Statistical Parameters Estimation of Correlated Millimeter-Wave MIMO Channel

Maziar Kakavand, Mohammadreza Hassannejad Bibalan*, Mina Baghani
Department of Electrical Engineering, Imam Khomeini International University, Qazvin, Iran
Email: maziarkakavand@edu.ikiu.ac.ir, mhbibalan@ikiu.ac.ir (Corresponding author), baghani@eng.ikiu.ac.ir

Received: 20 December 2022 Revised: 19 February 2023 Accepted: 26 March 2023

ABSTRACT:
This paper proposes a novel approach based on the copula models for estimating Millimeter-Wave channel parameters with Rician fading in a correlated MIMO system. Even though the use of these waves faces numerous challenges, having an accurate estimation of their parameters would be a crucial notion for the improved performance of this channel. Since the considered system in this paper is a MIMO system with nearby transmitter antennas, the components of the received signals are correlated. However, a general model is not available for this kind of correlation. By taking into account the correlation, this model enables a precise Joint Probability Density Function (JPDF) by employing copula models for the received signals in the antennas. We also obtain more precise channel parameter estimates using this density function. The multi-path fading parameters in a MIMO system and the correlation coefficients between the transmitter and receiver antennas are both presented in this paper. Some simulations are employed to assess the validity and reliability of estimating Millimeter-Wave channel parameters in a MIMO system.

KEYWORDS: MIMO System, Parameter Estimation, Millimeter-Wave Channel, Rician Fading, Copula Theory.

1. INTRODUCTION

The Millimeter-Wave frequency signal is beginning a new era in wireless communication. Compared to the frequency of the channels currently used in wireless systems, they have a much higher frequency spectrum between 30 and 300 GHz. As a result, these waves provide communication channels with higher bandwidth, capable of reaching data rates of several gigabits per second [1],[2]. Moving up to the Millimeter-Wave spectrum, with an enormous amount of available bandwidth, will satisfy the need for broadband wireless mobility. Numerous bandwidth issues could be quickly resolved with this frequency [3].

Most consumer wireless systems operate at carrier frequencies below 6 GHz. The larger spectral channels are the first and foremost advantage of moving to Millimeter-Wave carrier frequencies. As mentioned before, the bandwidth of these waves is not comparable to the spectrum of other common waves, which means this difference will significantly increase the speed of transmission data rates [4]. Therefore, having an accurate parameter estimation of Millimeter-Wave channels will be particularly important for its best implementation.

Due to their short wavelength, these waves need many antennas to transmit the signal from the transmitter to the receiver, so there is a line of sight (LOS) between them. Therefore, considering the Rician fading for Millimeter-Wave channels is reasonable, one of the significant differences between modeling this channel and other common channels. Also, since many antennas are used for Millimeter-Wave transmission, a MIMO system model is used for the Millimeter-Wave channel [5].

Higher capacity is achieved by transmitting data and signals using MIMO systems, significantly improving data throughput and connection range without needing extra bandwidth or transmission power. Another ideal assumption about channel coefficients is that they are considered independent and identically distributed (i.i.d) [6]. Nevertheless, the previous assumption is not adequate. Also, a correlation exists among the antennas in many practical and empirical conditions due to poor scattering conditions or physical proximity between the antennas [7]. Thus, it is important to investigate the behavior of MIMO systems in correlated fading environments [8].

This paper considers a 2×2 MIMO system for the Millimeter-Wave channel.

The transmitting antennas in this system are close together and correlated, but the receiving antennas are separated and independent of one another [9],[10]. This system has an infinite number of antennas that can be

Paper type: Research paper
DOI: 10.30486/mjee.2023.1975528.1033
How to cite this paper: M. Kakavand, M.. Hassannejad Bibalan and M. Baghani, **“Statistical Parameters Estimation of Correlated Millimeter-Wave MIMO Channel”**, *Majlesi Journal of Electrical Engineering*, Vol. 17, No. 2, pp. 47-57, 2023.

considered. The Rician model is also considered for the fading environment [11]. The parameters and correlation coefficients of the Millimeter-Wave MIMO channel for adjacent transmitters and isolated receivers have been calculated using a PDF-based method [12]. The proposed estimation approach is based on a PDF of the signals. The copula is a powerful instrument that considerably improves our proposed estimation method because the signals in this system are correlated. The copula model is appropriate when correlated with two or more random variables [13]. Therefore, the copula theory aids in calculating the overall PDF of the received signal at the receiver, which includes some correlated parts, and this PDF derived from the copula theory greatly aids in having better estimates of the desired channel parameters. The results of the estimation and the improvements made in this paper can be found in Section 4.

This paper is organized as follows: In Section 2, we discuss the Millimeter-Wave MIMO system and the design channel model. Section 3 presents the proposed method for estimating the channel and explains the copula theory and its applications in our case. In Section 4, simulations of the results are investigated, and finally, the results obtained in Section 5 will be concluded.

2. MILLIMETER-WAVE MIMO SYSTEM MODEL

Due to the different propagation properties, Millimeter-Wave channel models differ significantly from channel models applied at lower frequencies [14]. Standard features used in Millimeter-Wave channel models include low-frequency systems with various parameters for multi-path delay propagation, angle propagation, and Doppler shift (for example, cluster pathways that lead to increased scattering in the channel). In addition, to provide sufficient link margin in most Millimeter-Wave communication systems, an array of antennas should be used on both sides of the system channel, as shown in Fig. 1, which produces a MIMO communication system in the channel model.

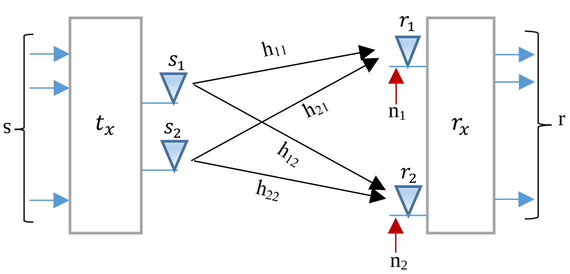

Fig. 1. A 2×2 MIMO system model.

