Application of De-noising in Identification of the Order of Viscoelasticity as a Function of Material State

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ABSTRACT:
Soft tissue modeling is a challenging issue in tissue engineering. Tissue is a complex environment. It is assumed to represent viscoelastic behavior. Therefore, a complicated process is required to model its stress-strain relationship. In this paper, a non-integer order model is considered for the tissue's mechanical behavior. The order indicates the amount that the tissue tends to behave as a pure viscous or pure elastic material. The main goal of the paper and the main contribution is to interpret the order as a function of the state of the material. To this aim, an experimental estimated model is used in which the order is considered as a function of time. Stress and strain signals are also available as functions of time. Then, an identification process is used to obtain the direct functionality of the order with respect to the state of the material (i.e. the momentary stress and strain). Data are gathered through an experimental setup. The stress signal calculated using a force sensor is highly noisy. Hence, de-noising is necessary. However, noise elimination may cause losing meaningful data. Also, a slight amount of noise enhances the generalization of the trained network in the identification process. Accordingly, a multi-level noise reduction method is used. The method is based on Empirical Mode Decomposition (EMD). To obtain the optimal noise reduction level, the noise reduction process is performed level by level and the best levels in train and test stages are chosen. Results show that supposing an explicit functionality between the order (as the amount of viscoelasticity) and the state of the tissue is reasonable. Also, it is verified that multi-level de-noising significantly improves the identification process.

KEYWORDS: Soft Tissue, Non-integer Order Calculus, ANFIS, EMD Multi-Level De-noising.

1. INTRODUCTION

Soft tissue modeling is still a challenging issue among both medicine and engineering researchers. The topic was firstly investigated in developing virtual surgery simulators [1]. An accurate model of soft tissue resembling the actual behavior would strongly enhance the quality of the learning process for beginners. Afterward, soft tissue modeling found a vast variety of applications, including needle insertion [2], blood sampling [3], seed planting [4-6], etc., where a needle must be inserted into the tissue to perform an operation. Most of the recent applications need a model to provide the state of the tissue at each moment of the process as a function of time. In other words, they need a dynamic rather than a steady-state model. As an example, in the seed planting process, the seed must be placed in the exact position. As the needle is inserted the tissue is deformed, therefore, the planting spot also deviates. The only method leading to exact planting is to have a dynamic model of the tissue and predict the deviation of the planting spot to have a precise seed planting process [4].

In a microscopic sight of view, soft tissue consists of two phases; liquids such as water or blood, and cell membranes. Accordingly, in the microscopic viewpoint, it is not a pure elastic or pure viscous material. The most common approach is to consider it viscoelastic [7-10]. Various approaches are used to model such a viscoelastic behavior. Some methods use the Kelvin-Voigt model and model each particle using a mass-spring equivalent [11-12]. To increase the accuracy, the Finite Element Method is used in several papers [12].

All modeling approaches consider the fact that a viscoelastic material behaves "between" viscous and elastic ones. Having an interesting interpretation for the word "between", non-integer order calculus is a recent alternative for modeling viscoelasticity [8,13].

Non-integer order calculus is an old topic with recent applications in modeling and engineering. The non-integer order derivation with order α in the sense of Caputo is defined as [14]:

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$$ {}^{C}D_{t}^{\alpha(t)}f(t)=\frac{1}{\Gamma(1-\alpha(t))}\int_{0}^{t}(t-\tau)^{-\alpha(t)}\frac{d}{d\tau}f(\tau)d\tau \quad (1)$$
$$0<\alpha(t)<1,\ 0<t$$

Where, $\Gamma(z)=\int_{0}^{\infty}e^{-u}u^{z-1}du$ is the extension of the factorial function to non-integer arguments. During the recent decades, the concept of non-integer order calculus was the main subject of many papers in different fields theoretical and practical in both theoretical and practical sights of view [14].

