Spectral Method of Identification of Peltier Thermoelectric Elements Based on Piecewise Linear Approximation

Gleb S.Vasilyev[1], Oleg .R. Kuzichkin[2], Dmitry I. Surzhik[3], Aleksandr V. Koskin[4]
1- Belgorod State University, Belgorod, 308015,Russia.
Email: vasilievgleb@yandex.ru (Corresponding author)
2- Belgorod State University, Belgorod, 308015, Russia.
Email: oldolkuz@yandex.ru
3- Vladimir State University, Vladimir, 600000, Russia.
Email: arzerum@mail.ru
4- Orel State University by I. S. Turgenev 95 Komsomolskaya Str., Orel, Russia.
Email: aleksandrkoskin@gmail.com

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ABSTRACT:
The functioning of a thermoelectric system in a stationary mode is often inefficient because it does not allow flexible control of the temperature regime. The design of thermoelectric devices and systems in transient modes requires identifying their dynamic models, first of all, the control object – the Peltier thermoelectric module. As a rule, well-known identification techniques involve calculating the parameters of a fractional-rational transfer function (or, equivalently, an autoregressive model) of the object of control. At the same time, the increase in the accuracy of identification requirements is associated with significant computational costs. The proposed identification technique requires the determination of the time dependencies of the control current and temperature deviation, the calculation of their spectra based on piecewise linear approximation, and the calculation of the transfer coefficient as the ratio of the spectra of the output and input signals. The calculated relations of piecewise linear approximating functions, spectral densities, and time forms of input and output parameters of the model under study are presented. The amplitude and phase frequency response of the Peltier module are calculated. The low RMS error in the identification of the amplitude-frequency characteristic and phase-frequency characteristic showed the effectiveness of the proposed identification technique.

KEYWORDS: Peltier Element, Transition Mode, Piecewise Linear Function, Spectral Method.

1. INTRODUCTION

Existing and prospective thermoelectric systems in various fields of application operate on the basis of standard Peltier thermoelectric modules [1-6]. The operation of a thermoelectric system in a stationary mode is often inefficient, because it does not allow flexible control of the temperature regime. The design of thermoelectric devices and systems in transient modes requires identification of their dynamic models, first of all, the control object – a Peltier thermoelectric module or a thermoelectric generator [7]. As a rule, well-known identification methods involve calculating the parameters of a fractional-rational transfer function (or, equivalently, an autoregressive model) of the object of control [8,9]. Block models have become important for modeling this type of installations [10], among them the Hammerstein model, the Wiener model, and the Hammerstein-Wiener model. Optimization techniques for Hammerstein models include the cuckoo algorithm [11], swarm optimization [12], the gravitational algorithm [13] and the sine-cosine algorithm [14]. At the same time, the increase in the accuracy of identification requires is associated with significant computational costs. At the same time, simple 1st order linear models have low accuracy or limited scope [15,16].

The aim of the article is to propose a spectral technique for the identification of Peltier thermoelectric elements using approximation by piecewise linear function of the control current and the resulting temperature deviation.

2. NECESSARY EXPRESSIONS

The object of research is a thermoelectric Peltier module shown in Fig. 1. The cold and hot sides of the module are in contact with ceramic plates acting as insulators. A structure formed by n-type and p-type

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semiconductors is located between the plates. Module sides contact with p-n junctions or n-p junctions.

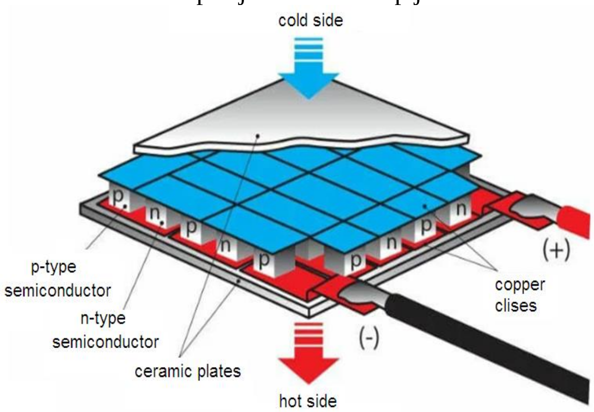


Fig. 1. The scheme of the thermoelectric Peltier module.

The unsteady temperature field created by the Peltier module as a result of the influence of the control current is described as

$$T = f(x, y, z, t),$$

Where x, y, z - the spatial coordinates, t is time. A temperature distribution caused by internal heat sources might be represented by Fourier differential equation of the form [17]

$$\frac{\partial T}{\partial t} = a\nabla^2 T + \frac{q_V}{c\rho}, \tag{1}$$

Where $a = \dfrac{\lambda}{c\rho}$ is the thermal conductivity, λ is the temperature transfer coefficient, c is specific heat capacity, ρ is density, $\nabla^2 T = \dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2}$ is Laplace operator, q_V is the heat released per unit volume of the environment per unit time.

