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Implementation of class topper optimized fractional order proportional-integral-derivative controller for cart-inverted pendulum system

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Abstract:

Controlling a cart-pendulum system involves both swinging the pendulum up and maintaining its upright position. Typically, separate controllers are used for the swing-up and stabilization phases. This paper presents the implementation of a Fractional Order Proportional-Integral-Derivative (FOPID) controller for the cart-pendulum system. A novel metaheuristic method, Class Topper Optimization (CTO), is utilized to design and optimize the FOPID controller's performance for both the cart and pendulum. The study focuses on the pendulum angle and arm angle as key parameters. Results indicate that the proposed approach outperforms existing methods such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Ant Colony Optimization (ACO).

Keywords: Inverted cart pendulum; Non-linear controllers; Fractional order PID controller; Class topper optimization; Performance enhancement

1. Introduction

The inverted pendulum represents the prototypical classical problem in control engineering because it is a nonlinear and unstable system. This system is used as a test model to gauge the efficacy of various control strategies due to its nonlinear and multivariable dynamic. The cart may swing back and forth along the x-axis because of an inverted pendulum with only one dimension [1]. The degree of freedom is expanded to incorporate both the cart's location and the pendulum's swing direction. It takes just one input and produces several results. The angle of the pendulum and the position of the cart are the system's outputs as a function of the force applied to the cart. Examples of real-world issues that may be applied to this simulation include robotic arms, missiles, rocket launches, segways, bicycles, and a huge crane hoisting cargo [2, 3]. The system is stable while the pendulum is pointing down, and unstable when it is pointing up. The only way to achieve vertical equilibrium is to apply a constant force. The system is divided into two parts for regulation purposes: the first part is responsible for bringing the pendulum to a resting position from the awaiting, unstable state. Stability, the second type of control, maintains the pendulum in a vertical position for an extended time [3]. A PID controller has been presented to achieve the swing up control with tuned parameters, and LQR is used to enhance system parameters and keep the pendulum in a steady state [4]. In this approach, the genetic algorithm is used to generate Q and R. Using the proposed control techniques, this article analyses the system's performance characteristics.

The study of control systems, particularly the cart-inverted pendulum system, has been a fundamental topic within the field of dynamic systems and control engineering. This system, characterized by its inherent instability and nonlinear dynamics, presents a significant challenge and serves as an ideal testbed for evaluating various control strategies. Among these strategies, the Fractional Order Proportional Integral Derivative (FOPID) controller has gained considerable attention due to its superior flexibility and robustness in handling complex control tasks [5–9]. This paper explores the optimization of the FOPID-based cart-inverted pendulum system using the Class Teaching Optimization (CTO) algorithm. This optimization technique has not been applied for the cart-inverted pendulum system as per the best explored literature by authors and hence by leveraging the search mechanism inherent in CTO, the research aims to enhance the performance and stability of the FOPID controller, providing a more efficient and effective solution for managing the intricate dynamics of the cart-inverted pendulum system.

The study is structured into five main parts. As mentioned earlier, Section 1 aims to offer a comprehensive overview of the entire project. Following this, Section 2 presents a review of relevant literature. Sections 3 and 4 delve into detailing the proposed algorithm and presenting the results, respectively. The paper wraps up with Section 5, which serves as the conclusion. Additionally, the concluding section of the analysis provides a concise summary of the study's findings.

2. Literature review

There are various inverted pendulum systems mentioned in published works. Some typical examples of inverted pendulums [10] are the cart inverted pendulum [11], the double inverted pendulum [12], the revolving single arm pendulum [13], and the rotational two link pendulum [14]. The cart inverted pendulum has been used often over the last seven decades as a control benchmark problem and one such mechanism is a control that follows a pendulum. From a stable equilibrium position, the pendulum will swing upward when it reaches an unstable equilibrium position [15]. The cart may be directed in the right direction after the inverted pendulum has been stabilized around the unstable equilibrium point [16]. Different strategies for managing inverted pendulum systems have been extensively researched in books and articles. It may be subdivided into several distinct forms of management. In [17, 18], a conventional self-tuning PID controller is used to operate the inverted pendulum rod. Artificially intelligent controllers are not the only kind available. The design incorporates controllers based on neural networks and fuzzy logic. In [19], a single input rule module is connected to a fuzzy inference model to keep the inverted pendulum in balance. In [20], a neural network is used to instruct an inverted pendulum system in its proper operation.