Technically, modeling the mechanical behavior of a material is equivalent to proposing an equation describing the stress-strain relationship [13]. As qualitative definitions, stress (denoted by σ) is the normalized force on each particle of the material, and strain (denoted by ε) is the relative displacement of a particle [15]. The non-integer order stress-strain relationship is suggested as

$$\sigma=\eta D^{q}\varepsilon \quad (2)$$

In which, η is a constant, however the order q may be varying [13,16]. The concept beyond the above suggestion is the fact that: 1. For a pure elastic material, the stress is linearly directed to the strain (or, in other words, to its 0th differentiation), $\sigma=E\varepsilon=ED_{t}^{0}\varepsilon$. 2. For a pure viscous material the stress is linearly directed to the first differentiation of the strain $\sigma=b\dot{\varepsilon}=bD_{t}^{1}\varepsilon$. Accordingly, as an interpretation for the word “between”, in a viscoelastic material, the stress is linearly directed to the qth differentiation of the strain. This is the reasoning explaining Equation (2).

The recent modeling concept has attracted researchers to apply identification methods to verify the modeling approach. In [17], a variable order diffusion model is used to model the tension in the cable. The model is compared to the real experimental data. In [18], data extracted from a robotics system is used to model soft tissue deformation. The model considers the order as a function of time, $q=q(t)$, and approximates it using a piece-wise constant function. Then, the Genetic Algorithm is used to estimate the constant coefficients. Convergence of an adaptive order/parameter estimation method is proven in [13], where, a non-integer order model is suggested for the one-dimensional tissue deformation, having time-varying order. The estimation results are shown in Fig. 1. The data used in this paper for identification purposes are captured from the recent paper.

In practice, considering the order as a function of time works. The reason is the fact that the states of the material are time-varying. Therefore, the order -as an explicit function of the states- would be, in turn, an implicit function of time. However, there is a significant drawback in considering the order as a function of time: The order cannot be predicted.

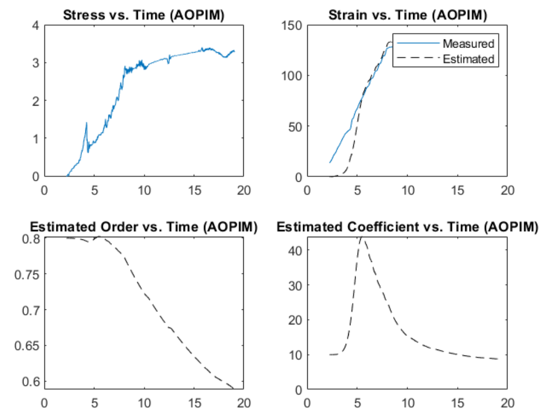

Fig. 1. Strain, stress, estimated strain and order with AOPIM (Experiment number 3) [13].

The point is that the order, itself, is not a function of time. Therefore, it is not possible to consider a function with a time argument and expect it to calculate the order in each instant. However, based on mechanical facts, the amount of viscoelasticity is dependent on the status of the material [19]. The status of the material is interpreted as the value of its stress and strain in each spot. In our model, it is denoted by the order, as tending the order to zero leads to elasticity, and order 1 implies pure viscosity. Therefore, the order is a function of the stress and strain, in each point, in each time instant.

However, the challenge is that how this functionality may be found. To this aim, the former non-integer one-dimensional model is used with the Adaptive Order/Parameter Identification Method (AOPIM) proposed in [13]. Data extracted from the experiments performed in [13] are used to build train and test data set. The AOPIM gives the stress/strain pair and the order as some functions of time. To remove the time argument, an ANFIS network is fed with the stress-strain pair of a training experiment as the input and the order of the same experiment as the output. After the network is trained, data extracted from another experiment is considered as the test dataset and the identification process is evaluated.