The MIMO system used for the desired channel model will differ due to the different channel characteristics and additional hardware issues at the Millimeter-Wave frequencies. So, this inevitable relationship between MIMO and Millimeter-Waves is the real goal that we emphasize in signal processing for Millimeter-Wave MIMO systems [4].

The wireless channels are considered a model for the MIMO system, which is based on the following equation:

$$r = Hs + n, \tag{1}$$

Where r represents the received signals, s represents the transmitted signals, H is the $N_r \times N_t$ channel matrix with random entries, and n represents noise or disturbances and unwanted signals in the path between the transmitter and receiver. As shown in Fig. 1 and from the MIMO system model in (1), the channel matrix for a 2×2 system is determined as follows [6]:

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \tag{2}$$

Where R represents the received signals and element $h_{11}, \dots, h_{22}$ represents the gain of the paths between the transmitter and receiver antennas, the vector s is the transmitted signals, and the parameters n exist as noise in this system. It should be noted that this work can be generalized to dimensions much more significant than size 2×2 and is not limited to this number of antennas and can be expanded.

2.1. Millimeter-wave channel model

There are typically two-channel models for Millimeter-Wave: parametric and nonparametric. The nonparametric model is taken into account in this paper. Due to the high frequency of the Millimeter-Wave, the wavelength of that signal is too small. Hence, several antennas are required to convey the signals from the transmitting antenna to the receiving antenna. The Millimeter-Wave channel fading is the Rician kind because there is most likely a line of sight (LOS) between them [5].

If H^l is a matrix with dimensions $N_r \times N_t$, it will contain all $h^l_{n_t,n_r}$ whose channel attenuation has a complex MIMO system [15],[16].

$$H^l = R_{rx}^{1/2} H_{Rician} R_{tx}^{1/2}, \tag{3}$$

Where R_{tx} and R_{rx} are the transmitter and receiver correlation matrices, respectively, H_{Rician} is a matrix of elements with a small-scale Rician distribution with K = 10 dB, [17].

Also, from [18], the correlation matrix R_{tx} and R_{rx} can be computed as follow:

$$R_{u,v} = e^{-j\theta}\left(0.9e^{-|u-v|d} + 0.1\right). \tag{4}$$

The parameters u and v represent the row and column elements of the correlation matrix elements calculated from (4). The parameter d is the distance between the transmitter and receiver antennas. θ is the angle whose uniform distribution is between $[-\pi,\pi)$.

3. CHANNEL PARAMETER ESTIMATION

The Millimeter-Wave MIMO channel parameters were generated using the PDF of the received signals, as specified by the PDF-based concept. Estimates of the signals PDF must be created using analytical and empirical methods. Before the analytic methods can produce the PDF of the received signal, the path gains between the transmitter and receiver antennas or the channel matrix elements must be identified. This section will look at the process used to calculate it.

To compute the path gains $h_{11},\dots,h_{22}$, in a 2×2 MIMO system, we must write (3) as a matrix:

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} R^r_{11} & R^r_{12} \\ R^r_{21} & R^r_{22} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \begin{bmatrix} R^t_{11} & R^t_{12} \\ R^t_{21} & R^t_{22} \end{bmatrix}. \quad (5)$$

All random variables $x_{11},\dots, x_{22}$ have a Rician distribution and are independent of one another due to channel fading. After multiplying these matrices, we will get the path gains between the transmitter and receiver antennas according to Fig. 1, in which h_{11} is one of these path gains and can be seen below:

$$h_{11} = (R^t_{11}\, R^r_{11})\, x_{11} + (R^t_{11}\, R^r_{12})\, x_{12} + (R^t_{21}\, R^r_{11})\, x_{21} + (R^t_{21}\, R^r_{12})\, x_{22}. \quad (6)$$

For simplicity, we can write (6) as:

$$h_{11} = a_{11}x_{11} + b_{11}x_{12} + c_{11}x_{21} + d_{11}x_{22}. \quad (7)$$

The random variables that determine the path gains in (7) have a Rician distribution multiplied by the correlation coefficients. Furthermore, all these coefficients, $a_{11}, b_{11}, c_{11}, d_{11}$, are considered positive and real in this paper. The coefficients multiplied by the random variables and can be derived in the same manner are the only differences between the rest of the channel matrix elements $h_{12},\dots,h_{22}$, which have the same form. To obtain the PDF of each element, we must modify the random variables from [19] in the method described below:

$$P(Y \approx y) = p(y)\,|\Delta y| = \sum_{i=1}^{k} p(x_i)\ |\Delta x_i|, \quad (8)$$

And then, we can compute $p(y)$ as the following.

$$p(y) = \sum_{i=1}^{k} \frac{p(x_i)}{\left|\frac{\Delta y}{\Delta x_i}\right|} = \sum_{i=1}^{k} \frac{p(x)}{|g'(x)|}\Bigg|_{x=x_i}, \quad (9)$$

Where $p(y)$ is a modified PDF of $p(x)$ when x multiply by a constant coefficient. Since all random variables have a Rician distribution, from [19], its PDF writes as follows:

$$p(x) = \frac{x}{\sigma^2}\, e^{-\frac{(x^2+v^2)}{2\sigma^2}}\, I_0\left(\frac{x\,v}{\sigma^2}\right), \quad (10)$$

Where x is the random variable, v is the mean, and σ is the standard deviation parameter or the same scale parameter, both parameters have a positive value.