Even while intelligent controllers have gained popularity, there is no assurance that they will provide a reliable system because of how they were built. Using a Lyapunov-based controller, [21] aims to bring the inverted pendulum under control. Additionally, [22] investigates sliding mode control. Several studies in recent years have looked at the question of how to regulate a pair of inverted pendulums. In [23, 24], linear quadratic regulator (LQR) and feedback linearization methods were introduced as a means of controlling the swing-up and stability of a wheeled Robot. To regulate double inverted pendulum systems, a kernel-based dual heuristic programming strategy [25] may be used. Experiments in tracking a multivariate nonlinear system with unknown nonlinear functions and parameters were conducted utilizing an output-feedback adaptive backstepping control strategy [26].

An adaptive PI Hermite neural control system was proposed for uncertain nonlinear systems and evaluated using a simulated double inverted pendulum [27]. An online learning control system that does not need explicit knowledge of the plant was evaluated using the balance problem of a double inverted pendulum [28]. For a double pendulum to swing up and remain balanced, a feedback control system was given in the form of a bang-bang control torque delivered at the suspension point [29]. With the Model Reference Adaptive System technique [30], a multivariable adaptive control approach was developed to keep a cart-type double inverted pendulum in an up-up unstable equilibrium state. A strategy based on energy and intuition may be used to stabilize and raise a double inverted pendulum [31]. Using a neuro fuzzy controller that incorporates a double chain quantum genetic algorithm [32], a double inverted pendulum was brought under real-time control. It is well-documented that an inverted pendulum system mounted on a cart can be stabilized.

The earliest traditional control approach is proportional-integral-derivative (PID) control [33], followed by linear quadratic regulator (LQR) control [34], both of which need an explicit mathematical description of the system. Fuzzy logic, neural network, and adaptive PID control [35–37] are all examples of intelligent control approaches that can function well without an exact mathematical model. The initial solutions to the 2-Rate controller design problem were found in [38–40]. These papers present algorithms designed to guarantee that the 2-Rate controller's parameters achieve the desired closed-loop placement and loop-zero placement, guaranteeing a good response and enough robustness.

In this paper, a FOPID controller is employed, which offers greater flexibility and precision than PID. Although PID controllers are easier to implement and widely adopted but FOPID controllers can provide better performance in terms of stability and precision, especially in complex systems. FOPID controllers are employed by researchers to solve various problems. A case study on a non-linear continuously stirred reactor under model uncertainties is presented in [6], where a novel $FOI^{\lambda}D^{1-\lambda}$ controller is implemented and stability region is explored through the complex root boundary (CRB) analysis. The effectiveness of the identification of fractional-order models (IDFOM) approach to augment the same with a robust indirect fractional-order internal model control (IFOIMC) is illustrated by comparing the closed-loop performance achieved by IFOIMC controllers designed using two widely used identification strategies [8] A novel FDA-tuned FOPID-$(1+TD)$ cascade control strategy for a challenging two-area system is presented in [41]. A hybrid version of two metaheuristic algorithms (arithmetic optimization and African vulture's optimization algorithm) is developed and is used to tune a novel type-2 fuzzy-based proportional-derivative branched with dual degree-of-freedom proportional-integral-derivative controller for the LFC of a three-area hybrid deregulated power system [42]. [43] modify the RIMC (as a bi-loop RIMC proportional-derivative (RIMC-PD)) strategy to make it applicable to a class of unstable and integrating systems involving inactive