There are 3 experiments, among which a pair is selected, the first is used as the training experiment and the second is used as the test experiment. Considering different combinations would show that the concept of considering the order as a function of the stress-strain pair is reasonable and the estimation process efficiently works. Now, having such a trained network in hand, by measuring the stress-strain one can estimate the order and therefore, predict the soft tissue deformation

However, as it is depicted in the top-left graph in Fig. 1, the stress data extracted from a force sensor are heavily noisy. Noise plays a twofold role in training Neural Networks: It improves the generalization, while it reduces the accuracy [20]. Therefore, a certain "level" of de-noising may be useful in identification. Accordingly, different levels of de-noising should be considered to choose the best level, resulting in the least identification error.

There are different methods for noise reduction [21,27,28] among which, the multi-level noise reduction techniques are of high interest [20]. It will be shown that the identification process is improved as the proper level of de-noising is chosen. Accordingly, the following procedure is taken for obtaining the amount of viscoelasticity of a material as a function of its state:

1. The variable order stress-strain relationship is considered, where the order determines the amount of viscoelasticity.
2. Extracting stress and strain as functions of time from several experiments, AOPIM is used to estimate the order as a function of time, as well.
3. A level of de-noising is considered by which the stress signal is de-noised.
4. The state of the material (momentary stress and strain signals) and the amount of viscoelasticity (the momentary order) are fed to an ANFIS network. The network is trained to estimate the input-to-output functionality.
5. After applying a data organizing approach similar to the former step, another experiment is used as a test dataset. The test error is calculated.
6. Steps 3 to 5 are done considering different levels of de-noising and the best DNL –lead to the least error calculated in step 5- is chosen.
7. The whole procedure is performed by choosing different experiments as training and testing datasets to verify the modeling scheme.

Based on the above, the rest of the paper is organized as follows:

The next section investigates the Empirical Mode Decomposition method. Afterward, in Section 3, the identification process is proposed. The results are given in Section 4, where different choosing of the train and test dataset, and, the effect of de-noising level are discussed. Finally, Section 5 concludes the paper.

2. MULTI-LEVEL NOISE REDUCTION BASED ON EMPIRICAL MODE DECOMPOSITION (EMD)

Because of the measurement noise, inherited from the force sensor, the data express noisy behavior. The problem is that the power of the noise is unknown. This is where multi-level noise reduction idea is taken into account to improve modeling process. Among the methods used to de-noise the data, the empirical mode decomposition method is the most practical one, because it is basically data-driven and does not require special prior knowledge of the data compared to other methods. Since there is no systematic approach to determine the De-Noising Levels (DNL), several experiments are required to achieve to the appropriate level.

First we introduce EMD and Edited-EMD (EEMD). EMD is an iterative algorithm through which a signal is decomposed into a set of oscillatory components, referred to as Intrinsic Mode Functions (IMFs). An IMF waveform is symmetric with respect to the local mean and at the same time the number of its zero-crossings and extrema at most equals to one. [26].

The EMD algorithm is as follows:

1. Find the local extrema during a given signal x(t)
2. Interpolate between the local maxima and minima to create the upper and lower envelopes of the signals.
3. Find the mean of two envelopes derived from stage 2.

$$m_1(t) = \frac{u_1(t) + l_1(t)}{2} \tag{3}$$

   Where, $u_1(t)$ and $l_1(t)$ are the upper and lower envelopes of the signal, respectively.
4. Calculate $h_1(t) = x(t) - m_1(t)$
5. Calculate the stopping criteria for the sifting process, sum of difference, defined as:

$$SD = \sum_{t=0}^{T} \frac{\left|h_{j-1}(t) - h_j(t)\right|^2}{h_j^2(t)} \tag{4}$$

   If $SD < THR$, the 1st IMF has been derived: $c_1(t)$, visit step 6, Else repeat steps 1-5 for $x(t) = h_1(t)$ THR is threshold level and frequently set between 0.2 to 0.3.
6. Form $r(t) = x(t) - c_1(t)$.
7. Check $r(t)$ for the number of extrema. If the number of extrema is one or less, the iteration is over. The original signal can be reconstructed by:

$$x(t) = \sum_{j=1}^{m} c_j(t) + r(t) \tag{5}$$

8. Else, return to step 1, by replacing $x(t)$ with $r(t)$.

In EMD-based de-noising strategy, the noisy signal first decomposed into IMFs. IMFs of the noisy signal are later filtered so as to get $\hat{c_j}(t)$. It is the estimation of the IMFs of original signal ($c_j(t)$). Therefore, the initial signal $x(t)$ are often reconstructed through

$$\hat{x}(t) = \sum_{j=1}^{m} \hat{c}_j(t) + r(t) \tag{6}$$

Where $r(t)$ is the residual signal.