Firstly, take new random variables from (7) as follow:

$$h_{11} = z_1 + z_2 + z_3 + z_4. \quad (11)$$

Then derive a PDF of each of these elements $z_1,\dots,z_2$ using (14) and (17) will be as follows:

$$z_1 = a_{11}x_{11}, \rightarrow g(x) = a_{11}\,x_{11}, \rightarrow g'(x) = a_{11}. \quad (12)$$

From (9) and (12), we can compute the PDF of each element, which is:

$$p(z_1) = \frac{p\left(\frac{z_1}{a_{11}}\right)}{a_{11}} = \frac{1}{a_{11}}\left(\frac{z_1}{a_{11}\sigma^2}\, e^{\left(-\left(\frac{\left(\frac{z_1}{a_{11}}\right)^2+v^2}{2\sigma^2}\right)\right)} I_0\left(\frac{z_1\,v}{a_{11}\sigma^2}\right) \right). \quad (13)$$

To determine the remaining elements, we can use the same equations. The new random variables $z_1,\dots, z_4$ are also independent because of the independence of the random variables $x_{11},\dots, x_{22}$, so by convolving them, we can obtain the PDF of the matrix elements.

$$p(h_{11}) = p_{z_1}(h_{11}) * p_{z_2}(h_{11}) * p_{z_3}(h_{11}) * p_{z_4}(h_{11}). \quad (14)$$

To solve the convolution in (14) we can reach to desire answer by the following equations:

$$p_1(h_{11}) = p_{z_1}(h_{11}) * p_{z_2}(h_{11}) = \int p_{z_1}(\alpha)\ p_{z_2}(h_{11} - \alpha)\, d\alpha, \quad (15)$$

$$p_2(h_{11}) = p_1(h_{11}) * p_{z_3}(h_{11}) = \int p_1(\beta)\ p_{z_3}(h_{11} - \beta)\, d\beta, \quad (16)$$

$$p(h_{11}) = p_2(h_{11}) * p_{z_4}(h_{11}) = \int p_1(\tau)\ p_{z_4}(h_{11} - \tau)\, d\tau. \quad (17)$$

By solving these three equations, $P(h_{11})$, the PDF of the channel matrix element between the transmitter and receiver antennas, is obtained.

Fig. 1 shows the model of the signal that receives at the first receiving antenna:

$$r_1 = h_{11}s_1 + h_{21}s_2 + n_1. \tag{18}$$

And a received signal in the second receiving is:

$$r_2 = h_{12}s_1 + h_{22}s_2 + n_2 , \tag{19}$$

Where r_1 and r_2 represent the signals in the first and second receiving antennas, respectively, n_1 and n_2 are independent, identically distributed additive white gaussian noise with zero mean and variance N, and s_1 and s_2 are transmitter antenna constant coefficients. In addition, $h_{11},\dots,h_{22}$ are the path gains between the transmitter and receiver antennas. The first receiver antennas received signal finishes h_{11} and h_{21}, and the methods for determining the path gain h_{11}, as indicated in (18). If these signals were independent, we could convolve their PDFs by $p(h_{11})$ and $p(h_{21})$ into each other to obtain the PDF of the received signal R_1 because we considered the antennas adjacent to each other, which is a more realistic assumption to make. As a result, the signals are correlated, and convolution cannot be used to achieve the remaining path gains. A valuable tool in this situation is the copula.

3.1. Copula

The copula theory is one of the most commonly used statistical methods for modeling parameter relationships. Sklar developed and applied this approach to mathematical problems [13]. The copula is a function that combines univariate PDFs to create a joint PDF with a particular dependency structure. Given that the signals received from the channel output in a MIMO system described in this paper include the sum of multiple correlated signals from transmitting antennas, we can use a copula to solve our problem. The PDF of the received signal is being used to estimate the Millimeter-Wave channel parameters. Because of the correlation of the received signal, we must determine the PDF of the signal, which is made up of numerous dependent components. As a result, the copula concept is the best approach for resolving our problem because it simplifies the PDF computation process and produces superior results. According to the Sklar theorem, a copula function connects any joint multivariate PDF and the corresponding marginal PDFs [6].

From [13], if F be an n-dimensional cumulative distribution function (CDF) with margins $P_1,\dots,P_n$, Then there is a function like C :

$$C : [0,1]^n \to [0,1]$$
$$P(\mathrm{x}_1,\dots,\mathrm{x}_n) = C\left(P_1(\mathrm{x}_1),\dots,P_n(\mathrm{x}_n)\right), \tag{20}$$

Where C is a copula CDF and $P_1,\dots,P_n$ is marginal CDFs, so the function P, which is defined in (20), is an n-dimensional CDF with margins $P_1,\dots,P_n$. Function C has some intrinsic properties that has been fully described in [20].

Finally, according to the copula's characteristics, a copula is a CDF defined in the range $[0,1]^n$ with uniformly distributed margins. The copula function-calculated multivariate CDF offers remarkable adaptability because we can choose the dependency relationship between them and the margins separately.

In [21], to reach the copula distribution function, we first calculate the joint PDF by taking the nth derivative of the function in (16) as follows:

$$P(x_1,\cdots,x_n) = \frac{\partial^n C\left(P_1(x_1),\dots,P_n(x_n)\right)}{\partial x_1 \dots \partial x_n}. \tag{21}$$

Then by applying the chain law in (21) the joint PDF is derived:

$$P(x_1,\dots,x_n) = \frac{\partial^n C\left(P_1(x_1),\dots,P_n(x_n)\right)}{\partial x_1 \dots \partial x_n} \times \prod_{i=1}^{n} \frac{dP_i(x_i)}{dx_i} = c(P_1(x_1),\dots,P_n(x_n)) \prod_{i=1}^{n} p_i(x_i), \tag{22}$$

Where c is the copula density function and $p_1(x_1),\dots,p_n(x_n)$ are the marginal PDFs. From (22), a multivariate PDF is created by multiplying a copula density function in a set of marginal PDF files in which the copula density function can be chosen independently of the margins.

There are two types of copulas: the first type is the family of elliptical copulas, which includes the Gaussian (Normal) and Student's t. The Archimedean copula is a second type of copula. Due to Rician fading and the fact that the Gaussian copula's distribution follows a normal distribution, it is mainly employed in this paper. The additional copulas used to assess estimating accuracy differences include Clayton, Student's t, and Gumbel. It is important to note that each copula function has a corresponding density function. Due to this, the normal copula density function is provided in the next section, and [13] contains information on the other three types of copulas and many other types, each of which has a variety of uses and characteristics.