time. A novel type-2 fuzzy proportional derivative-integral control tactic, tuned with an arithmetic optimizer (AO) is suggested for controlling load frequency in a deregulated power system with distributed generating (DG) units and plug-in electric vehicles (PEVs) [44]. A novel combination of fractional order Lyapunov (FOLyapunov) rule and fractional order proportional integral (FOPI)/two-degree of freedom FOPI (2DOF-FOPI) controller is proposed in [45]. [46] attempts to further demonstrate the capability of a highly effective metaheuristic approach known as Harris Hawk's optimization algorithm. The optimal tuning of a fractional order PID controller used in an AVR system is proposed by using the optimization technique called honey badger algorithm (HBA) is presented in [47]. [48] proposed a novel approach for designing a fractional order proportional-integral-derivative (FOPID) controller that utilizes a modified elite opposition-based artificial hummingbird algorithm (m-AHA) for optimal parameter tuning. To achieve a greater performance of the FOPID controller, a novel metaheuristic algorithm, which consists of a balanced structure in terms of explorative and exploitative phases, has also been developed by hybridizing the Hunger Games search algorithm with simulated annealing technique in [49]. The superiority of GOA-based FOPID controllers over HSFS-FOPID, GWO-FOPID, SMA-PID, AOA-HHO-PID and HASO-SA-PID controllers is demonstrated in [50]. The proposed modification in [51] is aimed to be used as an efficient tool to tune a fractional order proportional-integral-derivative (FOPID) controller to control a magnetic ball suspension system with greater flexibility.
Hence, in the proposed experiment, a FOPID controller is applied to an inverted pendulum cart. A novel Class Topper Optimization will be used to enhance performance.
Although, CTO-based controllers, while effective in many scenarios, do come with several drawbacks and limitations such as Complexity wherein CTO-based controllers often involve complex algorithms and require significant computational resources, making them challenging to implement and maintain. Due to their complexity and the need for advanced hardware, CTO-based controllers can be expensive to develop and deploy. As systems grow and complex, the scalability of CTO-based controllers can become an issue, potentially leading to performance degradation. These controllers may lack the flexibility to adapt to rapidly changing environments or unforeseen conditions, limiting their effectiveness in dynamic applications. Proper tuning and calibration of CTO-based controllers can be time-consuming and require expert knowledge, adding to the overall operational burden. In some cases, CTO-based controllers may be prone to failure if not effectively managed, which can impact on the reliability and safety of the systems they control. Despite the above-mentioned drawbacks, the controller is giving satisfactory results for the problem discussed in the manuscript. It is a new methodology. It will be more effective than other optimization methods, like GA [52], GA-LQR [4], PSO [53], ACO, GWO [54–57], Augmented Hunger Game Search Algorithm [58] and others.

3. Mathematical modelling of cart-inverted pendulum

Fig. 1 depicts the mathematically designed free body diagram of an inverted pendulum cart configuration [54, 59, 60]. Small mass and low friction are considered alongside a hinge and a Pendulum rod. m and M represent the pendulum and cart masses, respectively.

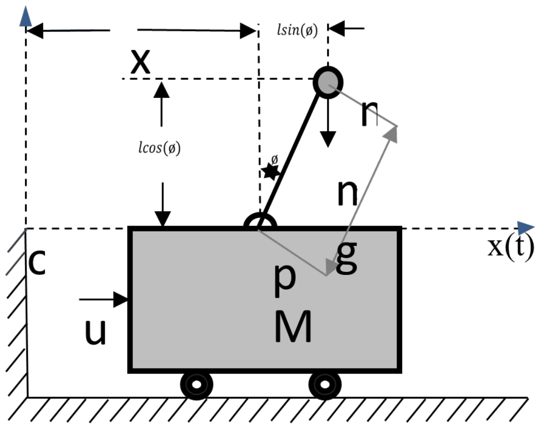

Figure 1. Cart-Inverted based Pendulum Arrangement.