In the second step of the above algorithm, cube spline function is employed which in the presence of noise it follows all the rapid and unwanted changes of the signal, therefore cube spline is an inflexible method and has no effect on noise reduction to form the upper and lower push curves of the signal. This paper uses a combination of two tricks to reduce the noise effect. The first is using the smooth spline function instead of using the cube spline function (known in the literature as the edited EMD method (EEMD)). A smooth spline for n points $(x_1.y_1).(x_2.y_2)....(x_n.y_n)$ is:

$$S(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \sum_{j=1}^{n} \theta_j (x - x_i)^3 \tag{7}$$

Where, S is a 3rd degree polynomial in each interval $[x_i.x_{i+1}]$ with first and second order derivatives. The $S(x)$ function must minimize the following expression:

$$J = p\sum_{i=1}^{n}\left(y_i - S(x_i)\right)^2 + (1-p)\int_{x_1}^{x_n}\left(\frac{d^2S}{dx^2}\right)^2 dx \tag{8}$$

The two above terms control the smoothness of the function, the closeness of the value of the function and the data points. It is done by smoothness parameter p which $p = 0$ is a straight line with the least squares method and $p = 1$ corresponds to a cubic spline. Experimentally the value $(1+\frac{h^3}{6})^{-1}$ is used, where h is the average distance between points.

Second, although different EMD-based de-noising methods use different filtering techniques, like soft-thresholding, hard-thresholding, adaptive filtering [22], instantaneous half period [22,23,24], in the above methods the noise level and the threshold level is estimated and new IMFs are calculated for reconstruction. To achieve a completely data driven method, based on [25] we filter out the first IMF and reconstruct the estimated signal using equation 6. Then, we apply the EEMD method to the estimated signal again. Thus, by applying the combination of the two methods to the data, the smoothness of the data increases and the unwanted changes caused by noise are reduced. The present numerical results show that using this strategy leads to a significant improvement.

3. IDENTIFICATION PROCESS

In this section, as the main part of the paper, different levels of de-noising are considered, using the de-noising method explained in the previous Section and the de-noised signals are fed to an ANFIS network. De-noising may be applied on both train and test inputs.

The output of the network is the order function, calculated using the AOPIM. It is noteworthy to mention that the order is not a measurable signal in model 2 and it must be estimated using an estimation method. AOPIM is a promising estimation algorithm with convergence proof [13]. This is the reason it is chosen in this paper to provide the network output. We aim to verify the hypothesis that the order has a meaningful functionality with respect to the stress-strain pair in each moment.

One approach to improve generalization error and to improve the structure of the mapping problem is to add random noise. Many studies have noted that adding small amounts of input noise (jitter) to the training data often aids generalization and fault tolerance.

First, to depict the effect of the EEMD method, it is applied to some signals. Fig. 2 compares the original signal and its de-noised version with 1 and 2 levels of de-noising. Obviously, each level of de-noising removes a certain part of the noise exceeding the threshold power.

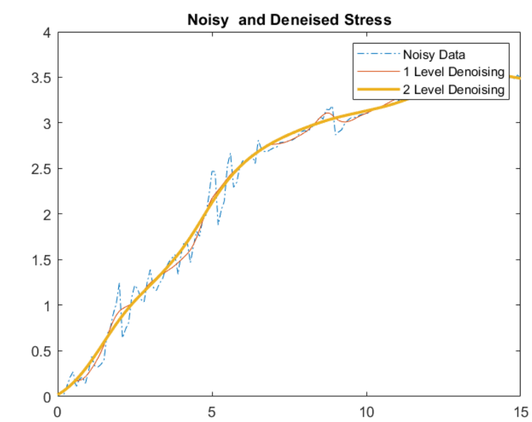

Fig. 2. Noisy and de-noised stress in two levels.

