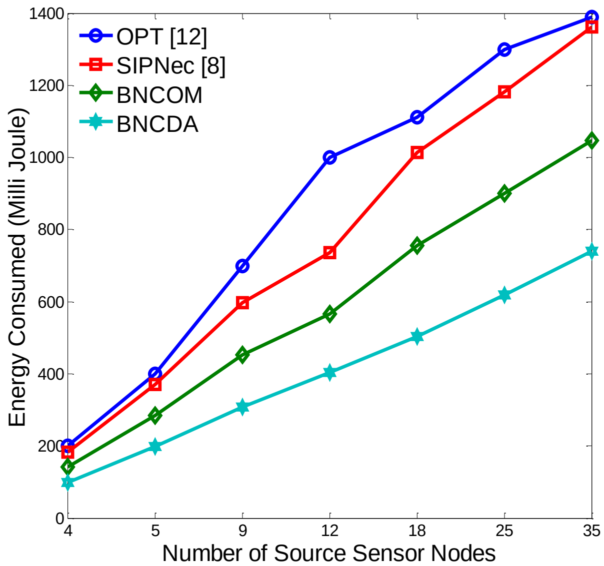

Fig. 10. Effect of the number of SSNs on total energy consumed in the network in Bytes with sending domain of 10 meters.

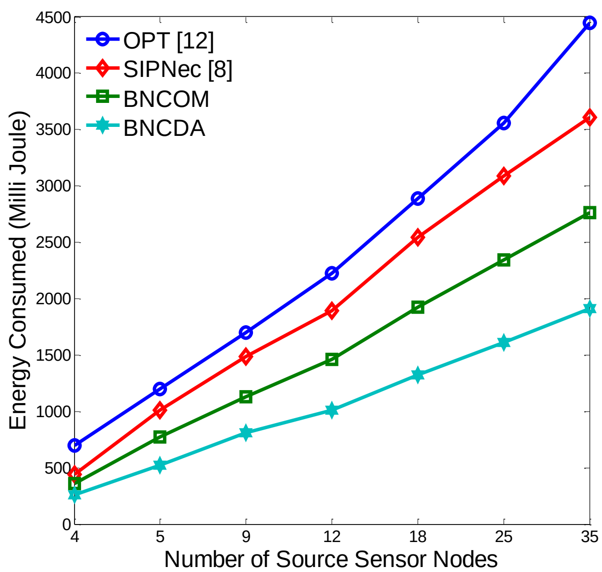

Fig. 11. Effect of the number of SSNs on total energy consumed in the network in Bytes with sending domain of 20 m.

5.5. Evaluation and Investigation of Network Lifetime

One of the most important goals in WSNs is the longer network lifetime. In this paper, like resources [8], [12], we define network lifetime since the deployment of SNs leads to depletion energy of the first SN. Because the energy consumption to send packets in SNs is very greater than the energy consumption to encode packets in SNs, thus lifetime of SNs has a reverse ratio to the rate of generate packet by the SNs. Fig. 12 shows the effect of increasing the number of SSNs over network lifetime in seconds. It has been observed that since all optimization models use optimal paths, they have almost the same trend, and with the increase in the number of SSNs, the amount of sent data increases, resulting in increased energy consumption, and also decrease of the network lifetime. It is also observed that because the model BNCOM only serves to balance the bandwidth in network's edges and send packets from edges that have more free bandwidth, the chance of NC in this model is less than the model SIPNec and lifetime of the model SIPNec is more than the model BNCOM. Also, because the algorithm BNCDA performs the routing of encoded packets between SNs in each step in a continuous and distributed manner, it has shorter lifetime than optimization models. Since the model OPT, only pays to determine optimal path for data transmission and does not use NC, due to the short packet paths, it has a longer lifetime than other models.