This type of heat exchange with convection and thermal conductivity might be described by a formula

$$\frac{\partial T}{\partial t} + w_x\frac{\partial T}{\partial x} + w_y\frac{\partial T}{\partial y} + w_z\frac{\partial T}{\partial z} = a\nabla^2 T, \tag{2}$$

Where w_x, w_y, w_z are the projections of the velocity on the coordinate axis *X*, *Y*, *Z*.

The operator transfer functions of the Peltier element *H*(*p*) represent the ratio of the temperature response of the element to the change in the control current $X(p) = \Delta I_{control}(p)$ that caused this response, and can have a high order and have nonlinear properties:

$$\acute{I}\ (p) = \Delta T(p) / X(p).$$

By replacing the differentiation *p* with frequency *jω*, we derive the transfer coefficient of the simulated system *H*(*jω*). The spectrum of deviation of the input parameter (control current) is denoted as $S_{in}(j\omega)$. Then the spectrum of deviation of the output parameter (temperature response)

$$S_{out}(j\omega) = S_{in}(j\omega)\cdot \acute{I}\ (j\omega). \tag{3}$$

Let us consider the problem of studying the transient process of the Peltier element when changing the control current of an arbitrary shape. In this case, the analytical expression of the spectrum of the control current is absent or has a bulky appearance. Numerical calculation of the spectrum based on the direct fast Fourier transform requires significant computational costs and does not make it possible to obtain generalized solutions. Piecewise linear approximation of the control current function from time (Fig. 2) allows us to obtain a generalized expression of its spectrum [18-20].

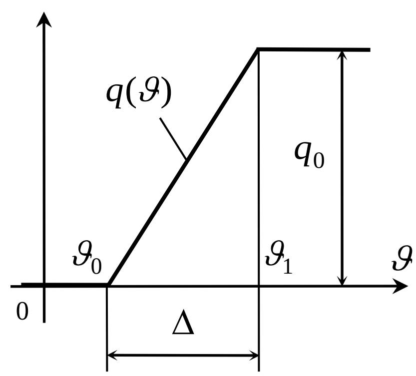


Fig. 2. Piecewise linear function approximating the dependence of the control current on time.

The expression of a piecewise linear function of the form (Fig. 2) has the form

$$q(\vartheta) = \frac{q_0}{2\Delta}\left(\left|\vartheta - \vartheta_0\right| - \left|\vartheta - \vartheta_1\right| + \Delta\right),$$

To obtain characteristics of various shapes, it is necessary to sum several functions with a shift along the argument axis:

$$q_i(\vartheta) = \frac{q_{0_i}}{2\Delta_i}\left(\left|\vartheta - \vartheta_i\right| - \left|\vartheta - \vartheta_i + \Delta_i\right| + \Delta_i\right),$$

Where q_{0_i} is the height of the i-th function (approximation coefficient), ϑ_{i} is the argument at the current segment , $\Delta_i = \vartheta_{\mathrm{i+1}} - \vartheta_{\mathrm{i}}$ is the current approximation step.

The approximation coefficients are determined by the difference in the values of the original function $\mathrm{f}(\vartheta)$ in neighboring nodes:

$$q_{0i} = \mathrm{f}(\vartheta_{i+1}) - f(\vartheta_i)\,.$$

The approximation formula based on the sum of piecewise linear segments has the form

$$q_{\Sigma}(\vartheta) = \sum_{\mathrm{i}=0}^{N-1} q_i(\vartheta) = \sum_{\mathrm{i}=0}^{N-1} \frac{q_{0_i}}{2\Delta_i}\left(\left|\vartheta - \vartheta_{\mathrm{i}}\right| - \left|\vartheta - \vartheta_{\mathrm{i}} + \Delta_i\right| + \Delta_i\right),$$

Where N is the number of approximation nodes.

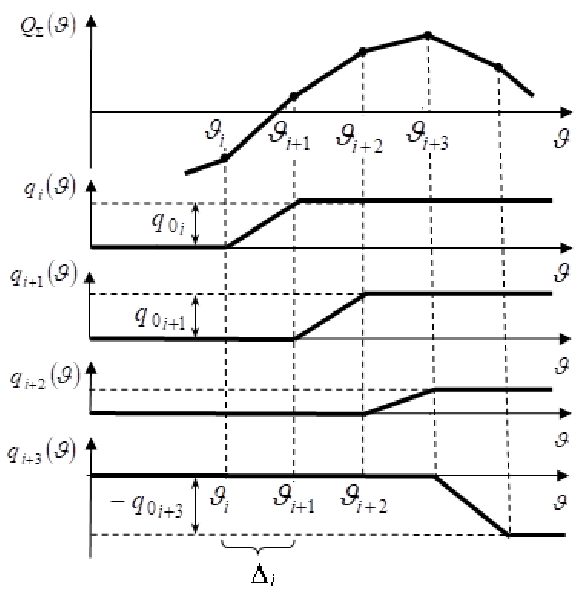


Fig. 3. Approximation of the dependence of the control current on time by the sum of piecewise linear functions.