3.2. Normal Copula

Due to its resemblance to a normal distribution, it is referred to as the Normal copula. The correlation between the variables is being used to calculate its dependence. However, the marginal distributions in the typical copula are arbitrary. After describing the copula idea and correlation modeling, the following section presents a correlated channel and estimates its parameters using the copula function. The Normal copula distribution function with the correlation matrix ρ, from [13], is expressed as follows:

$$\rho \in [-1,1]^{n\times n}$$

$$C_{\rho}^{Gauss}(x_1,\cdots,x_n) = \mathrm{P}_{\rho}\,(\,p^{-1}(x_1),\ldots,p^{-1}(x_n)), \quad (23)$$

Where, C_{ρ}^{Gauss} is Gaussian copula and P_{ρ} is the standard univariate standard normal distribution function, and p is the PDF of random variables with linear correlation coefficient ρ between the variables. The density function of this copula type can be written as the following:

$$C_{\rho}^{Gauss}(x_1,\cdots,x_n) = \frac{1}{\sqrt{det\,\rho}}\, e^{\left(-\frac{1}{2}\begin{pmatrix}P^{-1}(x_1)\\ \vdots \\ P^{-1}(x_n)\end{pmatrix}^T\cdot(\rho^{-1}-I)\cdot\begin{pmatrix}P^{-1}(x_1)\\ \vdots \\ P^{-1}(x_n)\end{pmatrix}\right)}. \quad (24)$$

More complete information on copulas is available at [13],[21]. Using the copula model to estimate the Joint PDF of elements which is $p(h_{11},h_{21})$, because when this joint PDF is computed, we can access the PDF of the received signal R_1 by integral from the Joint PDF as follow:

$$p(R_1) = \int_{-\infty}^{+\infty} p(h_{11}\,, R_1 - h_{11})\, dh_{11}. \quad (25)$$

By (25), we can analytically obtain the PDF of R_1. If the PDFs of the signals $P(h_{11})$ and $P(h_{21})$ are considered as the marginal density functions in the copula theory, the joint PDF $p(h_{11},h_{21})$ is obtained from (22):

$$p(h_{11},h_{21}) = p(h_{11})p(h_{21})c(P(h_{11}),P(h_{21});\rho_k), \quad (26)$$

The dependency criterion considered in this paper is Pearson's correlation or the linear correlation. In (26), $P(h_{11})$ and $P(h_{21})$ are the marginal CDFs of signals $h_{11}(t)$ and $h_{21}(t)$, respectively, and k represents the number of routes, and ρ_k is the linear correlation parameter among these signals. The linear correlation between h_{11} and h_{21} is the same as the linear correlation of the channel since the signals in the transmitter antennas are separately created, so estimating this parameter yields the channel correlation parameter.

This paper uses four kinds of copula for estimation: Normal, Clayton, Gumbel, and t-copula. For each copula, we have to calculate the linear correlation parameter ρ_k based on that related copula, and the linear correlation parameter ρ_k in (28) is not precisely the copula parameter.

One of the Gaussian copula's inputs is the ρ correlation matrix, which is mentioned in (23). These parameters are almost similar to the linear correlation parameters, which offer paired correlations between variables. However, the t-copula has two parameters, one of which is degrees of freedom, which in this simulation is equal to 5, and the other is similar to the parameter of the normal copula and is thus the same as the parameter of linear correlation [7].

From [6], In the Clayton copula, there is an α parameter, which is different from the linear correlation parameter, and the relationship between them for the bivariate case is given by:

$$\alpha = \frac{sin^{-1}(\rho_k)}{\pi - 2\,sin^{-1}(\rho_k)} \quad (27)$$

In a multivariate case, we can calculate α for each of them separately and consider the average of all obtained α values as the main Clayton copula parameter.

Thus far, the analytical approach has produced the PDF of the received signal $p(R_1)$. Based on the obtained analytical PDF of the received signal in the kth receiver, the parameters mean v and standard deviation σ, along with other desirable parameters between the transmitters and the receiver, can be calculated as follows:

The computed parameters are based on the Nonlinear Minimum Mean Square Error (NMMSE) estimator. Analytical and empirical PDFs of the signals received are necessary for this estimator. As shown in (26), empirical PDF is calculated using samples of the signal that was received and had the following structure [6]:

$$\hat{p}(R_k) = \frac{1}{N_k B}\sum_{i=0}^{N_k-1}\psi\,(\frac{p_k - p_{k_i}}{B}), \quad k = 1,2 \quad (28)$$

Where, (28) represents the kernel estimator's performance, a method for statistically estimating the PDF of the arbitrary signal. ψ is the kernel function that must integrate to 1, and B is the window width or bandwidth of the kernel. N_k is the number of the received samples in the kth receiver, and R_{k_i} is the value of the ith sample [6]. Using both analytical and Empirical obtained PDFs, NMMSE estimates the desired parameters of the channel as follows:

$$(\hat{v}_k,\hat{\sigma}_k) = \underset{v_k,\sigma_k}{argmin}\int|p(R_k) - \hat{p}(R_k)|^2 dR_k, \quad (29)$$

Where, $p(R_k)$ is a function of the PDF of the received signal through the analytical method obtained from (25) and $\hat{p}(R_k)$ is the empirical PDF of the received signal, which results from the kernel estimator and is derived from (28), parameters v and σ, are the main characteristics of the Millimeter-Wave MIMO channel, the accurate estimation of these parameters has particular importance for the implementation of this channel. Nevertheless, this method can be used to estimate the correlation coefficients that exist in the channel matrix (5).

It should be noticed, however, that the estimation of correlation coefficient parameters should not be expected to be very precise. Because changing the coefficient's locations might not change the PDF matrix element or path gains, which might make the estimator very sensitive to these parameters. Hence, we shall examine the findings from the suggested method to estimate channel parameters in the following section. We previously discussed the analytical and experimental methods for estimating Millimeter-Wave channel parameters and calculating the PDF of the received signal in the first receiver R_1 antenna. Therefore, some simulations are required to assess the proposed method for calculating channel parameters. We will discuss the simulation and the outcomes in the following section.

4. SIMULATIONS AND RESULTS

Determining the PDF of the matrix channel elements, which are the path gains between the transmitter and receiver antennas, is crucial to estimate the desired parameters of a Millimeter-Wave MIMO channel. As was already explained, the MIMO system in this paper has four random variables with a Rician distribution as part of its Millimeter-Wave channel matrices (6).