The proposed system makes use of the two forces of gravity (g) and externally imposed $u(t)$ to achieve its goals. Output variables to be manipulated (2-DOF) are presented in Fig. 1 for the cart's position $x(t)$ and the pendulum's angular displacement $\phi(t)$. To some extent, this system is underactuated since the externally provided signal $u(t)$ only controls $x(t)$ and leaves $\phi(t)$ uncontrolled. The combined energies of the cart-mass acceleration force and the corresponding x-direction pendulum acceleration force will be a good approximation of the total externally utilised force, as shown below [59].

$$M\frac{d^2}{dt^2}x + m\frac{d^2}{dt^2}x_m = u \tag{1}$$

where x and y represent the mass and direction of the pendulum, respectively, and

$$x_m = l\sin\phi + x \text{ and } y_m = l\cos\phi \tag{2}$$

The result is obtained by plugging in the values of the parameters in equation 4 into the relevant spots in equation 3.

$$M\frac{d^2}{dt^2}x + m\frac{d^2}{dt^2}(l\sin\phi + x) = u \tag{3}$$

More simplification may be achieved by resolving the following equations.

$$(M+m)\ddot{x} - (ml)\sin\phi\dot{\phi}^2 + (ml\cos\phi\ddot{\phi}) = u \tag{4}$$

By considering the effect of the pendulum's length and torque on the pendulum mass because of the acceleration and torque of gravitational force, we can deduce the torque steadiness equations of the arrangement (g). The results will be.

$$(u_x\cos\phi l) - (u_y\sin\phi l) = (mg\sin\phi l) \tag{5}$$

The components of external forces are derived as follows.

$$u_x = m\frac{d^2}{dt^2}x_m = m[\ddot{x} - l\sin\phi\dot{\phi}^2 + l\cos\phi\ddot{\phi}] \quad (6)$$

$$u_y = m\frac{d^2}{dt^2}y_m = -m[l\cos\phi\dot{\phi}^2 + l\sin\phi\ddot{\phi}] \quad (7)$$

Replacing equation 8 as well as the 9 in the torque equation 7,

$$m\ddot{x}\cos(\phi) + ml\ddot{\phi} = mg\sin\phi \quad (8)$$

The inverted pendulum may be in either of two states: a stable pendant position or an unstable upright position. To stabilize the pendulum in its upright posture, it must first swing from a steady to an imbalanced state and then maintain that state with as slight variation as possible. Displacement of the cart and the resulting change in pendulum angle make fine-tuning the controller more tedious. Further, transfer functions of the cart position and pendulum angle can be given by 9 and 10 respectively.

$$H_{cart} = \frac{X(s)}{U_(s)} = \frac{0.132(s^2 + 0.0378s - 12.98)}{0.339(s^3 + 0.058s^2 - 13.27s - 0.253)} \quad (9)$$

$$H_{angle} = \frac{\phi(s)}{U_(s)} = \frac{0.0874s^2}{0.339(s^3 + 0.058s^2 - 13.27s - 0.253)} \quad (10)$$

4. Implementing fopid for cart and pendulum using CTO

In this study, we provide a revised CTO method. The suggested CTO revision broadens the breadth of the student's education to the point where it can pick the brain of the best student in the class if necessary. The first CTO algorithm did not allow this to happen. The suggested change improves the classroom setting for all students. A new self-evaluation phase is included in the improved CTO. Exam results may be used as part of a self-evaluation process in which students can get insight into their own progress. Every student, based on their current and prior achievement, may plan their own educational path. Like the traditional CTO, the e-CTO has two learning phases. The first step is for the section top scorers (STs) to learn from the class top scorers (CT), and the second is for the pupils in each section to learn from their STs. For further information on the current CTO, one might look at [61].