Now, ANFIS is fed with the input (the stress-strain pair) and the output (the order), extracted from different experiments. Fig. 3 shows the results captured by considering results of experiment no. 2 as the training dataset and testing the network using the dataset extracted from experiment no. 1 with two levels of de-noising on both test and train data sets. Also, Figs. 4 to 8 depict some different choices of train and test experiments.

The figures definitely verify the main hypothesis of the paper. There exists a meaningful functionality between the order and the status of the material which is detectable by ANFIS

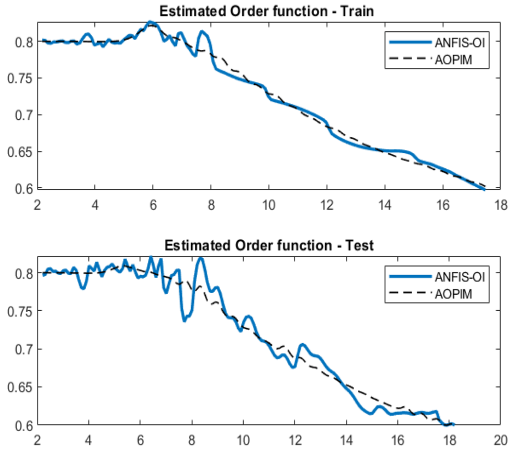

Fig. 3. Estimated order with ANFIS and AOIPM (Train: Experiment no.2. Test: Experiment no.1).

Fig. 4. Estimated order with ANFIS and AOIPM (Train: Experiment no.3. Test: Experiment no.1).

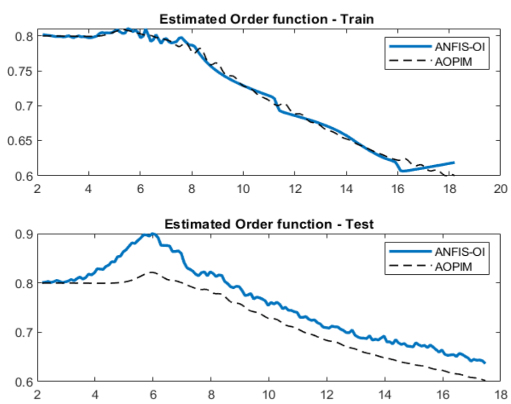

Fig. 5. Estimated order with ANFIS and AOIPM (Train: Experiment no.1. Test: Experiment no.2).

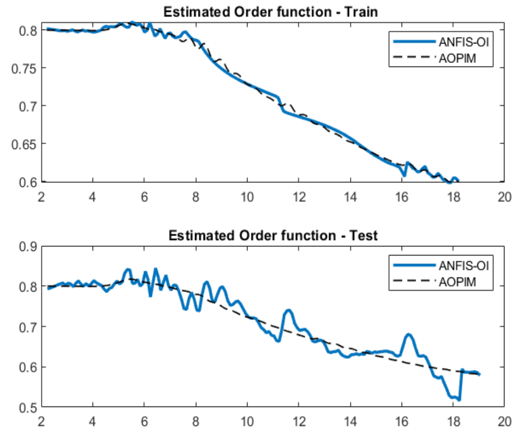

Fig. 6. Estimated order with ANFIS and AOIPM (Train: Experiment no.3. Test: Experiment no.2).