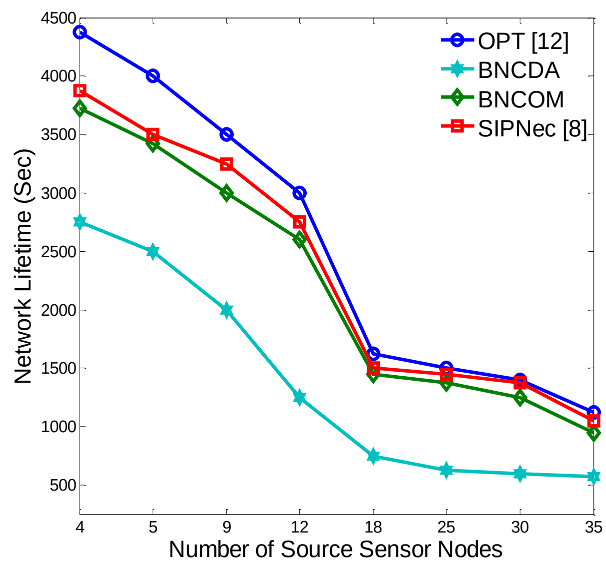

Fig. 12. Impact of the number SSNs on average of network lifetime in second.

5.6. Evaluation and Investigation of the Average End-to-end Latency

The end-to-end latency parameter is very important for real-time applications in the network, depending on the number of SSNs and the rate of generate data and the traffic load. Fig. 13 shows the results of the evaluation and investigation of effect of the number of SSNs on the average end-to-end latency in seconds. It has been observed that with increasing the number of SSNs, the amount of data at network is increased and the delay rate for sending encoded data increases. However, since the model BNCOM and the algorithm BNCDA use NC and balancing on the bandwidth consumption in network's edges, it reduces encoded packets and interferes and collapses data in the physical layer, resulting to have less latency than models SIPNec and OPT. Also, because the algorithm BNCDA is a repeating algorithm and requires several steps for packet routing, its delay is more than the model BNCOM.

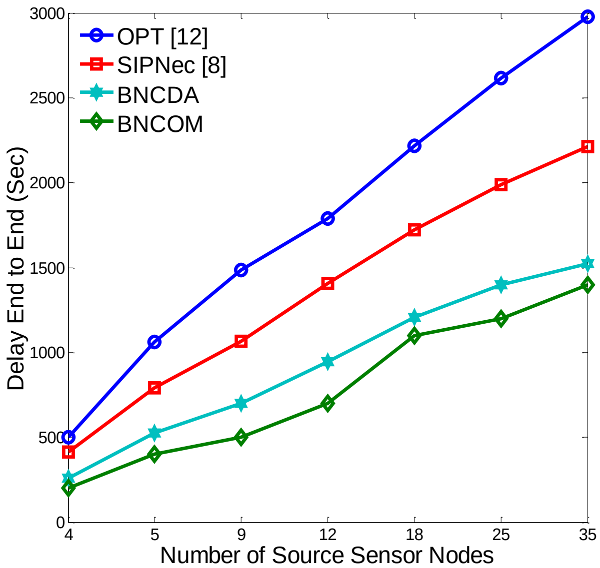

Fig. 13. Effect of the number SSNs on the average latency of end-to-end of network in seconds.

5.7. Evaluation and Investigation of the Time Complexity to Achieve Optimal Solution

Other factors in delay creation are the time complexity to solve the model and achieve optimal solution. The required energy to perform calculations in SNs is much less than the required energy to send packets. Although SNs calculates poorly and cannot perform complex calculations. Therefore, the less complexity of calculations, the efficiency of packet routing will be better. Fig. 14 shows the results of the evaluation and investigation of the number SSNs on the average required time to find the optimal path in seconds. It has been observed that the required time to find the optimal path in the algorithm BNCDA is far less than the optimization models BNCOM, SIPNec and OPT. Since this algorithm is distributed and does not require the collection of total network information at a central point and has simple computations, SNs can simultaneously and comfortably execute this algorithm. Therefore, this algorithm can be applied and is scalable in WSNs with a large number of nodes. Of course, the number of replications of the algorithm BNCDA depends on the speed of convergence. However, since the optimization models of BNCOM, SIPNec and OPT are centrally solved, the information of all the nodes is gathered in a central node, and then the optimization models are solved and the calculations are performed and then the computation result is sent to the other nodes. Therefore, the method of centralized solution for WSNs with a large number of nodes is very difficult and time consuming and almost impractical.