4.1 Controller Design

PID controllers are a popular controller and have been used in various applications which include industrial use, robotics, spacecraft and many more. This type of controller is simple in design and easy to implement and provides satisfactory performance which includes small settling time and low percentage overshoot. Because of this popularity and importance of PID controllers there has been a continuous process to improve robustness as well as the quality of such controllers. To make PID controller more robust, fractional order PID controller based on fraction calculus is introduced and has been accepted widely.

Fractional order PID controller was first introduced in [62] and since then it has applied over many applications. This type of controller has been implemented for inverted pendulum, robotic manipulator, distillation column [63] and many more. Such controller has integral of order λ and a differentiator of order μ and has proven better response than classic PID controller FOPID has advantage over classic controller as FOPID is less sensitive towards parameter change and has better control for dynamic system. The transfer function, $G(s)$, for FOPID can be given as:

$$G(s) = k_P + k_I\frac{1}{s^\lambda} + k_D s^\mu \quad (0 < \mu, \lambda < 1) \quad (11)$$

Where k_P is proportional gain, k_I is integral gain and k_D is differential gain.

Time domain representation of FOPID controller is given as:

$$u(t) = k_P e(t) + k_I D^{-\lambda} e(t) + k_D D^\mu e(t) \quad (12)$$

Where $e(t)$ is error signal and $u(t)$ are controller output.

In fractional calculus generalization of differentiation and integration to non-integer order is done through fundamental operator $a^{D_t^\alpha}$ and this operator can be given as:

$$a^{D_t^\alpha} = \begin{cases} \frac{d}{dt^\alpha}, & a > 0 \\ 1, & a = 0 \\ \int_0^t (d\tau)^{-\alpha}, & a > 0 \end{cases} \quad (13)$$

The block diagram of FOPID controller has been shown on Fig. 2.

Many researchers have published different definitions of fractional order integrator and fractional order differentiation. Popular methods for fractional order calculus are Grunwald-Letnikov (G-L) [64], Riemann and Liouville (R-L), Caputo definition, Oustaloup's approximation [65] and many more. In this paper, for implementation of fractional order, Oustaloup's approximation is used. This technique is preferred over others because of its ability to implement real hardware. The approximate transfer function provided

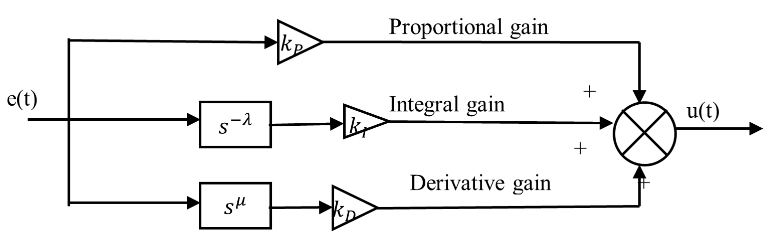

Figure 2. Block diagram for FOPID controller.

by Oustaloup [65] is as follows:

$$s^r = k_0 \prod_{k_0}^{N} = -N^{\frac{s+w_{k_z}}{s+w_{k_p}}} \quad (14)$$

Where r is real number, k_0 is gain, w_{k_z} and w_{k_p} are zeroes and poles respectively of the filter. These can be recursively calculated by the following equation.

$$w_{k_z} = w_b\left(\frac{w_h}{w_b}\right)^{\frac{k+N+\frac{1}{2}-\frac{r}{2}}{2N+1}} \quad (15)$$

$$w_{k_p} = w_b\left(\frac{w_h}{w_b}\right)^{\frac{k+N+\frac{1}{2}+\frac{r}{2}}{2N+1}} \quad (16)$$

$$k_0 = w_h^r \quad (17)$$

Where r is order of fractional differentiator-integrator, $2N+1$ is order of approximation and $[w_h, w_b]$ is fitting range.
Performance of the above approximation depends on value of N. Small value of N means simplicity in approximation with lesser complexity and higher value indicates more complexity. But if use smaller value of N then it can cause ripples in phase and gain behaviors [66].