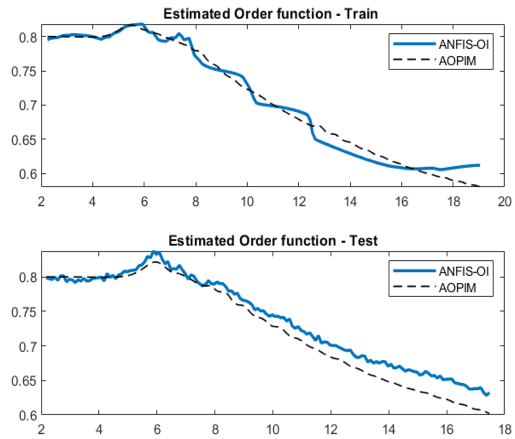

Fig. 7. Estimated order with ANFIS and AOIPM (Train: Experiment no.1. Test: Experiment no.3).

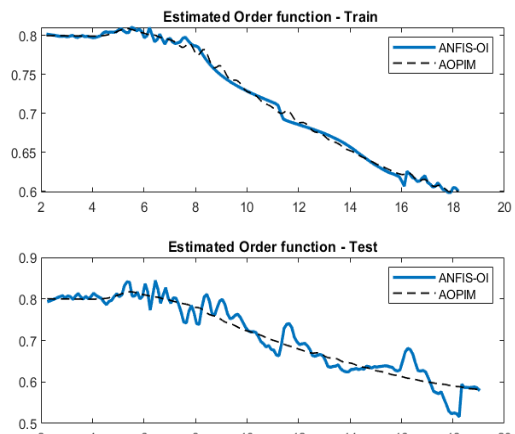

Fig. 8. Estimated order with ANFIS and AOIPM (Train: Experiment no.2. Test: Experiment no.3).

To discuss the effect of de-noising, the identification procedure is run several times on several pairs of experiments (as train and test pair), each considers different levels of de-noising in the training and testing process, in turn. The Mean of Squared Error between the ANFIS output and AOPIM output (as the reference output) is considered as the accuracy criteria. Table 1 shows the result.

The results shown in the table suggest that for the data gathered from the experiments 2 and 1 as the training and testing dataset, respectively, 2 De-Noising Level (DNL) for the test stress signal, because in all cases, the test error decreases from 0 to 2 DNL, however, it remains the same afterward. Training DNL is better to be set on 2, as well, since the training error does not decrease with more level of de-nosing.

Table 1. The MSE between ANFIS output and AOPIM output (as the reference output). Train: Experiment no.2. Test: Experiment no.1.
DNL: De-noising Level

Train DNL	Test DNL	Train MSE	Test MSE
0	0	0.01126	0.06528
	1		0.06421
	2		0.06269
	3		0.06269
	4		0.06269
1	0	0.00872	0.29401
	1		0.29142
	2		0.27677
	3		0.27677
	4		0.27677
2	0	0.00864	0.38191
	1		0.37914
	2		0.36057
	3		0.36057
	4		0.36057
3	0	0.00864	0.38191
	1		0.37914
	2		0.36057
	3		0.36057
	4		0.36057
4	0	0.00864	0.38191
	1		0.37914
	2		0.36057
	3		0.36057
	4		0.36057

Also, it is obvious that complete de-noising reduces the accuracy for the test error (resembling the generalization accuracy). The same result is obtained when other sets of data are chosen.

Table 2. The MSE between ANFIS output and AOPIM output (as the reference output). Train: Experiment no.2. Test: Experiment no.3.
DNL: De-noising Level

Train DNL	Test DNL	Train MSE	Test MSE
0	0	0.01126	0.035963
	1		0.035453
	2		0.032223
	3		0.032223
	4		0.032223
1	0	0.008718	0.076523
	1		0.073529
	2		0.057407
	3		0.057407
	4		0.057407
2	0	0.008638	0.124735
	1		0.120102
	2		0.096511
	3		0.096511
	4		0.096511
3	0	0.008638	0.124735
	1		0.120102
	2		0.096511
	3		0.096511
	4		0.096511
4	0	0.008638	0.124735
	1		0.120102
	2		0.096511
	3		0.096511
	4		0.096511

Table 3. The MSE between ANFIS output and AOPIM output (as the reference output). Train: Experiment no.1. Test: Experiment no.2.
DNL: De-noising Level