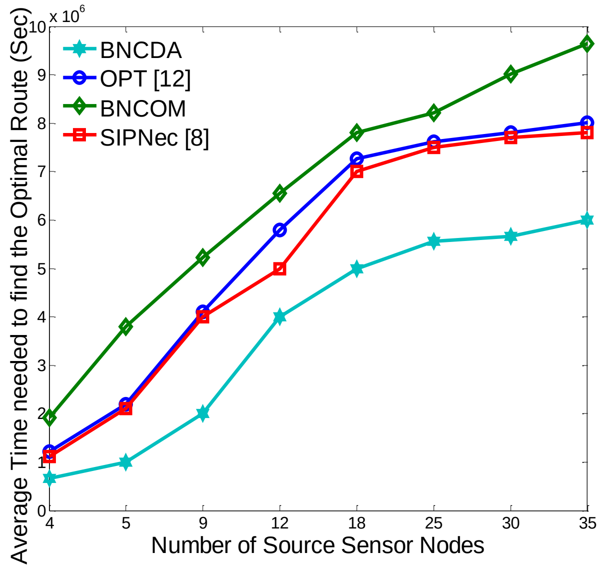

Fig. 14. Effect of the number of SSNs on the average required time to find the optimal route in seconds.

5.8. Evaluation and Investigation of the Total Amount of Virtual Flow in Network's Edges or *V*

The total amount of virtual flow in network's edges has a reverse ratio to the distance between the SNs and the longer the distance between SNs, the more virtual flow will be in network's edges. Fig. 15 shows the results of the evaluation and investigation of effect of the number of SSNs on the total amount of virtual flow in network's edges in Byte until the 20th minute. It has been observed that with increasing the number of SSNs, the total amount of virtual flow in network's edges or *V* increases in all models and the algorithm BNCDA. Because the models BNCOM, SIPNec and OPT are focused and use aware routing of encoded data in the network, they reduce the routing length between SNs and reduce the total amount of virtual flow in network's edges. The number of SSNs is less, the length of the data transmission path is increased, which reduces the total amount of virtual flow in network's edges. However, with the increase in the number of SSNs, the deployment of the SSNs in the center is increased and the routing length of encoded data is reduced and the total amount of virtual flow increases in network's edges. It is also observed that the total amount of virtual flow in network's edges in the algorithm BNCDA is more than optimization models. So, if the number of SSNs is 4, the distance between the algorithm BNCDA with two models BNCOM and SIPNec is about 1 KB and with model OPT is approximately 2 KB. If the number of SSNs is 35, the distance between the algorithm BNCDA and the two models BNCOM and SIPNec are almost unchanged, but the distance between the algorithm BNCDA and model OPT is much higher (15 times). That is the reason of lack of use of NC and balancing in bandwidth consumption in the model OPT.