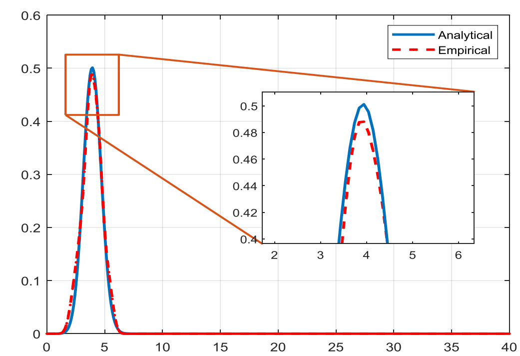

Fig. 2. PDF of h_{11} from Analytical and Empirical methods.

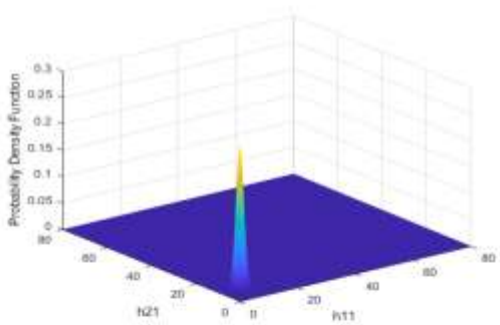

Fig. 3. Joint PDF of h_{11} and h_{21} from Copula.

Due to their independence, we may produce the desired PDF by convolution the random variables together. Due to the presence of correlation coefficients, we must acquire their modified form, described in (13), and then use those forms as the PDF of the path gains, whose calculations are expressed in (17). To do this, we convolve them two by two first, and then their results.

It should be emphasized that the correlation coefficients are assumed constant for the two path gains, h_{11} and h_{21}, according to Table 1. in this paper, the mean and standard deviation parameters from the Rician distribution are considered 2 and 0.7, respectively.

Table 1. Amounts of correlation coefficients in R_1.

h_{11}	Amount	h_{21}	Amount
a_{11}	0.96	a_{21}	0.54
b_{11}	0.56	b_{21}	0.12
c_{11}	0.21	c_{21}	0.88
d_{11}	0.12	d_{21}	0.44

In Table 1, According to (7), $a_{11},\ldots,d_{11}$ are correlation coefficients of h_{11} and $a_{21},\ldots,d_{21}$ are correlation coefficients of h_{21}.

Fig. 2 shows the PDF of h_{11} obtained from two analytical and empirical methods. The kernel estimator was employed in this figure to ensure the desired result was attained. The results show that the proper steps have been taken because the results of the two methods are comparable. As shown in (18), the received signal in the receiver antenna R_1 is composed of two path gains or signals, h_{11}, and h_{21}. To acquire the PDF of h_{21}, we will repeat the described method for the PDF of h_{11}. Because we considered both signals correlated in this paper, it is impossible to estimate the PDF of the received signal via convolution. Instead, we use the Gaussian copula model proposed in (24). Fig. 3 depicts the copula result, which reveals the type of Rician distribution of random variables.

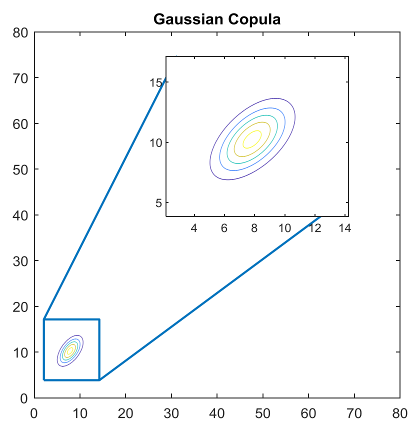

Fig. 4. Contour of Joint PDF h_{11} and h_{21}.

The Gaussian copula is used to calculate the correlation between these signals, and because of the existing correlation, the joint PDF is calculated.

In this paper, three other copula models will be mentioned to demonstrate the effect of different copulas on the accuracy of estimating Millimeter-Wave MIMO channel parameters. The counter of acquired joint PDF from the Gaussian copula of the h_{11} and h_{21} signals between the transmitter and receiver antennas is also shown in Fig. 4. Furthermore, this figure shows how the PDF layers are placed in the Gaussian model.

So far, we have shown how to use the copula model to obtain the joint PDF of these signals at the first receiver antenna and estimate the channel parameters using the method described in this paper. Nevertheless, first, we need to calculate the PDF of the received signal from the joint PDF. We use (25) to achieve this PDF, which leads to $P(R_1)$. Fig. 5 depicts the result of this integration. Two PDFs are required from this received signal to estimate channel parameters according to (29). The PDF created in this manner is employed as a tool for an analytical approach. Fig. 5 also shows the kernel estimator's outcome to calculate the PDF from the empirical approach. A precise estimate of the channel parameters could be expected because, as shown from the result, these two PDFs generated by the analytical and empirical approaches are somewhat similar.

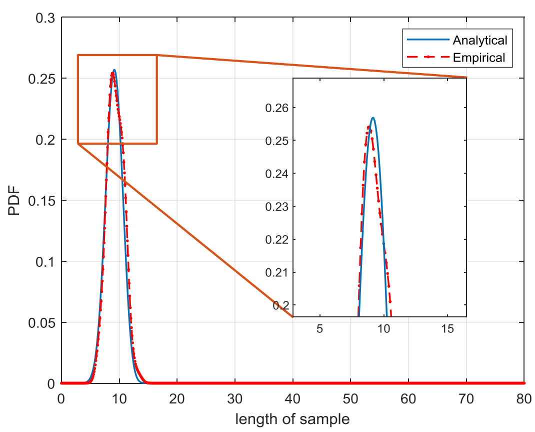

Fig. 5. PDF of R_1 with using Gaussian Copula.

Table 2. Estimation of S in first received antenna R_1.