4.2 Enhanced learning in e-CTO

The process of learning is a means by which the average student in a group might enhance their performance by picking up information from the more seasoned competitors in that group. The e-CTO is like the current CTO in the first stage of learning, but the second stage of learning is improved to provide each student with more opportunities to gain experience.
Specifically, there are two distinct types of learning that make up the second stage: internal learning and external learning. The number of students in each classroom (S). It is possible for there to be in different subsections inside a given class, and each one of them will need its own ST. The CT is just one kind of ST. Each group of students benefits from learning not just from their own ST but also from the other STs, but only if the other ST has a higher level of comparison knowledge. Having CT available in this manner allows for the education of all students. Each student tries to learn not only from the student who finished first in his or her own section (internal learning), but also from the students who finished first in all other sections (external learning). Consequently, this alternative learning environment provides every student with an opportunity to gain experience directly from CT, something that was previously impossible inside the traditional classroom environment. In what follows, we will discuss the differences between learning from inside and learning from beyond.

4.3 Internal and external learning

Every single student in every single class uses both internal and external learning strategies. The planned e-CTO instructs students to begin their education by classifying themselves as either an internal or external applicant (Toppers). These are the best students in that student's section, both from inside the school and from outside it. Internal learning is defined as knowledge gained from inside an organization, whereas external learning is information gained from outside organizations. There is only ever one internal candidate, and that person is the top performer in the class being attended by the student. There can be no outside candidates, no one, or many. Only the best students from outside the institution, who have more expertise overall, are considered for the external positions. Each ST is given a ranking based on how well it performed on the test. Now, a student's ST rating determines how he or she will be taught in a certain class. This student strives to learn not just from their own section's ST but also from the STs of other, higher-ranked sections. Each student's learning from his or her section's top performer is considered internal learning, while each student's learning from the top performers of other sections is considered external learning. In Fig. 3 we see a CTO flowchart.

5. Result

We ran twenty simulations to determine the best settings for the adaptability and PID gains. The adjusted controllers closed-loop effects on the CTO system are shown in figure 3. It should be noted that trial and error was used to establish the settings of all optimization techniques and controller parameters. According to figure 4, all the cost functions exhibit similar tendencies. But when comparing overshoot and settling time, the proposed approach comes out on top. The proposed approach is compared with the PID controllers in terms of stabilization of pendulum angle in Fig. 5-6. Findings show that the pendulum angle ($u2$), arm angle ($u1$), and control signal (u) for suggested controllers and standard PID controllers reach a steady state at around the same time. Overshoot, rising time, and settling time for both outputs of CTO system are all improved in comparison to traditional controllers (two PIDs) during transient reaction (pendulum

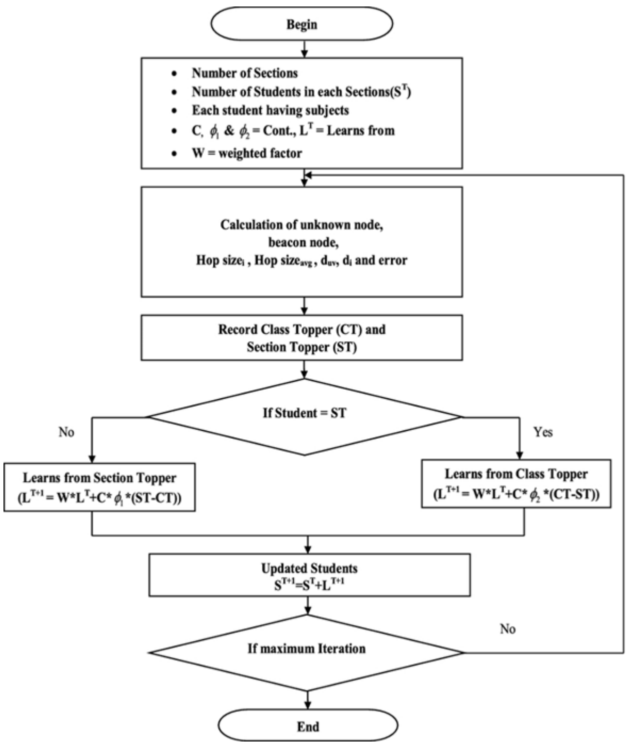

Figure 3. CTO Flowchart.