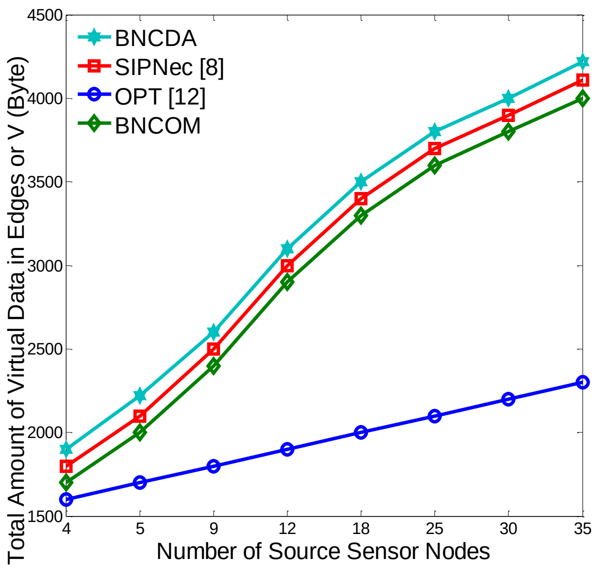

Fig. 15. Effect of the number of SSNs on the total amount of virtual flow in network's edges or V in Bytes.

6. CONCLUSION

One of the limitations in WSNs is the amount of available bandwidth on the network. The problem of finding the optimal route for data transmission and balancing in network bandwidth consumption is a NP-hard problem. In this paper, an optimization model was proposed to determine the optimal route for data transmission and balancing bandwidth consumption using NC in WSNs. In order to solve the optimization model, the information must be sent to a central node, a distributed and repeatable algorithm is proposed in WSN to solve this problem. The proposed algorithm is based on the Sub-gradient method and method of separation of network flows that dynamically and continuously determines the optimal path for sending encoded data, based on the bandwidth of the SNs. The effectiveness of the proposed optimization model and the proposed distributed algorithm with several simulations performed in terms of the number of SSNs and Lagrangian coefficient and step size have been investigated. The results show that the proposed model and algorithm can improve parameters of the average of required time to find the optimal route, the total amount of virtual flow in network's edges, the average latency end-to-end of network, the consumed bandwidth, the average of network lifetime and the consumed energy, due to aware routing and NC and do not work very poorly on other models. Also, the proposed algorithm has great scalability, since computations are distributed and decentralized and there is a small dependency between nodes. In the optimization model and in this article, to simplify the problem, we assume that the environment is open and flat, and the radio coverage is completely regular, which the future works can be used in real-world conditions, such as inside building or under strict conditions, and irregular radio coverage with interference. In the simulation, it is assumed that SNs are simultaneously and transmitting and accessing of SNs to bandwidth is not interference that in the future works, we can simulate in more realistic conditions.

REFERENCES

[1] M. Khalily-Dermany, M. Shamsi, M. J. Najafi-Arani, **"A convex optimization model for topology control in network-coding-based wireless-sensor networks,"** *Ad Hoc Networks*, Vol. 59, pp. 1-11, 2017.

[2] C. Fragouli and E. Soljanin, **"NC Applications,"** *Now Publishers*, 2008.

[3] T. Ho and D. S. Lun, **"NC: an introduction,"** *Cambridge University Press*, 2008.

[4] Y. Fan, **"NC based information security in multi-hop wireless networks,"** *Doctor of Philosophy Electrical and Computer Engineering University of Waterloo Waterloo*, Ontario, Canada, 2010.

[5] V. Pandit, J. H. Jun, D. P. Agrawal, **"Inherent security benefits of analog NC for the detection of byzantine attacks in multi-hop wireless networks,"** *In IEEE 8th International Conference on Mobile Adhoc and Sensor Systems (MASS*), pp. 697-702, 2011.

[6] Z. Ding, Z. Ma, and K. K. Leung, **"Impact of NC on system delay for multisource-multi destination scenarios,"** *IEEE Transactions on Vehicular Technology*, Vol. 59, No. 2, pp. 831-841, 2010.

[7] M. Xiao and T. Aulin, **"Optimal decoding and performance analysis of a noisy channel network with NC,"** *IEEE Transactions on Communications*, Vol. 57, No. 5, pp. 1402-1412, 2009.

[8] P. Gjanci, C. Petrioli, S. Basagni, C. A. Phillips, L. Bölöni, and D. Turgut, **"Path finding for maximum value of information in multi-modal underwater WSNs,"** *IEEE Transactions on Mobile Computing*, Vol. 17, No. 2, pp. 404-418, 2018.