S = 2, σ = 0.7	100 random variables		1000 random variables	
	$\hat{S}$	var	$\hat{S}$	var
s_0 = 1	2.0019	0.0592	2.0001	1.02×10^{-4}
s_0 = 2.5	1.9997	0.0030	2.0004	1.2×10^{-4}

These PDFs allow us to predict the Millimeter-Wave MIMO channel's desired parameters accurately. We attempt to reduce the error between these two PDFs by setting the target parameter or parameters as unknown in the empirical approach. It should be emphasized that the computational space is two-dimensional due to the usage of the copula model, and extensive calculations are conducted, necessitating the employment of a robust system to accomplish the results of these calculations. In this paper, calculations are done one hundred times and averaged to obtain reliable results with excellent analytical and empirical precision methods for estimating channel parameters.

In Table 2, the mean parameter S is estimated alone, and the column var represents the error variance of the parameter estimation. Also, the results obtained in this table are present for different initial guesses s_0, and the effect of the number of samples on their accuracy of parameter estimation and error variance will be reviewed. The results of Table 2 show that we have a highly accurate estimate even though our initial estimations were all significantly off-base. Additionally, as the number of random variables increases, so does the estimation accuracy of this parameter, and the error variance is noticeably reduced. Table 3 estimates the parameter σ for 100 random variables using various initial hypotheses. Further suggests that we will have accurate estimations of this parameter for the number of samples considered.

Table 3. Estimation of σ in R_1 .

S = 2, σ = 0.7	100 random variables	
	$\hat{\sigma}$	var
σ_0 = 1	0.6987	0.0016
σ_0 = 0.5	0.7259	0.0029

Table 4. Simultaneously estimation of S and σ in R_1 .

S = 2, σ = 0.7	100 random variables			
	$\hat{S}$	var $\hat{S}$	$\hat{\sigma}$	var $\hat{\sigma}$
$[s_0, \sigma_0]$ = [1.5, 0.5]	2.0063	0.0013	0.7153	0.0033
$[s_0, \sigma_0]$ = [3, 1.5]	2.0281	0.0038	0.7151	0.0036

Moreover, the suggested estimator can estimate the channel parameters simultaneously. The outcomes of the simultaneous estimation of S and σ parameters for the various initial guesses s_0, σ_0 are shown in Table 4. From the results of this table, it is clear that we will have precise estimates given the number of random samples. Table 5 shows the results of the simultaneous estimation of correlation coefficients a_{11} and b_{11} for the various initial guesses a_0 and b_0 in the $h_{11}(t)$.

Table 5. Estimate Simultaneous a_{11} and b_{11} in R_1 .

a_{11}= 0.96 b_{11}= 0.56	100 random variables			
	a_{11}	var a_{11}	b_{11}	var b_{11}
$[a_0, b_0]$ = [0.8, 0.4]	0.9336	0.0367	0.5818	0.0514
$[a_0, b_0]$ = [1, 1]	0.9142	0.0556	0.6144	0.0621

The results of Table 5 demonstrate that although we should not expect the calculation of correlation coefficients to be very accurate, the estimations we have for these parameters using the provided method are nonetheless quite reliable. In many works, the dangerous assumption is that path gains are independent when estimating channel parameters. In Fig. 6, unlike Fig. 5, the path gains are independent, and the two PDFs need to fit precisely.

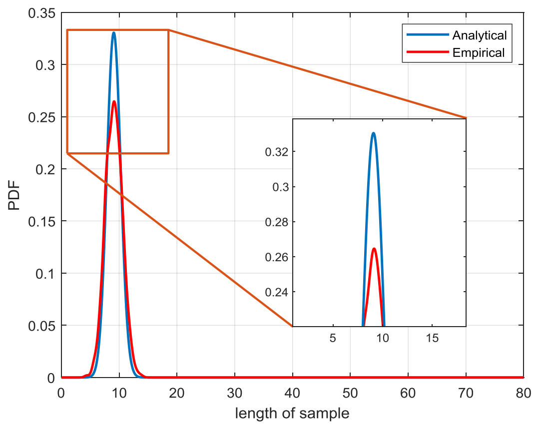

Fig. 6. PDF of R_1 when path gains are independent.

Table 6. Comparing the accuracy of estimating S and σ with the assumption of correlation and independence.

100 R.V.		s_0, σ_0 = [1,1]	s_0, σ_0 = [2.5,1]
Correlated	$\hat{S}$	2.0001	2.0004
	var $\hat{S}$	1.02×10^{-4}	1.2×10^{-4}
	$\hat{\sigma}$	0.7021	0.6948
	var σ	1.03×10^{-4}	1.17×10^{-4}
Independent	$\hat{S}$	1.9535	1.9546
	var $\hat{S}$	6.05×10^{-4}	5.65×10^{-4}
	$\hat{\sigma}$	0.9091	0.9075
	var $\hat{\sigma}$	0.0014	4.76×10^{-4}

We compare the simultaneous estimates of S and σ parameters for these two scenarios in Table 6.

Table 7. Comparing the accuracy of estimating a_{11} and b_{11} with the assumption of correlation and independence.

a_{11}= 0.96 b_{11}= 0.56 100RV		a_0, b_0= [0.8,0.4]	a_0, b_0 = [1,0.5]
Correlated	a_{11}	0.9336	0.9709
	var a_{11}	0.0367	0.0566
	b_{11}	0.5818	0.5596
	var b_{11}	0.0514	0.0716
Independent	a_{11}	1.5207	1.5787
	var a_{11}	0.0095	0.0239
	b_{11}	0.1017	0.1055
	var b_{11}	0.0056	0.0033

It is evident from the data in this table that the correlated assumptions have substantially better estimates of the standard deviation parameter than independent assumptions, in addition to having less error variance. This comparison is made for correlation coefficients in Table 7. the results reveal that we will have significantly better estimates in the correlated condition than in the independent condition considering the number of random variables used, even while the error variance in the independent mode decreases as the number of random variables increases, in another simulation, we will investigate the precision of estimating the channel's parameters using a few commonly used copulas.

As was previously indicated, in addition to the Gaussian type, we also employ t-copula, Clayton, and Gumbel types, each of which has specific properties and constraints.