The expressions obtained make it possible to approximate the complex dependencies of the control current in the modes of continuous or pulsed control of a thermoelectric system. The choice of a large number of linear fragments makes it possible to represent substantially nonlinear dependencies and impacts with high accuracy.

The spectral density of the input signal parameter (deviation of the control current) or the output parameter (temperature changes caused by this control action) is found using the direct Fourier transform. To represent the impact of a complex form, it is necessary to apply the sum of piecewise functions. In general, the expression for the sum of N functions will take the form:

$$q_{\Sigma}(\mathrm{t}) = \sum_{\mathrm{i}=0}^{N-1} q_i(t) = \sum_{\mathrm{i}=0}^{N-1} \frac{q_{0_i}}{2\Delta_i}\left(\left|t - t_{\mathrm{i}}\right| - \left|t - t_{\mathrm{i}} + \Delta_i\right| + \Delta_i\right) \quad (4)$$

In general, the spectrum of a non-periodic signal based on the Fourier transform [20] has the form:

$$S(\mathrm{j}\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\cdot\omega\cdot t}dt\,.$$

Thus, based on expression (4), the expression for calculating the signal spectrum takes the form:

$$S(\mathrm{j}\omega) = \int_{-\infty}^{\infty}\sum_{\mathrm{i}=0}^{N-1} \frac{q_{0_i}}{2\Delta_i}\left(\left|t - t_{\mathrm{i}}\right| - \left|t - t_{\mathrm{i}} + \Delta_i\right| + \Delta_i\right)\cdot e^{-j\cdot\omega\cdot t}dt$$

We take out the sign of the sum from the integral:

$$S(\mathrm{j}\omega) = \sum_{\mathrm{i}=0}^{N-1} \frac{q_{0_i}}{2\Delta_i}\int_{-\infty}^{\infty}\left(\left|t - t_{\mathrm{i}}\right| - \left|t - t_{\mathrm{i}} + \Delta_i\right| + \Delta_i\right)\cdot e^{-j\cdot\omega\cdot t}dt\,.$$

We obtain an expression for the spectrum of one element of the sum:

$$S_{\mathrm{i}}(\mathrm{j}\omega) = \frac{q_{0_i}}{2\Delta_i}\int_{-\infty}^{\infty}\left(\left|t - t_{\mathrm{i}}\right| - \left|t - t_{\mathrm{i}} + \Delta_i\right| + \Delta_i\right)\cdot e^{-j\cdot\omega\cdot t}dt$$

We take into account $t_{i+1} = t_i + \Delta_i$ and represent the integrand as follows:

$$\left|t - t_i\right| - \left|t - t_i + \Delta_i\right| + \Delta_i = \begin{cases} 0, t < t_i \\ t - t_i, t_i \le t < t_{i+1} \\ 1, t > t_{i+1}. \end{cases}$$

Based on expression (62), integral (61) is divided into two. Let's write down an analytical expression for the spectrum of the entire sum:

$$S(\mathrm{j}\omega) = \sum_{\mathrm{i}=0}^{N-1} \frac{q_{0i}}{\Delta_i\omega^2}\left[e^{-j\omega(\mathrm{t}_i+\Delta_i)} - e^{-j\omega t_i}\right]. \quad (5)$$

Expression (5) can be used for analytical calculation of the spectra of complex control actions and TEM output parameters for any number of approximation nodes N, which determines the convenience of its application for the study of thermoelectric devices and systems by the spectral method. The choice of an adaptive approximation step expands the scope of its application and allows to achieve high accuracy of calculations with fewer points.