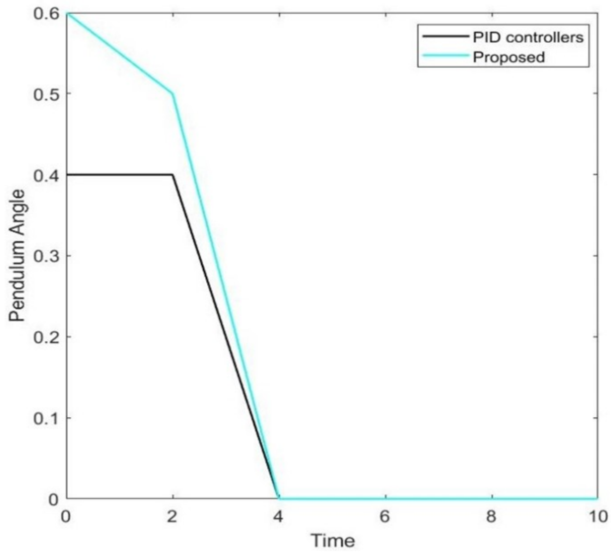

Figure 4. Closed loop responses.

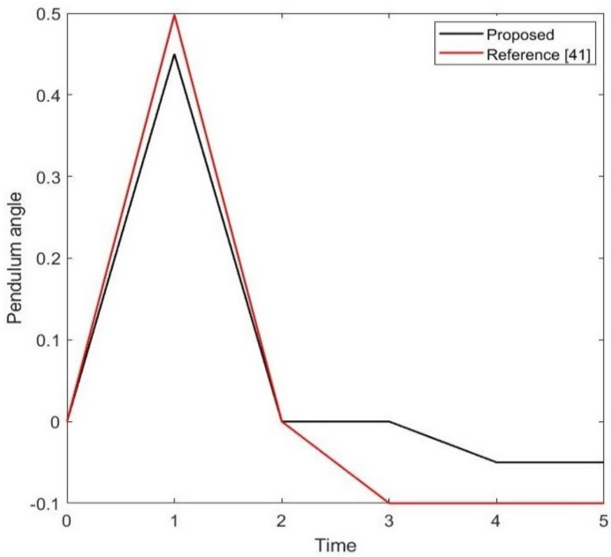

Figure 5. Stabilization of the pendulum angle.

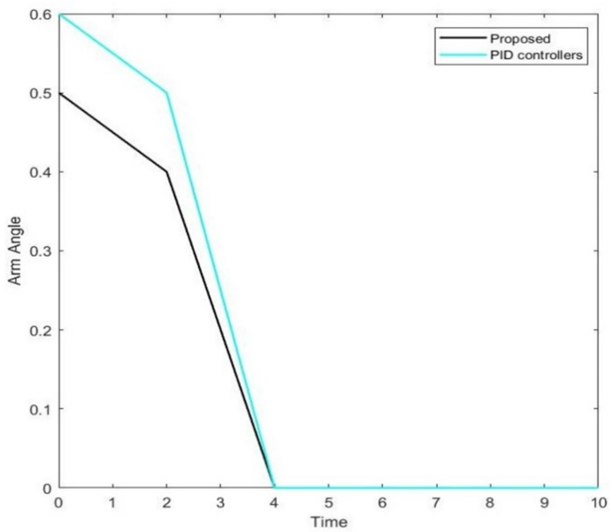

Figure 6. Stabilization of the arm angle.

angle, arm angle).
The convergence of the proposed CTO approach is analyzed and compared with other approaches in Fig. 7. We put each suggested algorithm through its paces to see how well it performs. Results are shown in terms of their convergence tendencies. All the details of the suggested algorithms' statistical evaluations and optimization execution times are summed up here.
The proposed work is compared with the existing approaches in terms of comparative response of pendulum angle in Fig. 8. We find that CTO can provide optimum design parameters for the suggested controllers by comparing their simulation results. The CTO-based adaptive controller, however, outperforms the other algorithms in terms of how well they function.