The outcome of the simultaneous estimation of S and σ parameters for the same starting guess S_0 and σ_0 for different copulas in the first receiver antenna R_1 is shown in Table 8. It is evident from the results in this table that while we have accurate estimates of these parameters in both the Gaussian and t-copula types, the Gumble type's error variance in estimating the S

parameters is very low and precise due to the number of random variables.

A similar procedure for the parameters of the correlation coefficient is visible in Table 9. The Gaussian type of copula has a better simultaneous estimation of the correlation coefficients than the other copulas in this table, but its estimation error is also higher.

Table 8. Estimate Simultaneous S and σ in R_1 with different Copulas.

$S = 2$ $\sigma = 0.7$ $[S_0, \sigma_0] = [2,2]$	100 random variables			
	$\hat{S}$	var $\hat{S}$	$\hat{\sigma}$	var $\hat{\sigma}$
Gaussian	2.0019	0.0028	0.6980	0.0031
t-Copula	1.9991	0.0032	0.6926	0.0042
Clayton	1.9506	0.0043	0.7442	0.0081
Gumbel	2.0395	9.56×10^{-4}	0.6861	0.0042

Table 9. Estimate Simultaneous a_{11} and b_{11} in R_1 with different Copula.

a_{11}= 0.96 b_{11}= 0.56 $[A_0, B_0] = [2,1]$	100 random variables			
	a_{11}	var a_{11}	b_{11}	var b_{11}
Gaussian	0.9721	0.0782	0.5436	0.0940
t-Copula	1.0656	0.0702	0.4981	0.0760
Clayton	1.1461	0.0633	0.3443	0.0534
Gumbel	0.9541	0.0653	0.6322	0.0491

The Millimeter-Wave MIMO channel parameter estimations with multiple modes were investigated in this section. The proposed model will have significantly better estimates of this parameter.

The error variance of the estimation of different copulas is shown in the following figures to understand the accuracy better.

Fig. 7 shows the accuracy of the mean parameter estimation error for the different numbers of samples for random variables N. As it is clear, the Gaussian copula has the best performance for parameter estimation. After that, Clayton, t-copula, and Gumble, are the other copulas that perform better in estimation accuracy and error variance.

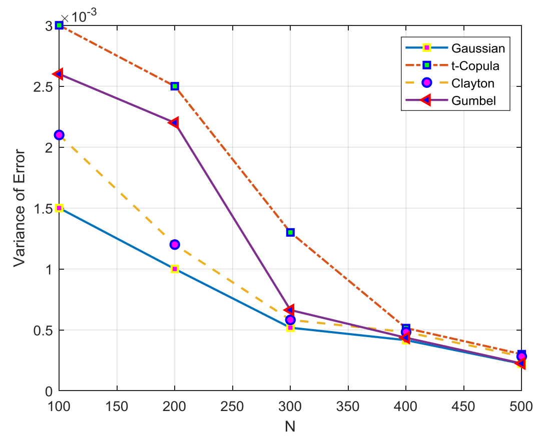

Fig. 7. Error variance of S estimation using four types of copulas for S = 2 and σ = 0.7.

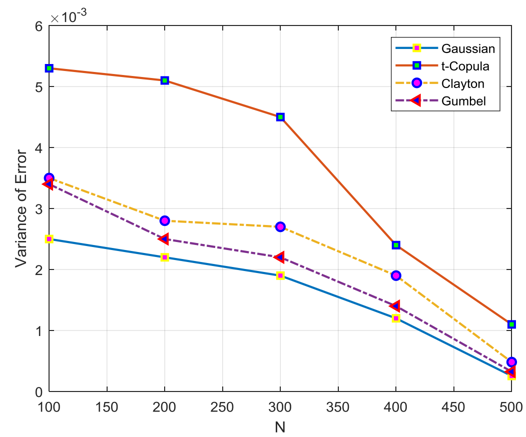

Fig. 8. Error variance of σ estimation using four types of copulas for S = 2 and σ = 0.7.

Fig. 8 shows the error variance of the estimated standard deviation σ parameter for various random variables, demonstrating that the Gaussian copula fared better in this field.

Increases in the standard deviation parameter limit might produce erroneous results and decrease estimation accuracy. Fig. 9 shows the error variance of mean parameter S estimation when S is constant and equals two and σ changes in a specific interval.

Fig. 10 also shows the error variance of σ by keeping the σ fixed and changing the mean parameter S within a specific interval. As it is clear from this figure, the estimation error has increased with the increase in variance here.

According to the figures and tables, the Gaussian type copula performs better in estimating the parameters of the desired channel for the Rician type of channel considered in this paper, as is concluded from the Gaussian distribution.

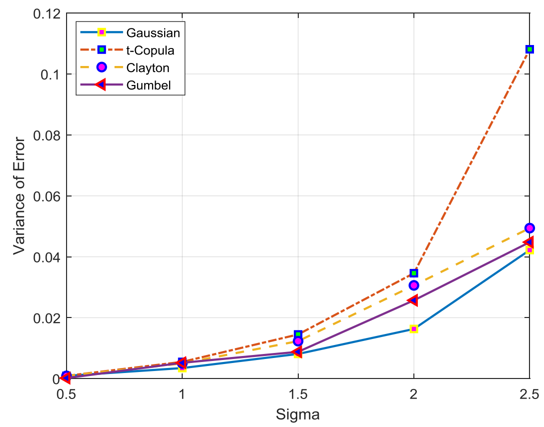

Fig. 9. Error variance of S estimation using four types of copulas with S = 2 and different values of σ.