[9] S. Jaggi, P. A. Chou, and K. Jain, **"Low complexity algebraic multicast network codes,"** *In IEEE International Symposium on Information Theory*, pp. 368-368, 2003.

[10] R. S. Youail, C. Wenqing, and T. Shaoguo, **"Cost minimization for multi-source multi-sink NC,"** *Zhang Jia Jie, Hunan*, pp. 253- 258, 2008.

[11] M. Khalily-Dermany, M. Sabaei, and M. Shamsi. **"Topology control in network–coding–based–**

multicast WSNs," *International Journal of Sensor Networks,* Vol. 17, No. 2, pp. 93-104, 2015.

[12] M. Khalily-Dermany, and M. Nadjafi-Arani, **"Itinerary planning for mobile sinks in network-coding-based WSNs,"** *Computer Communications*, Vol. 111, pp. 1-13, 2017.

[13] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, **"XORs in the air: practical wireless NC,"** *IEEE/ACM Transaction on Networking*, Vol. 16, No. 3, pp. 497-510, 2008.

[14] S. Y. R. Li, R. W. Yeung, and C. Ning, **"Linear NC,"** *IEEE Transactions on Information Theory*, Vol. 49, No. 2, pp. 371-381, 2003.

[15] Z. Yang, M. Li, and W. Lou, **"A NC approach to reliable broadcast in wireless mesh networks,"** *In 4th International Conference on Wireless Algorithms, Systems, and Applications*, WASA, Boston, MA, 2009.

[16] K. Fan, L. X. Li, and D. Y. Long, **"Study of on-demand COPE-aware routing protocol in wireless mesh networks,"** *Tongxin Xuebao/Journal on Communications*, Vol. 30, No. 1, pp. 128-134, 2009.

[17] K. Chi, X. Jiang, and S. Horiguchi, **"A more efficient COPE architecture for NC in multihop wireless networks,"** *IEICE Transactions on Communications*, Vol. E92-B, No. 3, pp. 766-775, 2009.

[18] M. Khalily-Dermany, and M. J. Nadjafi-Arani, **"Itinerary planning for mobile sinks in network-coding-based WSNs,"** *Computer Communications*, Vol. 111, pp. 1-13, 2017.

[19] D. S. Lun, **"Minimum-cost multicast over coded packet networks,"** *IEEE Transactions on Information Theory,* Vol. 52, No. 6, pp. 2608-2623, 2006.

[20] V. Shah-Mansouri, and V. W. S. Wong, **"Maximum-lifetime coding subgraph for multicast traffic in WSNs,"** *In IEEE GLOBECOM Global Telecommunications Conference, New Orleans*, pp. 497-502, 2008.

[21] S. Boyd and L. Vandenberghe, **"Convex Optimization,"** *Cambridge University Press*, 2004.

[22] E. Kharati, M. Khalily-D, and H. Kermajani, **"Optimized sink control to increase the lifetime of underwater wireless sensor networks".** *Computer and Knowledge Engineering*, 2019. 9.

[23] M. Khalily-Dermany, **"A convex programming for range assignment to optimize lifetime in network-coding-based-wireless-sensor networks,"** *International Journal of Wireless Information Networks*, pp. 1-6, 2017.

[24] E. Kharati, , M. Khalily-D, and H. Kermajani, **"Increasing the Value of Collected Data and Reducing Energy Consumption by Using Network Coding and Mobile Sinks in Wireless Sensor Networks".** *AUT Journal of Modeling and Simulation*, 2019.

[25] E. Kharati, , M. Khalily-D, and H. Kermajani, **"Increasing the Amount of Collected Data using Network Coding and Continuous Movement of Mobile Sinks in Wireless Sensor Networks"** , *IET Networks*, 12pp. DOI: 10.1049/iet-net.2019.0031.