Train DNL	Test DNL	Train MSE	Test MSE
0	0	0.015367	0.383632
	1		0.284527
	2		0.265624
	3		0.265624
	4		0.265624
1	0	0.011786	0.410846
	1		0.398655
	2		0.411355
	3		0.411355
	4		0.411355
2	0	0.009738	0.550254
	1		0.521141
	2		0.550397
	3		0.550397

	4		0.550397
3	0	0.009738	0.550254
	1		0.521141
	2		0.550397
	3		0.550397
	4		0.550397
4	0	0.009738	0.550254
	1		0.521141
	2		0.550397
	3		0.550397
	4		0.550397

Table 4. The MSE between ANFIS output and AOPIM output (as the reference output). Train: Experiment no.1. Test: Experiment no.3.
DNL: De-noising Level

Train DNL	Test DNL	Train MSE	Test MSE
0	0	0.015367	0.38038
	1		0.258379
	2		0.183072
	3		0.183072
	4		0.183072
1	0	0.011786	0.232344
	1		0.215015
	2		0.210798
	3		0.210798
	4		0.210798
2	0	0.009738	0.330804
	1		0.289379
	2		0.296879
	3		0.296879
	4		0.296879
3	0	0.009738	0.330804
	1		0.289379
	2		0.296879
	3		0.296879
	4		0.296879
4	0	0.009738	0.330804
	1		0.289379
	2		0.296879
	3		0.296879
	4		0.296879

Table 5. The MSE between ANFIS output and AOPIM output (as the reference output). Train: Experiment no.3. Test: Experiment no.1.
DNL: De-noising Level

Train DNL	Test DNL	Train MSE	Test MSE
0	0	0.017514	0.050746
	1		0.050393
	2		0.050166
	3		0.050166
	4		0.050166
1	0	0.016967	0.072878
	1		0.072421
	2		0.072371
	3		0.072371
	4		0.072371
2	0	0.015826	0.162172
	1		0.161527
	2		0.158765
	3		0.158765
	4		0.158765
3	0	0.015826	0.162172
	1		0.161527
	2		0.158765
	3		0.158765
	4		0.158765
4	0	0.015826	0.162172
	1		0.161527
	2		0.158765
	3		0.158765
	4		0.158765

Table 6. The MSE between ANFIS output and AOPIM output (as the reference output). Train: Experiment no.3. Test: Experiment no.2.
DNL: De-noising Level

Train DNL	Test DNL	Train MSE	Test MSE
0	0	0.017514	0.062192
	1		0.051407
	2		0.050374
	3		0.050374
	4		0.050374
1	0	0.016967	0.072397
	1		0.054823
	2		0.053549
	3		0.053549
	4		0.053549
2	0	0.015826	0.077863
	1		0.054982
	2		0.053269
	3		0.053269
	4		0.053269
3	0	0.015826	0.077863
	1		0.054982
	2		0.053269
	3		0.053269
	4		0.053269
4	0	0.015826	0.077863
	1		0.054982
	2		0.053269

	3		0.053269
	4		0.053269

4. CONCLUSION

In this paper, a non-integer order model describing the stress-strain relationship of soft tissue in one dimension was investigated and the hypothesis that in such a modeling approach the order is a function of tissue status was considered. The tissue status refers to its moment stress and strain. For this purpose, ANFIS was used to estimate the functionality.

To provide output in supervised training, obtained with AOPIM was taken into account. In this optimization process, the order is assumed to be variable with time. After training, the neural network is obtained as a function of stress and strain. In this process, in fact, the time argument is removed from the input (stress and strain) and output (order), and direct functionality is extracted.

Noise and de-noised data were used to train the neural network, using the EEMD method with different levels. The results of various experiments showed that performing two levels of noise reduction in the network training stage leads to minimum error. On the other hand, in order for the trained network to be able to act more accurately on the test data, we need to perform two-level noise reduction.

Future work may concentrate on estimating the order and predicting the soft tissue deformation during a procedure like surgery or needle insertion, using the approach proposed in this paper.

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