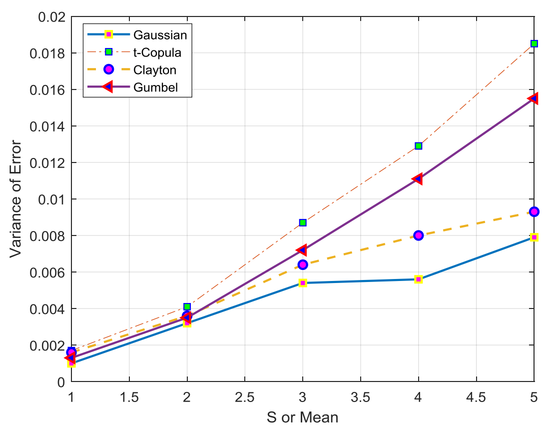

Fig. 10. Error variance σ of estimation using four types of copulas with σ = 1 and different values of S

5. CONCLUSION

This paper proposes a new method for estimating Millimeter-Wave MIMO channel parameters and correlation coefficients. This method uses copula theory to calculate the PDF of the received signal. When there is a correlation between signals, the copula model makes it simpler to calculate the PDF. Its significance will become apparent once we have a more precise density function from this model. Additionally, we compared the estimation accuracy of a few copula models. As the results show, given the reasonable assumption of correlation in the MIMO system, we will have better estimations of the channel parameters than the uncorrelated or independent condition. However, no model has been examined for this correlation, and no relationship exists.

As a result of the parameter estimate of channels using copula, a new model for detecting a MIMO system with Rician fading is created. If there is a correlation, then this model can be used to estimate the parameters of other channel types more accurately. Its findings may allow for a more precise calculation of the capacity of these channels. The copula selection could be reconsidered to gain a better and more precise estimation of the channel's numerous features and attain the best results.

REFERENCES

[1] Z. Pi and F. Khan, **"An introduction to millimeter-wave mobile broadband systems,"** *IEEE Commun. Mag.*, Vol. 49, No. 6, pp. 101–107, Jun. 2011.

[2] T. S. Rappaport *et al.*, **"Millimeter wave mobile communications for 5G cellular: It will work!,"** *IEEE Access*, Vol. 1, pp. 335–349, May 2013.

[3] M. K. Samimi and T. S. Rappaport, "**3-D millimeter-wave statistical channel model for 5G wireless system design,"** *IEEE Trans. Microw. Theory Techn.*, Vol. 64, No. 7, pp. 2207–2225, Jul. 2016.

[4] R. W. Heath, N. Gonzalez-Prelcic, S. Rangan, W. Roh, and A. Sayeed, **"An overview of signal processing techniques for millimeter wave MIMO systems,"** *IEEE J. Sel. Top. Signal Process.*, Vol. 10, No. 3, pp. 436–453, Apr. 2016.

[5] A. Younis, N. Abuzgaia, R. Mesleh, and H. Haas, **"Quadrature spatial modulation for 5G outdoor millimeter wave communications: Capacity analysis,"** *IEEE Trans. Wireless Commun.*, Vol. 16, No. 5, pp. 2882–2890, May 2017.

[6] Mohammad Hossein Gholizadeh, Hamidreza Amindavar, and James A Ritcey. **"On the capacity of MIMO correlated Nakagami-m fading channels using copula,"** *EURASIP Journal on Wireless Communications and Networking* 2015, Article number: 138.

[7] Mohammad Hossein Gholizadeh, Hamidreza Amindavar, and James A Ritcey. **"Analytic Nakagami fading parameter estimation in dependent noise channel using copula,"** *EURASIP Journal on Advances in Signal Processing*. 2013, Article number: 129.

[8] Z Xu, S Sfar, RS Blum, **"Analysis of MIMO systems with receive antenna selection in spatially correlated Rayleigh fading channels,".** *IEEE Trans VehTechnol.* **58**, pp. 251–62, 2009.

[9] M Kang, MS Alouini, **"Capacity of correlated MIMO Rayleigh channels,"**. *IEEE Trans Wireless Commun*. 5, pp. 143–55, 2006.

[10] Eduru and N. Rangaswamy, **"Ber analysis of mimo zf receiver with imperfect channel estimation under correlated rician fading channel,"** *AEU International Journal of Electronics and Communications*, Vol. 126, p. 153401, 2020.

[11] I. Boukhedimi, A. Kammoun, and M.-S. Alouini, **"LMMSE receivers in uplink massive MIMO**

systems with correlated Rician fading," *IEEE Trans. Commun.*, Vol. 67, No. 1, pp. 230–243, Jan. 2019.

[12] Q. Ding, Y. Lian, and Y. Jing, **"Performance analysis of full-duplex massive MIMO systems with low-resolution ADCs/DACs over rician fading channels,"** *IEEE Trans. Veh. Technol*., Vol. 69, No. 7, pp. 7389–7403, Jul. 2020.

[13] RB Nelsen, **"An Introduction to Copulas,"**. 2nd edition. Springer, New York, 2006.

[14] T. S. Rappaport, R. W. Heath Jr., R. C. Daniels, and J. Murdock, **"Millimeter-Wave Wireless Communications,"**. Prentice-Hall, September 2014.

[15] A. Forenza, D. J. Love, and R. W. Heath, Jr., **"Simplified spatial correlation models for clustered MIMO channels with different array configurations,"** *IEEE Trans. Veh. Technol.*, Vol. 56, No. 4, pp. 1924–1934, Jul. 2007.

[16] M. Steinbauer, A. F. Molisch, and E. Bonek, **"The double-directional radio channel,"** *IEEE Antennas Propag. Mag.*, Vol. 43, No. 4, pp. 51–63, Aug. 2001.

[17] R. Y. Mesleh, **"Spatial modulation: A spatial multiplexing technique for efficient wireless data transmission,"** Ph.D. dissertation, Dept. Electr. Eng., Jacobs Univ. Bremen, Bremen, Germany, Jun. 2007.

[18] P. Karttunen, K. Kalliola, T. Laakso, and P. Vainikainen, **"Measurement analysis of spatial and temporal correlation in wideband radio channels with adaptive antenna array,"** in Proc. *IEEE Int. Conf. Universal Pers. Commun. (ICUPC)*, Vol. 1. Oct. 1998, pp. 671–675.

[19] A Papoulis, SU Pillai, **"Probability, Random Variables and Stochastic Processes,"**,4th edition. *McGraw-Hill, New York*, 2002.

[20] A Sklar, **"Fonctions de repartition a n dimensions et leurs marges,"**. Publ Inst Statist Univ Paris. **87**, pp. 229–31, 1959.

[21] RT Clemen, T Reilly, **"Correlations and copulas for decision and risk analysis,",** Manage Sci. 45(2), pp. 208–24, 1999.