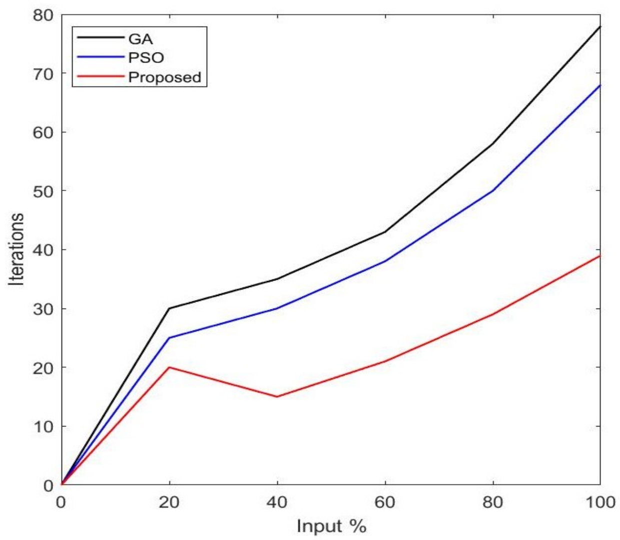

Figure 7. Convergence comparison of the proposed work.

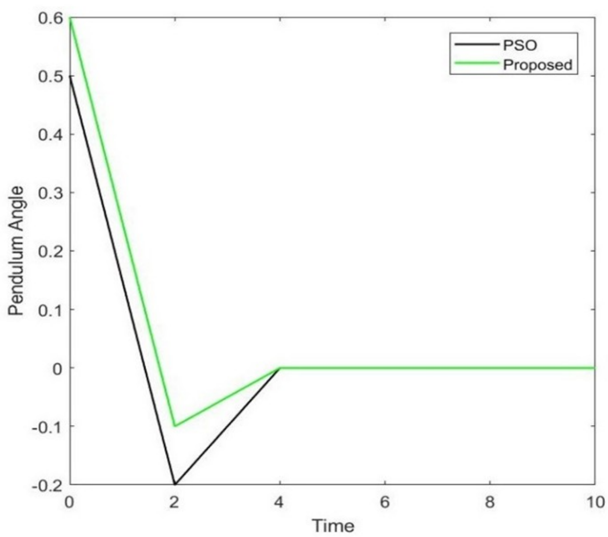

Figure 8. Comparative response of pendulum angle.

6. Conclusion

This study explores the control of a cart-pendulum system by stabilizing the pendulum in an upright position. While conventional methods often utilize separate controllers for swing-up motion and stabilization, this work applies to an FOPID controller to achieve both objectives. A novel optimization technique, Class Topper Optimization, is introduced to enhance the performance of the FOPID controller. This innovative methodology demonstrates superior outcomes compared to traditional optimization methods such as GA and PSO, considering the pendulum's arc and the arm's arc as key variables. The results highlight the effectiveness of the proposed approach, surpassing existing methods in achieving robust and efficient control. Future work will focus on expanding the scope of this research through additional experimental scenarios, including impulse and step disturbances as well as modeling variations, to assess the robustness of the proposed controller. A detailed quantitative and qualitative comparative analysis of the experimental results along with an evaluation of time-response errors and statistical validation will be done in future work. Further comparisons with additional optimization algorithms will be made to reinforce the findings presented in this study.

Authors contributions
All authors have contributed equally to prepare the paper.

Availability of data and materials
The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflict of interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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