In accordance with expression (3), the identification of the TEM transfer function is reduced to solving a problem of the form

$$\hat{Í}\ (j\omega) = \frac{S_{out}(j\omega)}{S_{in}(j\omega)}, \tag{6}$$

In this case, the identification error is determined by the difference between the estimated and true transfer function:

$$\Delta\hat{Í}\ (j\omega) = \hat{Í}\ (j\omega) - Í\ (j\omega). \tag{7}$$

3. MODELING OF IDENTIFICATION OF THE FREQUENCY RESPONSE OF THE PELTIER ELEMENT

Let's perform a simulation of the transition process of the Peltier element, represented by an experimentally verified model [21,22]:

$$H(p) = \frac{p + 0.1323}{p^2 + 0.5964p + 0.00855}.$$

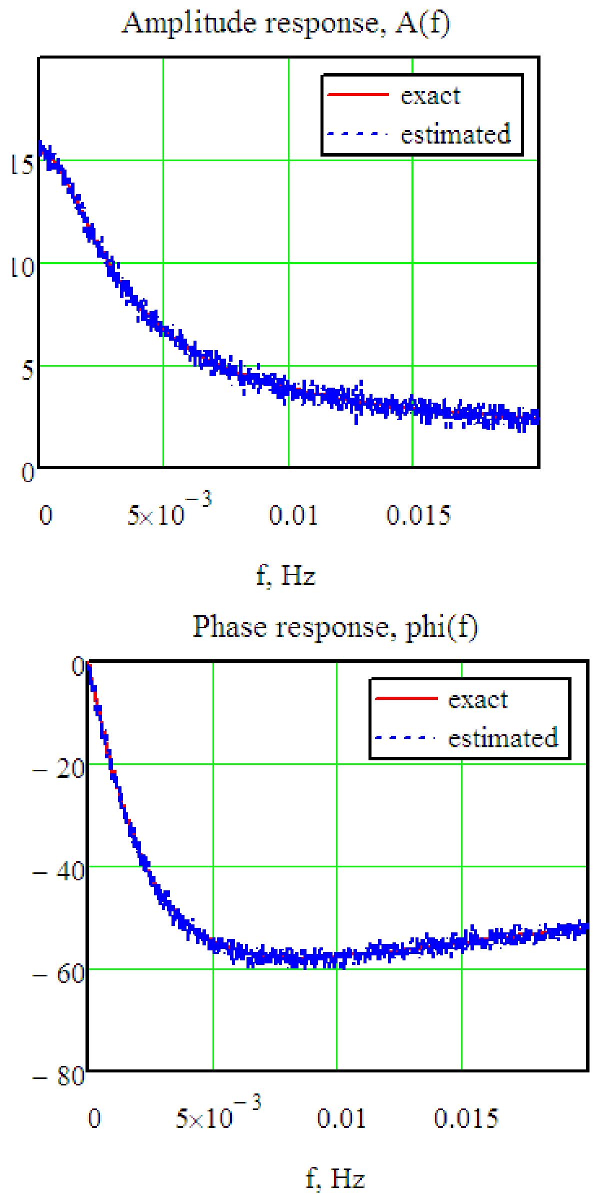
The control current of the Peltier element is set by an additive mixture of a sinusoidal function with an amplitude of 1 A and a frequency of 0.2 Hz, as well as centered Gaussian noise with a standard deviation of 0.4 A.

The exact value of the temperature response spectrum of the Peltier element in accordance with expression (3) is determined by the product of the spectrum of the control current and the transfer function of the device:

$$S_{out}(j\omega) = S_{in}(j\omega) \cdot H(j\omega) = \frac{10\pi(j\omega+0{,}1323)}{(-25\omega^2+4\pi^2)(-\omega^2+0{,}5964j\omega+0{,}00855)}.$$

To test the noise immunity of the method, a centered Gaussian noise with a standard deviation of 0.5 ^{0}C was added to the output parameter before performing identification. The specified noise levels correspond to the practical operating conditions of Peltier modules with comparable characteristics. The approximation of the control current action and the temperature response of the TEM was performed at 100 nodes in the range from 0 to 10 s.

Figure 3 shows the amplitude (a) and phase (b) frequency responses, calculated as a module and an argument of the exact transfer function of the module (exact), as well as estimated as a result of identification (estimated). The standard error of identification was: for frequency response – 0.3 ^{0}C, for frequency response – 1 degree. The low error shows the effectiveness of the proposed identification technique.



a)

Fig. 4. Amplitude-frequency (a) and phase-frequency (b) response of TEM: exact and estimated as a result of identification.

4. CONCLUSION

The developed method of identification of the dynamic model of the Peltier module can be used to study the operating modes of a thermoelectric device of arbitrary order with arbitrary dependencies of control currents and temperature response on the cold or hot side. In this case, the computational costs are determined by the number of nodes of approximation of time dependencies. The increase in computational costs with growing order of the model in this case manifests itself significantly less than in the known methods of identifying the coefficients of the fractional rational transfer function of the Peltier module. The developed approach makes it possible to study the characteristics and quality indicators of Peltier elements and thermoelectric systems in general. The approach is suitable for optimization and synthesis tasks of designed thermoelectric devices and systems. The technique can also be used to diagnose the condition of devices and systems during operation: in this case, the excess of the difference between the estimated and the reference characteristic of the device of the selected threshold indicates a malfunction of the tested element.

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